Supplemental Materials

Determining the stability of the ARX model

To determine the stability of an ARX model, Equation 1 from the main text is rearranged into:

$$\sum_{i=0}^{M} a_i \times QTI_{n-i} = \sum_{i=1}^{M} b_i \times RRI_{n-i} \qquad \text{Eq. 2}$$

The z-transform,[1] a widely-used engineering approach similar to the Laplace's transform, is then applied to Equation 2, resulting in:

$$QTI(z)\left(a_0 + a_1 z^{-1} + a_2 z^{-2} + ... + a_M z^{-M}\right) = RRI(z)\left(b_1 z^{-1} + b_2 z^{-2} + ... + b_M z^{-M}\right) \qquad \text{Eq. 3}$$

The last equation is re-written as:

$$\frac{QTI(z)}{RRI(z)} = \frac{b_1 z^{-1} + b_2 z^{-2} + ... + b_M z^{-M}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + ... + a_M z^{-M}} = \frac{(z-\beta_1)(z-\beta_2).........(z-\beta_M)}{(z-\alpha_1)(z-\alpha_2).........(z-\alpha_M)} \qquad \text{Eq. 4}$$

Equation 4 is the transfer function of the ARX model in the z-domain. Here α_i and β_i (i=1,..,M) are coefficients (constants) obtained from the weights a_i and b_i (i=1,..,M) in Equation 1; they are all complex numbers. QTI(z) and RRI(z) represent the QTI and RRI series in the z-domain. In Equation 4, when z is set equal to any of the α_i (i=1,..,M), one obtains a pole of the system; when z is set equal to any of the β_i (i=1,..,M), one obtains a zero of the system; the system has M pole-and-zero pairs. A pole is canceled if it is equal to a zero, which means this pole has no contribution to the stability of the system. In the present study, in practical terms, we assume that a pole is canceled by a zero if the distance between the pole and the zero is smaller than 0.05. According to the z-domain stability theory,[1] the system represented by the ARX has unstable dynamics if a pole that is not canceled falls outside of the unit circle |z|=1, i.e. the magnitude of the pole is >1. The z-transform operation and the calculation of poles and zeros are executed using Matlab functions.

