Supporting Materials for Landscape, Flux, Correlation, Resonance, Coherence, Stability and Key Network Wirings of Stochastic Circadian Oscillation

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Coherence of oscillation system

The robustness of the oscillation with respect to molecule number V can be quantified further by the phase coherence ξ, which measures the degree of periodicity of the time evolution of a given variable.(1) The phase coherence ξ quantitatively measures the degree of persistence of the oscillatory phase, and is defined as follows: First, the vector $N(t) = n_1(t)e_1 + n_2(t)e_2$ is shown in Figure 1. The unit vectors are $e_1 = (1, 0)$ and $e_2 = (0, 1)$, $n_1(t)$ and $n_2(t)$ are the concentration of the two kinds of protein molecules at time t. Then $\phi(t)$ is the phase angle between $N(t)$ and $N(t + \tau)$, where τ should be smaller than the deterministic period and larger than the fast fluctuations. $\phi(t) > 0$ to represent that the oscillation goes on the positive orientation (counterclockwise). The formula of ξ is shown as:

$$\xi = \frac{2\sum_i \theta(\phi(t))\phi(t)}{\sum_i |\phi(t)|} - 1 \tag{1}$$

where $\theta(\phi) = 1$ when $\phi(t) > 0$, and $\theta(\phi) = 0$ when $\phi(t) \leq 0$, and sums are taken over every time steps for the simulated trajectory. $\xi \approx 0$ means the system moves stochastically. The oscillation is most coherent as ξ is close to 1 . In the presence of fluctuations, the more periodic the evolution is, the larger the value of ξ is.

References

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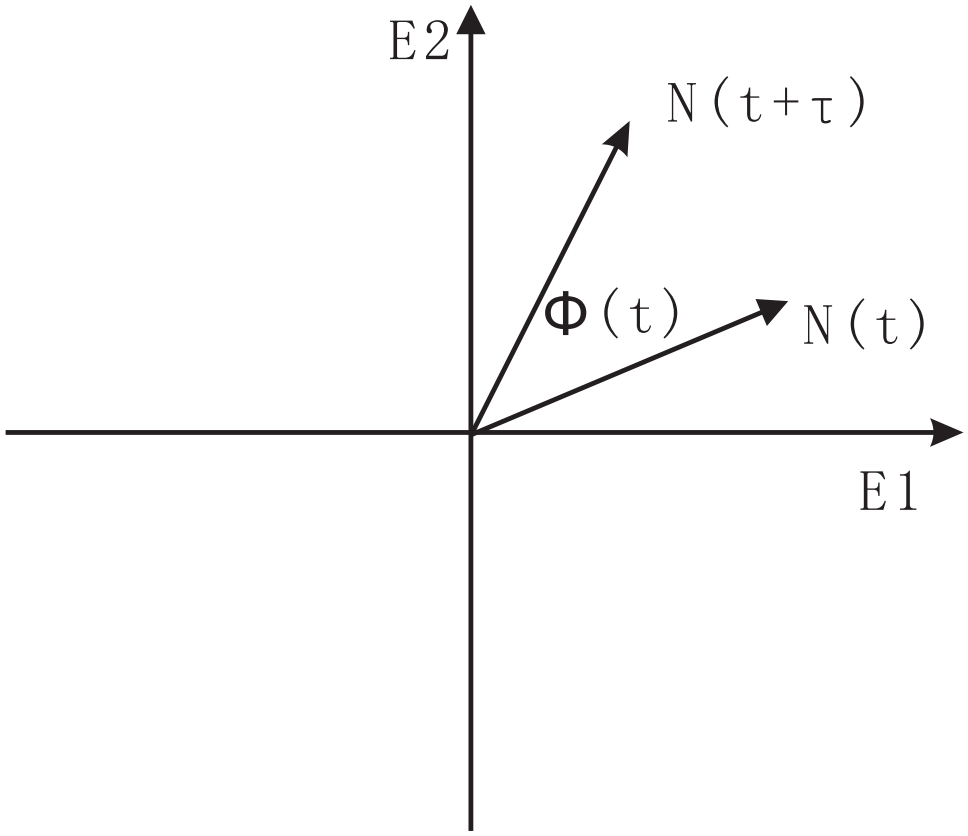

Figure 1. Definition of phase coherence.