Supporting Information

Zhao et al. 10.1073/pnas.1105239108

SI Text

Part I: Leaflet to the Human Experiments. There are totally 50 participants doing the experiments together. The experiment situation is the same for everyone. Once the experiments begin, any kind of communication is not allowed.

Together with other participants, you shall engage in a resource-allocation experiment. For the experiment, there are two virtual rooms (Room 1 and Room 2), and the amounts of virtual money in the two rooms are M_1 and M_2, respectively. The value of M_1/M_2 is fixed in one experiment, but is not announced. In each round, you have to choose to enter one of the two rooms, in order to share alike the virtual money inside the room. After everyone has made decision, those who earned more than the global average are regarded as winners of the round, and the room which the winners had entered as the winning room.

After you log in, you will see the choosing panel on the computer screen (as shown in Fig. S1*B*), buttons with numbers of 1 and 2 are used to choose Room 1 and Room 2. The left displays your current score (a) and the current experiment round (t). During the experiment, 60 s were given for making choice. If you could not decide your choice within 60 s, the experiment-control computer program would assign you a random choice with probability 50%. Nevertheless, the participant who borrowed the computer's choice twice would be automatically kicked out of the experiment. In each round of the experiment, the experiment-control computer program will update the score for each participant after all the participants have made their choices. If your score is added 1 point, it means that the room you have chosen happened to be the winning room. If the score keeps unchanged, it may have two possible interpretations: either the other room won or neither of the rooms won (i.e., the experiment ended in a draw).

The initial capital of each participant is 0 point and the total payoff of a participant is the accumulated scores (points) of all the experiment rounds. At the end of the experiments, as a premium, this payoff (points) will be converted to the monetary payoff in Renminbi with a fixed exchange rate 1∶1 (namely, one point equals to one Chinese Yuan). Try to win more points, and then you can get more premium.

Part II: The Open Complex Adaptive System (CAS)—Theoretical Analysis of the Agent-Based Modeling. Besides the simulations performed in the main text here we present some theoretical analysis for the same open system. It is reasonable to assume that, if P is not too small, the right column of a strategy filled in by 1 with probability L/P is equal to the one filled in 1 with the number of L. Hence strategies with the same preference number L can be regarded as the same. It is worth noting that if the situations vary in a random manner, the probability is L/P for a normal agent to choose Room 1 using a strategy with preference number L. Next, we assume that the preference number of the best strategy held by normal agent i at time T, is L_i. Denote the choice of room as x_i so that $x_i = 1$ if Room 1 is chosen and $x_i = 0$ otherwise. At the same time, let imitating agent j choose to follow the normal agent μ, the best agent (who has the highest score) in the group of size k ($1 \le k \le N_n$). For the imitating agent, its choice of room is $y_j = x_\mu$, and its preference number becomes $L_j = L_\mu$. With these definitions, the total number of agents in Room 1 at time T can be written as

$$N_1 = \sum_{i=1}^{N_n} x_i + \sum_{j=1}^{N_m} y_j. \quad \textbf{[S1]}$$

It is obvious that $\langle x_i \rangle = L_i/P$, which can be used to derive the expectation and the variance of the population in Room 1 as

$$\langle N_1 \rangle = \frac{1}{P}\left(\sum_{i=1}^{N_n} L_i + \sum_{j=1}^{N_m} L_j\right), \quad \textbf{[S2]}$$

$$\sigma_{N_1}^2 = \sum_{i=1}^{N_n} \sigma_{x_i}^2 + \sum_{j=1}^{N_m} \sigma_{y_j}^2 + \sum_{i=1}^{N_n}\sum_{j=1}^{N_m} (\langle x_i y_j \rangle - \langle x_i \rangle \langle y_j \rangle) + \sum_{p,q=1,p\neq q}^{N_m} (\langle y_p y_q \rangle - \langle y_p \rangle \langle y_q \rangle). \quad \textbf{[S3]}$$