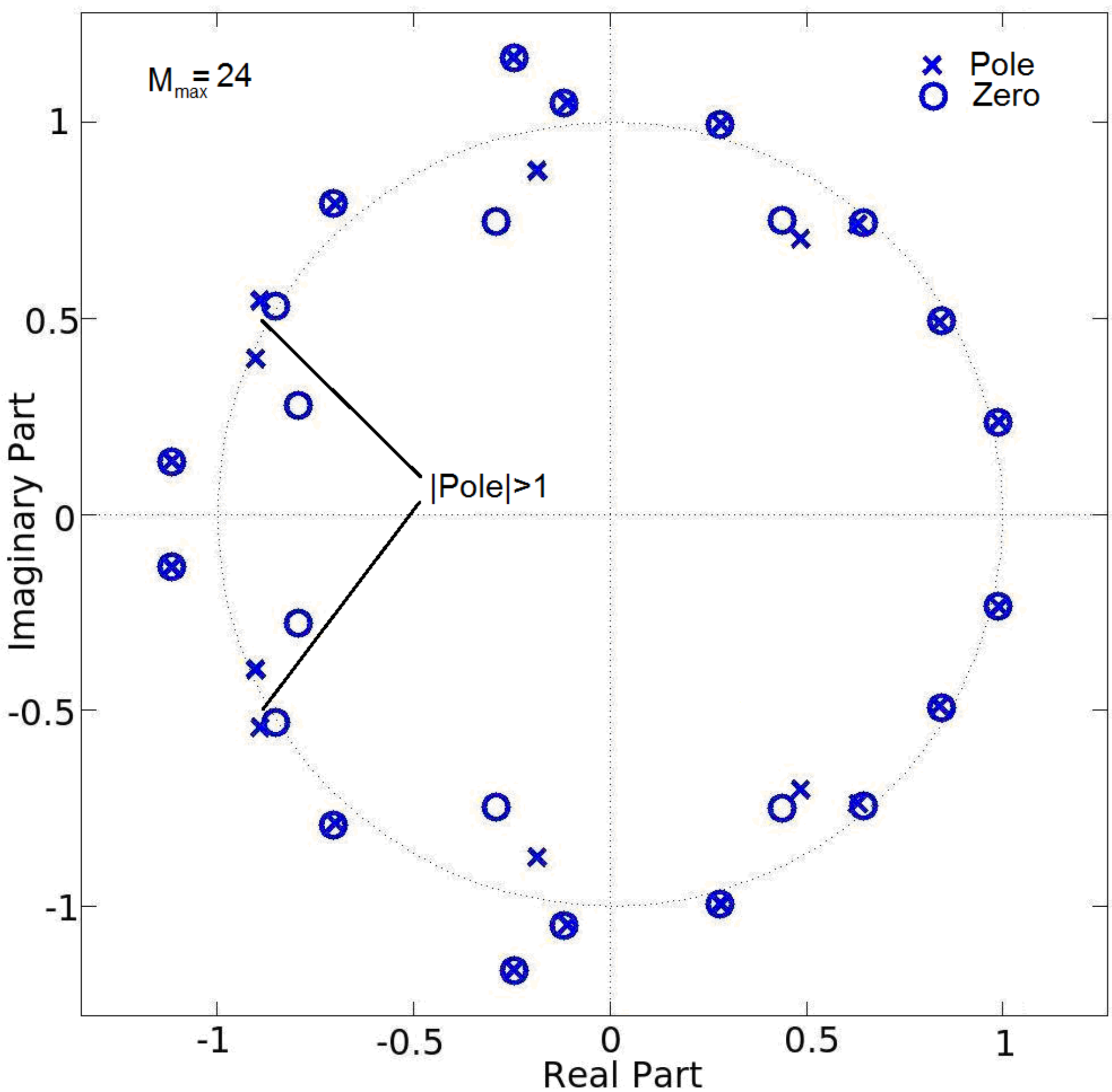
In this study, the pole of largest magnitude among all poles that are not cancelled constitutes the QTI stability index P_m. If the not-cancelled-pole-of-largest magnitude is within the unit circle, the system is stable, since all other poles are also within the unit circle. If the not-cancelled-pole-of-largest-magnitude is outside of the unit circle, the system is unstable, even if there are other not cancelled poles inside the unit circle. The not cancelled pole of the largest magnitude determines largely the stability of the system.

Stability analysis of QTI dynamics in the z-domain for the minECG used in Figure 3 is presented in Supplemental Figure 1 for M_{max}=24 and M_{min}=3. The QTI dynamics of this minECG is assessed as unstable because a pair of poles not canceled by zeroes is outside of the unit circle, as seen in Supplemental Figure 1A. The numerous cancelled poles do not contribute to instability. Supplemental Figure 1B shows that, although M_{min}=3 does not result in a good prediction of QTI dynamics (as seen in Figure 3B in the main text), QTI dynamics instability is nonetheless captured accurately, as evidenced by the presence of the two not cancelled poles outside of the unit circle. The “locations” of these two not cancelled poles are very close to those in Supplemental Figure 1A.

