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Fig. S1. Decay of the curvature radii as a function of the actin concentration. Independent of the actin concentrations the distributions decay according to  $p(r) = a_1 \cdot \exp(-r/l_1) + a_2 \cdot \exp(-r/l_2)$ , shown as solid lines. For the lowest actin concentration (1 µM, blue curve) parameters are  $a_1 = 0.59$ µm;  $a_2 = 0.41$ µm;  $l_1 = 9.34$ μm;  $l_2 = 24.62$ μm. For the intermediate actin concentration (2.6 μM, red curve)  $a_1 = 0.53$ μm;  $a_2 = 0.47$ μm;  $l_1 = 5.10$ μm;  $l_2 = 12.57$ μm was found and for the highest actin concentration (3.5 μM, green curve)  $a_1 = 0.59$ μm;  $a_2 = 0.41$ μm;  $l_1 = 8.29$ μm;  $l_2 = 15.01$ μm resulted. The motor density was adjusted to  $\sigma_m = 90$  nM and the fascin concentration was  $c = 0.5$   $\mu$ M.



Fig. S2. Time overlay of a moving actin-fascin string. Actin-fascin strings move on circular trajectories. The thicker the strings get, the higher their directional persistence. As a consequence variations of a given curvature happen increasingly less frequent and have less effect. The motor density was adjusted to  $\sigma_m$  = 90 nM, the actin concentration was  $\rho = 3$  μM and the fascin concentration was  $\epsilon = 0.5$  μM. The scale bar is 50 μm.



Fig. S3. Phase diagram as a function of the actin  $\rho$  and fascin concentration c. For high actin and fascin concentration the systems evolves into a frozen steady state that is characterized by coherently moving streaks or fibers (F). For intermediate concentrations of actin and fascin the frozen active state is given by the ring phase (R). At low fascin concentrations the steady state is characterized by persistent fluctuations on the single filament level and no frozen steady state emerges (o). All scale bars are 50 <sup>μ</sup>m and the motor density was adjusted to 90 nM.



Fig. S4. Cumulative radii distribution  $P(r)$  with and without aggregation. The stochastic process determined by Eq. 1 in the main manuscript leads to a powerlaw of slope -1 (black line) in the cumulative radii distribution  $\overline{P}(r)$  (red curve). In contrast, simulations that include growth stiffening according to Eqs. 3 and 4, as well as the possibility of merging events, exhibit a different shape (green and blue curve). Inset: Normalized noncumulative curvature distribution  $p(k)$  for stochastic processes according to Eq. 1 (red curve) and for the aggregation process according to Eq. 2 (blue boxes correspond to closed). In all simulation runs <sup>α</sup> was set to  $\alpha = 1.0$ . The parameters that determine the aggregation processes were set to:  $\lambda = 0.1$ ,  $\omega = 0.4$ ,  $\theta_c = 10^0$  (see blue and green curve).



Fig. S5. Dependence of Γ on growth rate λ for three critical merging angles  $\theta_c$ . The ratio of open to closed rings Γ increases with the thickening rate λ. Slow growth stiffening allows the emergence of longer structures leading to more closed ring, whereas fast growth stiffening freezes the structure*'*s orbits resulting in open rings spatially separated without any further interactions. The critical merging angle has only a minor impact on the ring statistics. The parameters characterizing the noise level are:  $\omega = 0.1$ ,  $\alpha = 1.0$ .

## The Ring Phase

video1 - supplement to Fig. 1



### parameters:  $\rho = 10 \mu M$ ,  $c = 0.1 \mu M$ ,  $\sigma_m = 90 \text{ nM}$

Movie S1. The addition of fascin leads to the emergence of a frozen steady state of rotating rings (actin concentration  $\rho = 3 \mu M$ , motor concentration σ<sub>m</sub> = 90 nM, fascin concentration  $c = 0.2$  μM, labeling ratio 1:16).

[Movie S1 \(MOV\)](http://www.pnas.org/lookup/suppl/doi:10.1073/pnas.1107540108/-/DCSupplemental/SM01.mov)

**Ring Closure** 

#### video 2 - supplement to Fig. 2

parameters:  $\rho = 10 \mu M$ ,<br>c = 0.1 $\mu$ M,  $\sigma_m$  = 90 nM

Movie S2. The emergence of stable curvatures relies on two mechanisms: the ring closure and the freezing of a current curvature (actin concentration  $\rho = 3$  μM, motor concentration  $\sigma_m = 90$  nM, fascin concentration c = 0.2 μM, labeling ratio 1:16).

[Movie S2 \(MOV\)](http://www.pnas.org/lookup/suppl/doi:10.1073/pnas.1107540108/-/DCSupplemental/SM02.mov)

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# **Simulations**

video 3 - supplement to Fig. 3

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### parameters:  $\omega = 0.1$ ,  $\lambda = 0.4$ ,  $\alpha = 1.0$ ,  $\Theta_c = 10^{\circ}$

Movie S3. Emergence of rotating rings in the cellular automaton simulations and coexistence of open and closed rings ( $\omega$  = 0.1,  $\lambda$  = 0.4,  $\alpha$  = 1.0 and  $\theta_c = 10^0$ ).

[Movie S3 \(MOV\)](http://www.pnas.org/lookup/suppl/doi:10.1073/pnas.1107540108/-/DCSupplemental/SM03.mov)

**Coherently Moving Streaks** 

video 4 - supplement to Fig. S3



parameters:  $\rho = 10 \mu M$ , c = 0.5 $\mu$ M,  $\sigma_m = 90 \text{ nM}$ 

Movie S4. Above a certain material density curved trajectories are no longer possible and the actin-fascin structures are forced on straight trajectories; extended actin-fascin streaks evolve that successively merge and coarsen, leading to a large scale symmetry breaking (actin concentration  $\rho = 10 \mu$ M, motor concentration  $\sigma_m = 90$  nM, fascin concentration  $c = 0.5$  µM, labeling ratio 1:32).

[Movie S4 \(MOV\)](http://www.pnas.org/lookup/suppl/doi:10.1073/pnas.1107540108/-/DCSupplemental/SM04.mov)