Owing to the specific method for the construction of strategies in the resource-allocation model, the covariance between the choices of different normal agents can be neglected. On the right-hand side of Eq. **S3**, the third term is the correlation between choices of the normal agents and those of the imitating agents who followed them. The fourth item is the correlation between the choices of different imitating agents who followed the same normal agent. Both terms should always be positive, which means that adding the imitating agents could cause large fluctuations (volatility) in the resource-allocation system. It should be emphasized here that the stability defined in the main text is different from the traditional definition of variance. The former characterizes both the deviation and the fluctuation to the idealized room population in the balanced state, while the latter only represents the fluctuation to the mean value of the time series. When the resource distribution is comparable ($M_1/M_2 \approx 1$), because normal agents are able to produce the idealized population or $\langle N_1 \rangle / \langle N_2 \rangle \approx M_1/M_2$, these two kinds of definitions are approximately equal. This condition explains why the stability can be destroyed when imitating agents are involved in situations with a nearly unbiased resource distribution. However, when the system environment becomes difficult for the normal agents to adapt to, the difference between the "variance" and the "stability" cannot be neglected. If no imitating agents are involved, the normal agents alone cannot make the system reach the balanced state. In that case, even if the fluctuation of N_1/N_2 to its average value could be made small, the deviation to the idealized population ratio can still be very large. This situation would make the system suffer from a higher dissipation. If an appropriate portion of imitating agents is added, the deviation of N_1/N_2 to the idealized room population diminishes, leaving only some fluctuations around M_1/M_2, which could result in a reduction of waste in the resource allocation.

Then, we study the performance of different strategies (namely, strategies with different preference numbers). We also assume the condition of $M_1/M_2 \ge 1$, as used in the main text. Assume that at time T, the winning rate of Room 1 is $\alpha(T)$. The expectation of the increment of score for the strategy with the preference number L should be $1 - \frac{L}{P} + (\frac{2L}{P} - 1)\alpha(T)$. Then the expectation of the cumulative score for this strategy from $t = 1$ to $t = T$ can be expressed as

$$f(L,T) = \left(1 - \frac{L}{P}\right)T + \left(\frac{2L}{P} - 1\right)\sum_{t=1}^{T}\alpha(t). \qquad \textbf{[S4]}$$

From this expression, we can calculate the dependence of the cumulative score on the preference number as

$$\frac{\Delta f}{\Delta L} = \frac{2}{P}\sum_{t=1}^{T}[\alpha(t) - 0.5]. \qquad \textbf{[S5]}$$

It is easy to find from Eq. **S3** that if $\sum_{t=1}^{T}[\alpha(t) - 0.5] > 0, f$ should be a monotonically increasing function with L. Now we assume that $[\alpha(T) - 0.5]$ is always positive, which is not a too stringent condition as long as M_1 is large enough. As the experiment evolves under this assumption, the gap of scores among different strategies of different preference numbers will become larger and larger. Eventually, the best performed strategy owned by a normal agent would be the one with the largest L in its strategy book. As a consequence, imitating agents will choose to follow those who own the strategy with the largest preference number $L_{\max}$. From Eq. **S2**, it is obvious that $\langle N_1\rangle$ will also reach its maximum value $\langle N_1\rangle_{\max}$, when both L_i and L_j reach their maximum values. With this maximum value of the expected population in Room 1, we can propose the following two conditions:

- If $\langle N_1\rangle_{\max} < \frac{M_1}{M_1+M_2}N$, the system can never reach the balanced state.
- If $\langle N_1\rangle_{\max} > \frac{M_1}{M_1+M_2}N$, the system can fluctuate around the balanced state.

