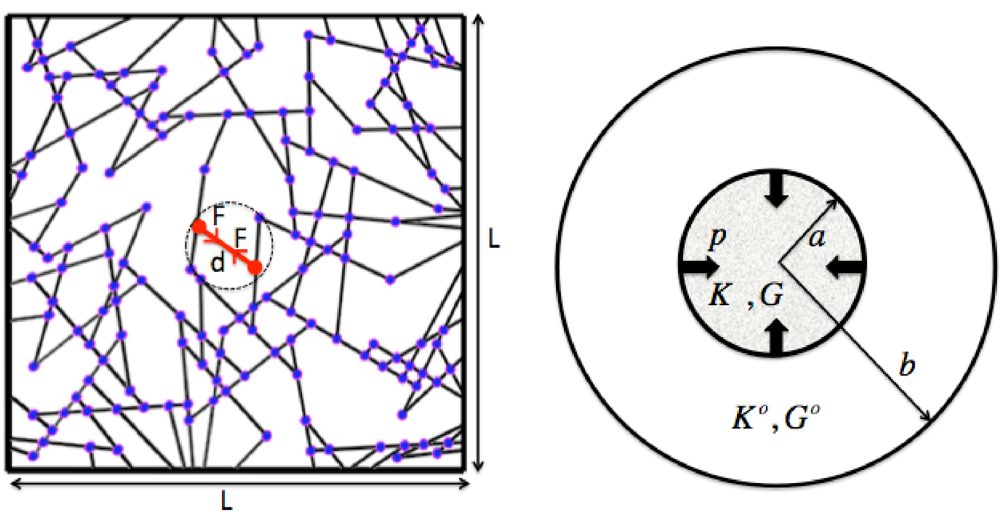
SUPPLEMENTARY MATERIAL FOR "GENERAL MECHANISM OF ACTOMYOSIN CONTRACTILITY"

DERIVATION OF THEORETICAL WALL STRESS σ_{th}

Here we obtain an approximate expression for the stress on the fixed boundary of a two-dimensional actin network due to an active myosin minifilament, as shown in Fig.1a. We treat the minifilament as a force dipole. To simplify our calculations we consider a circular region and assume that the effect of the force dipole is equivalent to that of a uniform inward pressure P along a boundary at a radius a that is half the size of the force dipole (see Fig. 1b).. For generality, we first consider a layered system having two different elastic moduli inside and outside $r = a$: κ , G from $r = 0$ to $r = a$ and κ^o and G^o from $r = a$ to $r = b$; later we will treat the myosin-actin system as a special case.



(a) (b)

FIG. 1: (a) An actin network with a myosin minifilament (dumbbell) represented as a force dipole acting on the network. (b) Circularly layered system with different material properties in two regions. (Here a and b in (b) correspond to $d/2$ and $L/2$ in (a) respectively).

The boundary conditions for the displacement U and the stress σ are as follows. Because of the assumption of a fixed boundary, $U_r(r = b) = 0$, and because there is no singularity at the origin, $U_r(r = 0)$ is finite. Furthermore, because there are no gaps in the material,

TABLE I: **Notation Used**

κ, κ^o	Bulk modulus of the material
G, G^o	Shear modulus of the material.
$\vec{U}(\vec{r})$	Displacement vector at position r
U_r	Radial component of the displacement vector
η_{ij}	ij^{th} component of the strain tensor
σ_{ij}	ij^{th} component of the stress tensor

$U_r(r = a^+) = U_r(r = a^-)$. Finally, the application the pressure at $r = a$ leads to a discontinuity in σ, so that $\sigma_{rr}(r = a^+) - \sigma_{rr}(r = a^-) = P$.

To obtain the functional form of the solution, we note that circular symmetry and the absence of body forces imply that $\vec{U}(\vec{r}) = U_r(r)\hat{r}$ and $\vec{\nabla}(\vec{\nabla} \cdot \vec{U}) = 0$ in both regions. Thus the solution has the form $U_r(r) = Ar + B/r$ for $r < a$ and $U_r(r) = Cr + D/r$ for $a < r < b$, where $A, B, C,$ and D are constants to be determined. The boundary condition that $U_r(r = 0)$ is finite implies that $B = 0$, and the condition that $U_r(r = b) = 0$ implies that $D = -Cb^2$. Then the condition that $U_r(r = a^+) = U_r(r = a^-)$ implies that $Aa = C(a - b^2/a)$ so that $A = -C(b^2/a^2 - 1)$, and the solution becomes

$$U(r) = \begin{cases} -Cr(b^2/a^2 - 1) & \text{for } r < a \\ -Cr(b^2/r^2 - 1) & \text{for } a < r < b \end{cases}$$

