Supporting material

The following material shows how R_0 was derived, and determines the stability of disease-free equilibrium when $\phi = 0$.

Theorem: If $P_U^0, P_C^0 \geq 0$, the solutions are non-negative and remain bounded in the positive cone of R^6. If R_0 <1, the disease-free steady state E_0 is locally asymptotically stable. If R_0 >1, E_0 is unstable.

Proof: Let $T(t) = P_U(t) + P_C(t) + H_U(t) + H_C(t) + V_U(t) + V_C(t)$. Then

$$\frac{dT(t)}{dt} = \lambda - \delta_U P_U(t) - \delta_C P_C(t) \leq \lambda - \min\{\delta_U, \delta_C\} T(t)$$

Thus, the solutions remain bounded in the positive cone of R^6 where the system induces a global semi-flow. The procedure introduced by van den Driessche and Watmough[26] or Diekmann *et al.*[27] can be used to evaluate R_0, and determine the stability of the disease-free equilibrium. After re-ordering, the disease-free steady state can be written as:

$$E_0 = (P_U, H_U, V_U, P_C, H_C, V_C) = (\frac{\lambda}{\delta_U}, 1, 1, 0, 0, 0)$$

Then:

$$F = \begin{bmatrix} \left[\frac{(1-\eta)}{N}\beta_{PH} H_c(t) + \frac{(1-\xi)}{N}\beta_{PV} V_c(t)\right] P_U(t) \\ \frac{(1-\eta)}{N}\beta_{PH} P_C(t) H_U(t) \\ \frac{(1-\xi)}{N}\beta_{PV} P_C(t) V_U(t) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and:

$$V = \begin{bmatrix} \delta_C P_C(t) \\ \gamma_H H_C(t) \\ \gamma_V V_C(t) \\ \left[\frac{(1-\eta)}{N}\beta_{PH} H_c(t) + \frac{(1-\xi)}{N}\beta_{PV} V_c(t)\right] P_U(t) + \delta_U P_U(t) - \lambda \\ \frac{(1-\eta)}{N}\beta_{PH} P_C(t) H_U(t) - \gamma_H H_C(t) \\ \frac{(1-\xi)}{N}\beta_{PV} P_C(t) V_U(t) - \gamma_V V_C(t) \end{bmatrix}$$

Since:

$$F = \begin{bmatrix} \frac{\partial F_1}{\partial P_C} & \frac{\partial F_1}{\partial H_C} & \frac{\partial F_1}{\partial V_C} \\ \frac{\partial F_2}{\partial P_C} & \frac{\partial F_2}{\partial H_C} & \frac{\partial F_2}{\partial V_C} \\ \frac{\partial F_3}{\partial P_C} & \frac{\partial F_3}{\partial H_C} & \frac{\partial F_3}{\partial V_C} \end{bmatrix} = \begin{bmatrix} 0 & \frac{(1-\eta)}{N}\beta_{PH} P_U(t) & \frac{(1-\xi)}{N}\beta_{PV} P_U(t) \\ \frac{(1-\eta)}{N}\beta_{PH} H_U(t) & 0 & 0 \\ \frac{(1-\xi)}{N}\beta_{PV} V_U(t) & 0 & 0 \end{bmatrix}$$

it follows that:

$$F|_{E_0} = \begin{bmatrix} 0 & \frac{(1-\eta)}{N}\beta_{PH}\frac{\lambda}{\delta_U} & \frac{(1-\xi)}{N}\beta_{PV}\frac{\lambda}{\delta_U} \\ \frac{(1-\eta)}{N}\beta_{PH} & 0 & 0 \\ \frac{(1-\xi)}{N}\beta_{PV} & 0 & 0 \end{bmatrix}$$

Similarly:

$$V = \begin{bmatrix} \delta_C & 0 & 0 \\ 0 & \gamma_H & 0 \\ 0 & 0 & \gamma_V \end{bmatrix}$$

and:

$$V^{-1} = \begin{bmatrix} \frac{1}{\delta_C} & 0 & 0 \\ 0 & \frac{1}{\gamma_H} & 0 \\ 0 & 0 & \frac{1}{\gamma_V} \end{bmatrix}$$

So:

$$FV^{-1} = \begin{bmatrix} 0 & \frac{(1-\eta)\beta_{PH}\lambda}{\delta_U \gamma_H N} & \frac{(1-\xi)\beta_{PV}\lambda}{\delta_U \gamma_V N} \\ \frac{(1-\eta)\beta_{PH}}{\delta_C N} & 0 & 0 \\ \frac{(1-\xi)\beta_{PV}}{\delta_C N} & 0 & 0 \end{bmatrix}$$

Thus, the Jacobian matrix is:

$$XI - FV^{-1} = \begin{bmatrix} x & -\frac{(1-\eta)\beta_{PH}\lambda}{\delta_U \gamma_H N} & -\frac{(1-\xi)\beta_{PV}\lambda}{\delta_U \gamma_V N} \\ -\frac{(1-\eta)\beta_{PH}}{\delta_C N} & x & 0 \\ -\frac{(1-\xi)\beta_{PV}}{\delta_C N} & 0 & x \end{bmatrix}$$

Since $x^3 - \left[\frac{(1-\eta)^2 \beta_{PH}^2 \lambda}{\delta_U \delta_C \gamma_H N^2} + \frac{(1-\xi)^2 \beta_{PV}^2 \lambda}{\delta_U \delta_C \gamma_V N^2} \right] x = 0$, we obtain that:

$$R_0 = \rho(FV^{-1}) = \sqrt{\frac{(1-\eta)^2 \beta_{PH}^2 \lambda}{\delta_U \delta_C \gamma_H N^2} + \frac{(1-\xi)^2 \beta_{PV}^2 \lambda}{\delta_U \delta_C \gamma_V N^2}} .$$

By the results in van den Driessche and Watmough,[26] the stability of the disease-free equilibrium is obtained.

Author queries

Collection of data on MRSA, para 2: please supply city for bioMerieux