Denoting the population ratio $\langle R_1\rangle = \langle N_1\rangle/N$, we need to calculate $\langle R_1\rangle_{\max} = \langle N_1\rangle_{\max}/N$, to evaluate the conditions above. As the normal agents construct their strategies in a random way, a strategy with an arbitrary preference number may be picked up with a uniform probability $1/(P+1)$. Thus, among the S strategies of a normal agent, the probability to have $L_{\max} = \tilde{L}$ is $p(\tilde{L}) = (\frac{\tilde{L}+1}{P+1})^S - (\frac{\tilde{L}}{P+1})^S$. Because an imitating agent would choose the best normal agent among the k group members, the probability to have $(L_{\max})_{ks} = \tilde{L}$ should be $p'(\tilde{L}) = (\frac{\tilde{L}+1}{P+1})^{kS} - (\frac{\tilde{L}}{P+1})^{kS}$. With these probabilities, we obtain the population ratio as

$$\begin{aligned}\langle R_1\rangle &= \frac{1}{NP}\left(\sum_{i=1}^{N_n}L_i + \sum_{j=1}^{N_m}L_j\right) \\ &= \frac{1}{NP}\left[N_n\sum_{\tilde{L}=1}^{P}\tilde{L}p(\tilde{L}) + N_m\sum_{\tilde{L}=1}^{P}\tilde{L}p'(\tilde{L})\right] \\ &= 1 - \frac{1}{(\beta+1)P}\sum_{\tilde{L}=1}^{P}\left[\left(\frac{\tilde{L}}{P+1}\right)^s + \beta\left(\frac{\tilde{L}}{P+1}\right)^{ks}\right]. \qquad \textbf{[S6]}\end{aligned}$$

Part III: A Closed CAS—Simulations Based on the Agent-Based Modeling. For the open system discussed in the main text, if there are too many imitating agents in the resource-allocation system, it may still become a disturbing factor to the system. For the completeness of the study, here we consider a closed system in which the number of normal and imitating agents is fixed at $N = 150$ with the parameter β being varied. As shown in Fig. S2, in the larger M_1/M_2 region, situations with the imitating agents ($\beta = 2.0$ and 4.0) are generally better than those without the imitating agents (β =0), similar to cases of the open system. Meanwhile, there clearly exists an optimized β (=4.0 in the current case) with which the best state of the closed system can be realized in the aspects of the efficiency (which, herein, only describes the degree of balance of resource allocation in the model system) and the stability. When $\beta = 9.0$, the system seems to be disturbed by the imitating agents and the performance (except the system unpredictability) becomes even worse than the case of $\beta = 2.0$. The reason for this phenomenon may be explained as follows. If too many imitating agents join the system, even the best normal agents may be confused. Typically the best normal agents might have wrong estimations about the system situation and then make incorrect decisions. When the best normal agents' decisions are learnt by the imitating agents, the herd will overconsume the arbitraging opportunities in the system as a result of the distribution of biased resources, thus yielding a less efficient (or equivalently less balanced) and less stable but still unpredictable state.

Part IV: An Alternative Approach to Analyzing Preferences of Normal Agents and Imitating Agents in the Agent-Based Modeling: Analysis of the Shannon Information Entropy. In order to study the agents' preferences and their estimation of the system, the Shannon information entropy (S1) may be introduced to our agent-based modeling. The information entropy S_I of a discrete random variable X with possible values $\{x_1,\ldots x_n\}$ is defined as $S_I(X) = -\sum_{i=1}^{n}P(x_i)\ln P(x_i)$, in which $P(x_i)$ denotes the probability mass function of x_i. In the agent-based model, the information entropy for a normal agent is $S_{Ii} = -\frac{L_i}{P}\ln\frac{L_i}{P} - \frac{P-L_i}{p}\ln\frac{P-L_i}{P}$, where L_i stands for the preference of the current strategy. If the normal agent would choose two rooms with an equal probability, this information entropy could reach the maximum value of $\ln 2$. On the other hand, the information entropy S_{Ij} for imitating agent j will be the same as that of the normal agent he/she follows in the local group. Thus the averaged information entropy of all the agents (i.e., normal agents and imitating agents) can be calculated as

$$S_I = \frac{1}{N}\left(\sum_{i=1}^{N_n}S_{Ii} + \sum_{j=1}^{N_m}S_{Ij}\right), \qquad \textbf{[S7]}$$