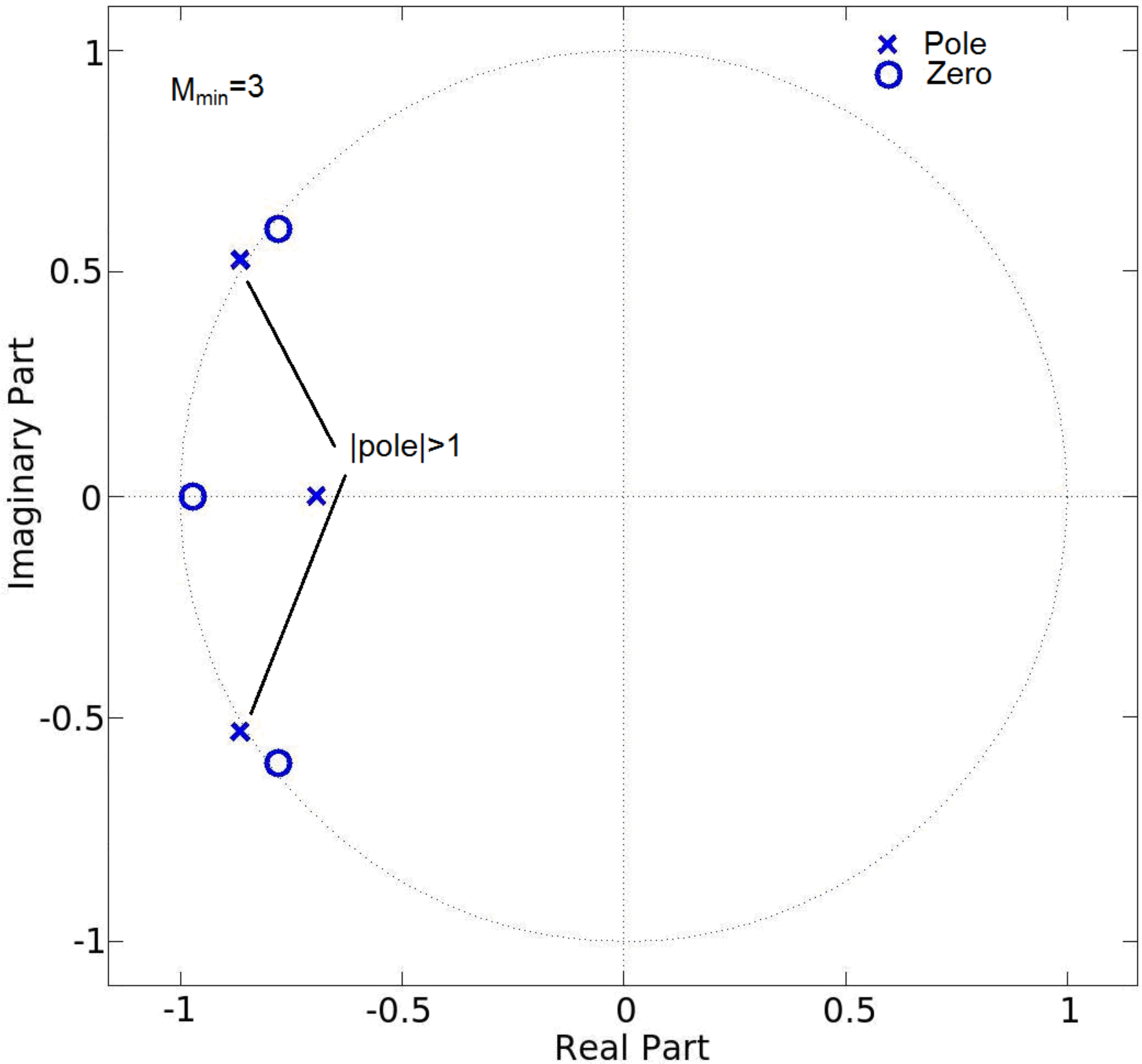
It is important to underscore that the present methodology for representing the ECG signal as an ARX uses different extents of activation history depending on the specific application of the methodology. If the goal of an application is to achieve an accurate (mean square error <5m^2) prediction of QTI dynamics, then the largest number of beats, M_{max}, needs to be included in the ARX model (Fig.4A). If the value of M in the ARX model is less than M_{max}, then the prediction of QTI dynamics is less accurate. Clearly, M_{max} is the full activation history; including a number of beats larger than M_{max} does not improve the prediction of QTI dynamics.

Alternatively, if the goal of an application is to determine whether QTI dynamics is stable or unstable, then using M_{min} number of beats in the ARX model is sufficient for this purpose, and this saves computational time. M_{min} is the value of M at which the QTI instability is first manifested; M_{min} is always much smaller than M_{max}. M_{min} represents the number of beats (out of the entire activation history, M_{max}) that are the major determinants of stability or instability in QTI dynamics. If QTI dynamics is unstable at $M_{min,}$ the contribution of the beats with numbers between M_{min} and M_{max} is to only change the degree of instability in QTI dynamics; it may change the number and magnitude of the not-cancelled-poles located outside of the unit circle, but will not change the fact that QTI dynamics is unstable.

Supplemental Figure 1A



Supplemental Figure 1B



Supplemental Figure Legend

Supplemental Figure 1. Locations of pole-zero pairs with respect to the unit circle in the z-domain for the minECG used in Figure 3 of the main text for **A.** M_{max}=24 and **B.** M_{min}=3. The poles that are not cancelled by zeros and contribute to the instability of the QTI dynamics are located outside of the unit circle.

REFERENCE

1. Lathi BP. Linear systems and signals. *Berkeley-Cambridge Press.* 1992:227.