To impose the boundary condition that $\sigma_{rr}(r = a^+) - \sigma_{rr}(r = a^-) = P$, we first calculate the strains, using the general result $\eta_{rr} = \frac{\partial U_r}{\partial r}$, $\eta_{\phi\phi} = \frac{U_r}{r}$ and $\eta_{r\phi} = 0$ (Ref. [1]), Eq. (1.7):

$$\begin{aligned} \eta_{rr} &= -C(b^2/a^2 - 1), \eta_{\phi\phi} &= -C(b^2/a^2 - 1) \ (r < a) \\ \eta_{rr} &= C(b^2/r^2 + 1), \eta_{\phi\phi} &= -C(b^2/r^2 - 1) \ (a < r < b) \end{aligned} \tag{1}$$

The stress is given in terms of the strain as follows (Ref. [1], Eq. 4.6)):

$$\begin{aligned} \sigma_{rr} &= (\kappa + \tfrac{4}{3}G)\eta_{rr} + (\kappa - \tfrac{2}{3}G)\eta_{\phi\phi} \\ \sigma_{\phi\phi} &= (\kappa + \tfrac{4}{3}G)\eta_{\phi\phi} + (\kappa - \tfrac{2}{3}G)\eta_{rr} \end{aligned} \tag{2}$$

Thus for $r < a$

$$\begin{aligned} \sigma_{rr} &= -2C(\kappa + \frac{G}{3})(\frac{b^2}{a^2} - 1) \\ \sigma_{\phi\phi} &= -2C(\kappa + \frac{G}{3})(\frac{b^2}{a^2} - 1) \end{aligned} \tag{3}$$

and for $a < r < b$

$$\begin{aligned} \sigma_{rr} &= 2C[\kappa^o + (\frac{1}{3} + \frac{b^2}{r^2})G^o] \\ \sigma_{rr} &= 2C[\kappa^o + (\frac{1}{3} - \frac{b^2}{r^2})G^o] \end{aligned} \tag{4}$$

Then the stress boundary condition, $\sigma_{rr}(r = a^+) - \sigma_{rr}(r = a^-) = P$, implies that

$$2C[\kappa^o + (\frac{1}{3} + \frac{b^2}{a^2})G^o] + 2C(\kappa + \frac{G}{3})(\frac{b^2}{a^2} - 1) = P \tag{5}$$

so that

$$C = \frac{P}{2[\kappa^o + (\frac{1}{3} + \frac{b^2}{a^2})G^o + (\kappa + \frac{G}{3})(\frac{b^2}{a^2} - 1)]} \tag{6}$$

Finally, for $a < r < b$ we have

$$\sigma_{rr} = \frac{[\kappa^o + (\frac{1}{3} + \frac{b^2}{r^2})G^o]P}{[\kappa^o + (\frac{1}{3} + \frac{b^2}{a^2})G^o + (\kappa + \frac{G}{3})(\frac{b^2}{a^2} - 1)]} \tag{7}$$

We now assume that the two regions consist of the same material, so that $\kappa^o = \kappa$. Furthermore, for actin networks, Poisson's ratio is close to 0.5 [2], so that we take $G^o = G = 0$. Finally, we assume that $b >> a$. Then we obtain at $r = a$

$$\sigma_{rr} \simeq \frac{Pa^2}{b^2} \tag{8}$$

For the geometry of Fig.1a, we have $a = d/2,\ b = L/2$. Since the magnitude of the contraction induced by a force distribution $f_{vec}(\vec{r})$ is measured by its force dipole moment $\int \vec{r} \cdot f_{vec}(\vec{r}) d^3r$, we choose the value of P to have the same dipole $-Fd$ as the pair of myosin forces. Since the force density associated with P is $-\hat{r}P\delta(r-a)$, we obtain $-2\pi Pa^2 = -Fd$, so $P = \frac{2F}{\pi d}$. Thus

$$\sigma_{th} = \frac{Fd}{2\pi(L/2)^2} \tag{9}$$

[1] L. D. Landau and E. M. Lifshitz, *Theory of Elasticity* (Pergamon press, New York, 1986), chapter 1, 3rd ed.

[2] M. L. Gardel, M. T. Valentine, J. C. Crocker, A. R. Bausch, and D. A. Weitz, Phys. Rev. Lett. **91**, 158302 (2003).