and the results are shown in Fig. S3*A*. As the averaged information entropy decreases as M_1/M_2 becomes larger, a clear-cut average preference of agents emerges as the distribution of resources gets more biased. This observation agrees with the analysis of participants' preferences in the human experiments; see Fig. 1 in the main text. Furthermore, the information content of agent i can be defined as $I_i = (\ln 2 - S_{Ii})/\ln 2$. Note that a larger I_i indicates that the agent has more confidence in a certain room. The averaged information content for all the normal agents (I_n) and imitating agents (I_m) are shown in Fig. S3*B*. In this figure, I_n decreases with the increase of the population of imitating agents when M_1/M_2 is small. This observation means that normal agents can be confused by the actions of imitating agents in a rather uniform distribution of the resource. When M_1/M_2 gets larger, I_n is nearly a constant implying that imitating agents will no longer affect the estimation of the normal agents. All of these arguments go well with the analysis of the experimental results in Fig. 1. The averaged information content of imitating agents has a rather drastic change as the environment varies. When $M_1/M_2 = 1$, I_m is pretty low, even lower than that of the normal agents, a fact indicating that imitating agents have almost unbiased preferences when the resource distribution is uniform. As M_1/M_2 increases, imitating agents are apt to flood into a specific room and thus form the herd in the modeled system.

Part V: A Different Agent-Based Modeling in Which Imitating Agents Follow the Majority, Rather than the Best Agent: an Open CAS vs. a Closed One. To make our work more general, a different modeling of the formation of herd is studied. Following the most successful person is often seen in daily life, and there is another common case following the majority. For example, people often decide on which store or restaurant to patronize on the basis of how popular they seem to be. In this sense, we make some changes to our agent-based modeling adopted in the main text. Models' main structures are similar, but the difference between them is that the "imitating agent" now follows the majority decision of his/her group (namely, the current imitating agents act as local majority-followers). This way of imitation means if more than half of the group members choose Room 1, he/she will enter Room 1 accordingly.

As shown in Fig. S4 and Fig. S5, the system behaviors are similar as those of "imitating the best." The main difference between Fig. 4 and Fig. S4 lies in Fig. S4*C*. When M_1/M_2 is small, adding imitating agents (local majority-followers) will cause relatively larger fluctuations. From Fig. S5 we can find that in the closed system of "following the majority," there also exists a proper β for one certain system environment M_1/M_2. Compared with Fig. S2, Fig. S5 shows that following the majority works not that well as following the best and this is easy to understand. The best normal agent owns the best strategy and is usually much more sensitive than common ones, while the majority reflects the average level of all the normal agents in the group. Although quantitatively different, the main conclusions arising from the two kinds of "following" are similar. Thus, the mechanism on how to form the herd is not an essential problem, and both imitating the best and following the majority can lead to similar conclusions.

1. Shannon CE (1951) Prediction and entropy of printed English. *The Bell System Technical Journal* 30:50–64.

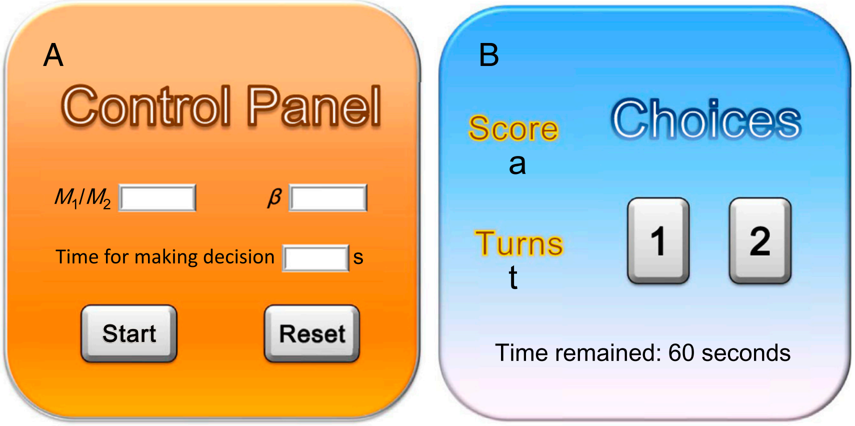

Fig. S1. The desktops of the experiment-control computer program used in the computer-aided human experiments: The control panel (*A*) for the coordinator and (*B*) for the human participants.

