Supporting Information

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SI Text

Bayes' Theorem and Why a Bayesian Framework Is Not Useful in Understanding Perception or Predicting What We See. As briefly noted in the text, Bayes' theorem is used to assess the probability of an event, A, given another event or condition, B. The theorem states that the posterior probability (i.e., the probability of A given B) is equal to the probability of B given A (called the likelihood) multiplied by the probability of A absent information about B (the prior for A) divided by the probability of B absent any information about A (the prior for B). The theorem thus equates two related conditional probabilities in terms of the ratio of their independent probabilities and is expressed as (Eq. **S1**)

$$P(A|B) = \frac{P(B|A) \bullet P(A)}{P(B)}. \quad \textbf{[S1]}$$

An example makes relating conditional probabilities in this way clearer. Let A be rainfall and B be clouds in the sky.

In terms of Bayes' theorem (Eq. **S2**),

$$\text{probability of rain given clouds} = \text{probability of clouds given rain} \times \text{independent probability of rain}/\text{independent probability of clouds}. \quad \textbf{[S2]}$$

Expressed in this way, Bayes' equation indicates that the chance of rain when it is cloudy depends on how often clouds are associated with rain, how often it is cloudy in some location, and how often it is rainy.

Giving values to the elements in the equation (Eqs. **S3**–**S6**),

$$\text{probability of rain given clouds} = \text{the unknown that is sought}, \quad \textbf{[S3]}$$

$$\text{probability of clouds given rain} = 1 \text{ (it will never rain when there are no clouds)}, \quad \textbf{[S4]}$$

$$\text{independent probability of rain} = 0.1 \text{ (an arbitrary but reasonable value for, say, a 1-h measuring period in a typical weather environment), and} \quad \textbf{[S5]}$$

$$\text{independent probability of clouds} = 0.5 \text{ (again, a reasonable value for a 1-h measuring period in a typical weather environment).} \quad \textbf{[S6]}$$

Therefore (Eq. **S7**),

$$\text{probability of rain given clouds} = 1 \times 0.1/0.5 = 0.2 \text{ or a 20\% chance of measurable rain during a 1-h period when the sky is cloudy in this locale.} \quad \textbf{[S7]}$$

When applied to vision, A typically stands for a physical state of the world (the sources of a retinal image) and B stands for a retinal image itself. Thus (Eq. **S8**),

$$\text{probability of a world state given a particular image} = \text{probability of the image given the world state} \times \text{independent probability of world state}/\text{independent probability of image}. \quad \textbf{[S8]}$$

Giving values to the elements in the equation (Eqs. **S9**–**S12**),

$$\text{probability of the world state given an image} = \text{the unknown that is sought}, \quad \textbf{[S9]}$$

$$\text{probability of the image given the world state} = 1 \text{ (a given world state will always produce the same image)}, \quad \textbf{[S10]}$$

$$\text{independent probability of the image} = \text{can be acquired from a database of natural images, and} \quad \textbf{[S11]}$$

$$\text{independent probability of the world state} = \text{obtained from repeated physical measurements of the reflectance values, illuminants, and transmittances from the environment.} \quad \textbf{[S12]}$$

The difficulty of obtaining the independent probability of the relevant world state by repeated physical measurements notwithstanding, the probability of a world state given an image could be determined. This approach, however, overlooks the inverse problem. As a result, the visual system cannot ascertain the real-world cause of any image. Once this obstacle is understood, the rationale for a Bayesian explanation of vision in terms of images and sources is nullified.

The wholly empirical theory of vision in Bayesian terms would be (Eq. **S13**)

$$\text{probability of a behavioral state given a particular image} = \text{probability of the image given the } \textit{behavioral state} \times \text{independent probability of the behavioral state}/\text{independent probability of the image}. \quad \textbf{[S13]}$$

Giving values to the elements in the equation (Eqs. **S14**–**S17**),

probability of the behavioral state given an image
= the unknown that is sought, **[S14]**

probability of the image given the behavioral state
= ? (not clear how to obtain values), **[S15]**

independent probability of the image
= can acquire from database of natural images **[S16]**

independent probability of behavioral state
= ? (not clear how to obtain values, because they would depend on evolution, development, and individual learning). **[S17]**

Without values for the likelihood and the behavioral prior, a Bayesian approach has little practical or theoretical value for understanding visual perception and its neural basis.