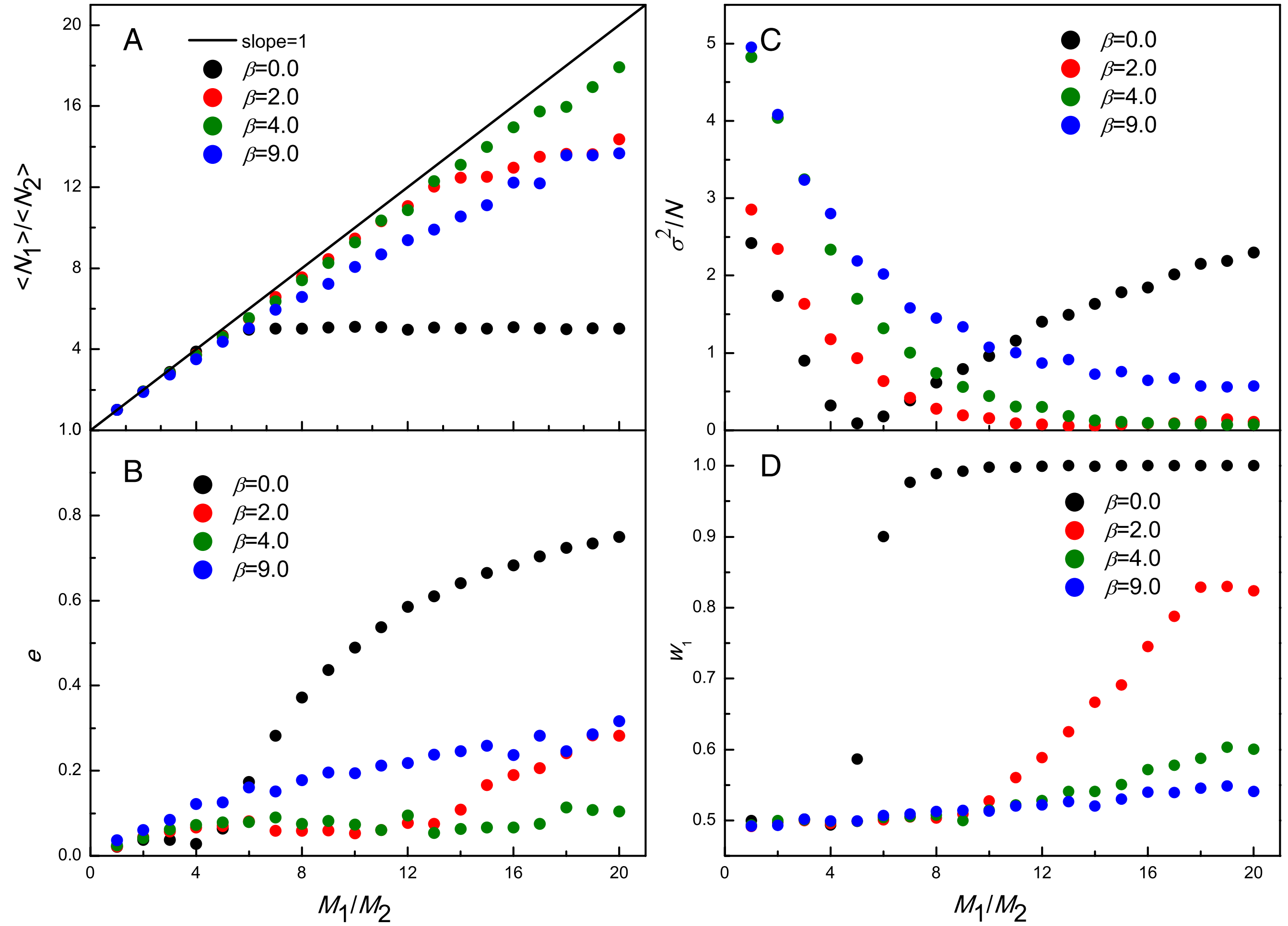

Fig. S2. (*A*) $\langle N_1 \rangle / \langle N_2 \rangle$, (*B*)e, (*C*) σ^2/N, and (*D*) w_1 as a function of M_1/M_2, for a closed system. Parameters: $N = 150$, $S = 4$, $P = 16$, $k = 5$, and $\beta = 0, 2.0, 4.0$, and 9.0. Simulations are run for 200 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). In (*A*), "slope = 1" denotes the straight line with slope being 1.

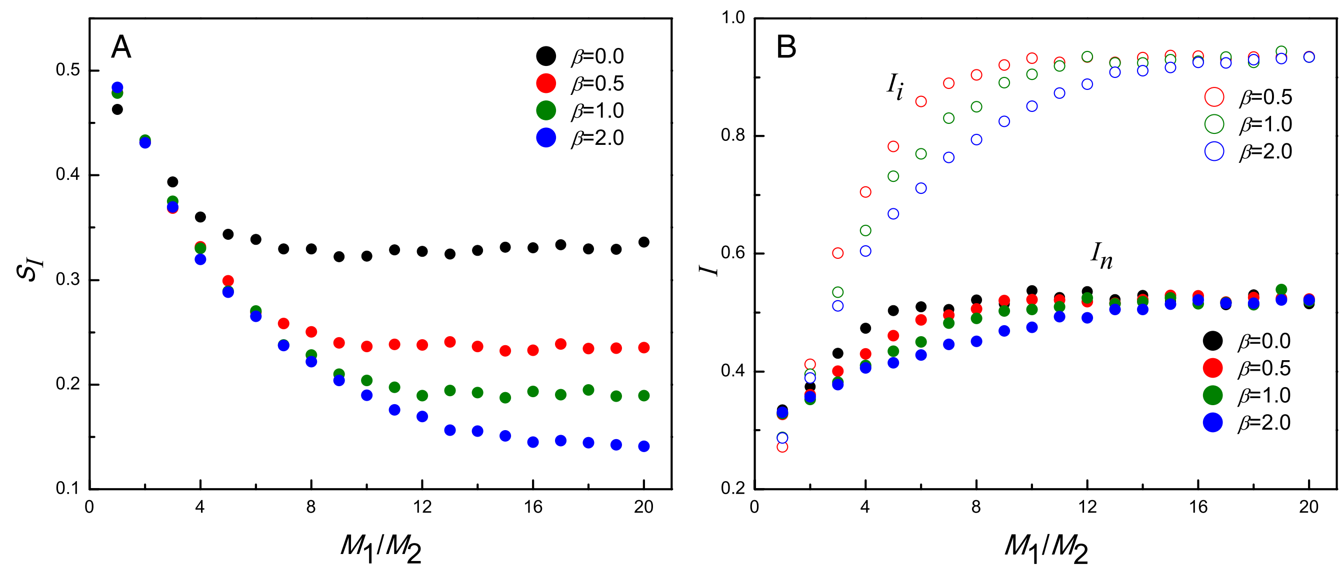

Fig. S3. The change of (*A*) the averaged information entropy (S_I) for all the agents including the normal agents and imitating agents and (*B*) the averaged information content for the normal agents (I_n) and imitating agents (I_m), respectively. Simulations are run for 200 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). Parameters: $N_n = 50$, $S = 4$, $P = 16$, and $k = 5$.

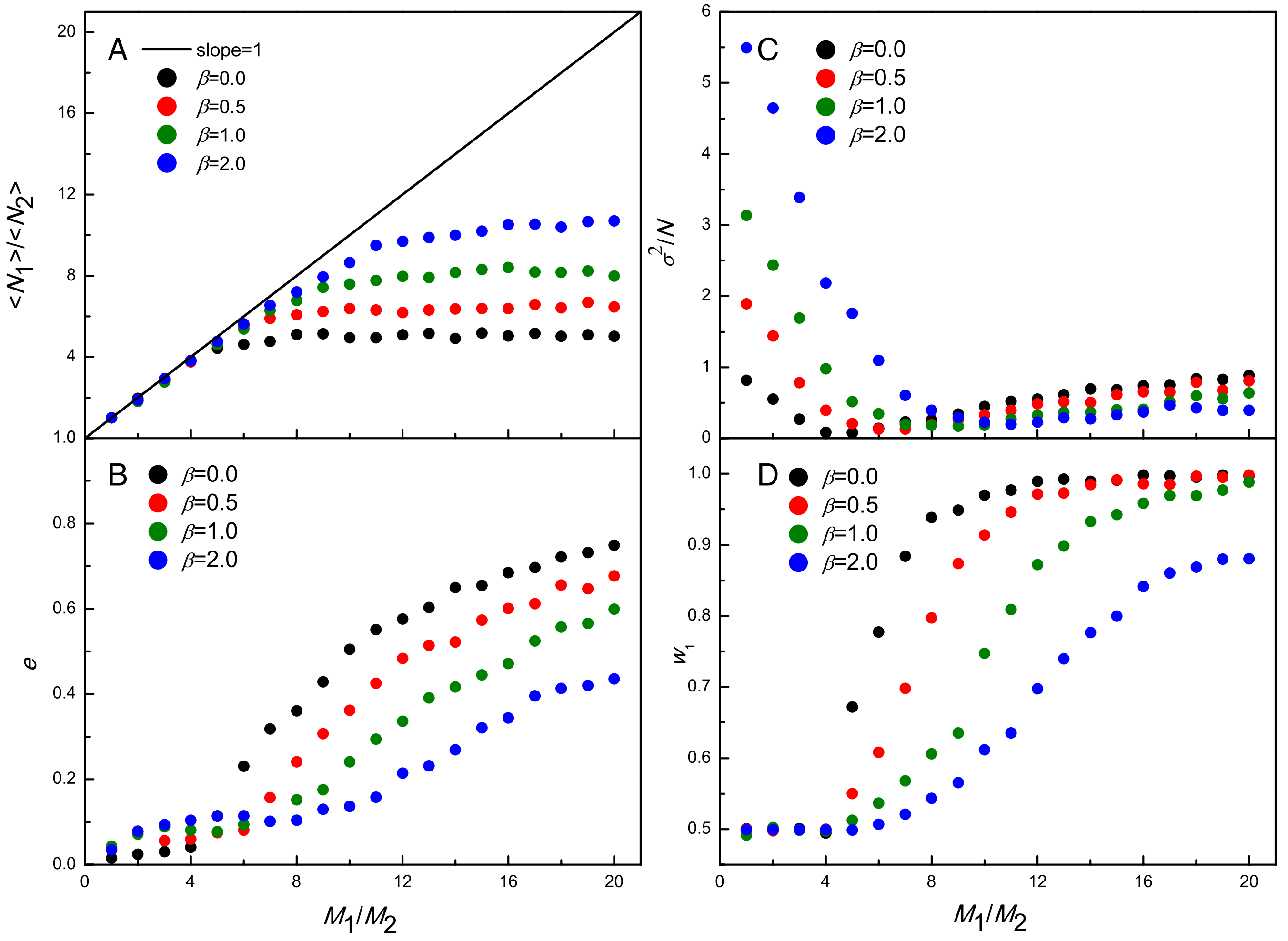

Fig. S4. (*A*) $\langle N_1\rangle/\langle N_2\rangle$, (*B*)e, (*C*) σ^2/N, and (*D*) w_1 as a function of M_1/M_2, for an open system. Parameters: $N_n = 50$, $P = 16$, $S = 4$, $k = 5$, and $\beta = 0, 0.5, 1.0$, and 2.0. The imitating agents follow the majority of their local groups. Simulations are run for 200 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). In (*A*), "slope = 1" denotes the straight line with slope being 1.

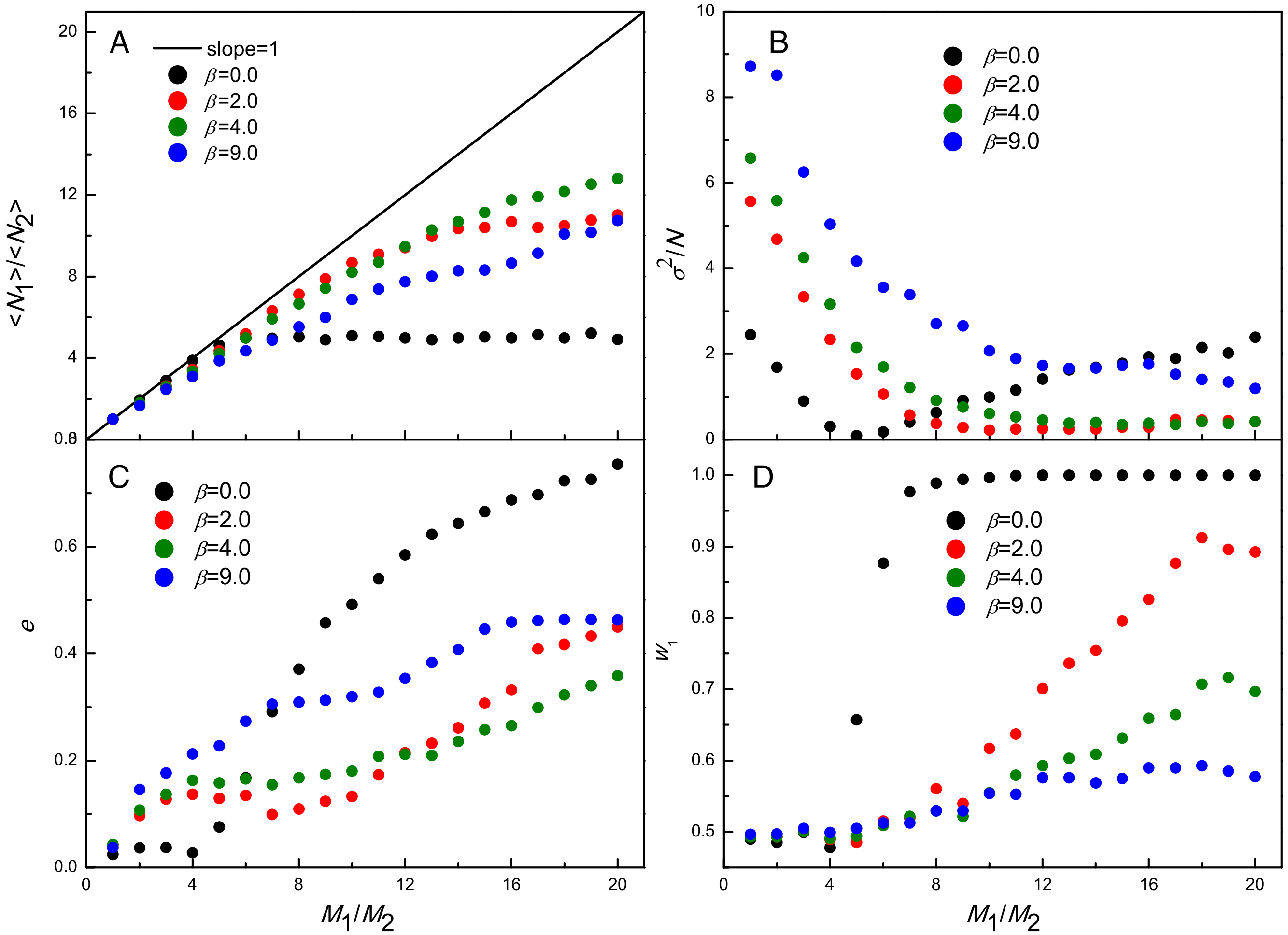

Fig. S5. (*A*) $\langle N_1\rangle/\langle N_2\rangle$, (*B*) e, (*C*) σ^2/N, and (*D*) w_1 as a function of M_1/M_2, for a closed system. Parameters: $N = 150$, $P = 16$, $S = 4$, $k = 5$, and $\beta = 0$,2.0,4.0, and 9.0. Others are the same as those information in Fig. S4.