Supplementary Material to

Cavitation induced by a surfactant leads to a transient release of water stress and subsequent "run away" embolism in Scots pine (Pinus sylvestris) seedlings

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1. Model description

The model used is a simplified version of the model described in Hölttä et al. (2009). The model plant is depicted in Fig. S1. In Hölttä et al. (2009) the model tree was divided into N numerical elements. Here we run the model only for one numerical element, as for simplicity we do not take into consideration the axial variation in hydraulic properties. We also confirmed that this simplification did not affect the results very much. All the equations presented below are from Hölttä et al. (2009).

The change in the mass (*m*, [kg]) of water in the model plant is

$$
\frac{dm}{dt} = Q_{in} - Q_{out} \tag{S1}
$$

where Q_{in} is the water uptake [kg s⁻¹] by the plant and Q_{out} is the water loss [kg s⁻¹] by the plant.

Water uptake is calculated by Darcy's law

$$
Q_{in} = k(P_{soil} - P) \tag{S2}
$$

where *k* is the hydraulic conductance between the soil and the leaves $[m^3Pa^{-1} s^{-1}]$, *P* is plant water potential [Pa], and *Psoil* is the soil water potential [Pa] (which is zero in our case).

Water loss from the plant equals the transpiration rate $(E, [kg s⁻¹])$

$$
Q_{out} = E \tag{S3}
$$

Loss in hydraulic conductance due to cavitation is calculated using the Pammenter equation (Pammenter & Willigen 1998)

$$
PLC = (1 + \exp(a(P - b)))^{-1}
$$
\n
$$
(S4)
$$

$$
k = k_0 (1 - PLC) \tag{S5}
$$

where *PLC* is the relative loss of hydraulic conductance due to cavitation, *b* is the PLC₅₀ value [Pa], i.e., the water potential for which half of the hydraulic conductivity is lost, and *a* is a parameter [Pa-¹] describing the steepness of the vulnerability curve in relation to water potential.

The change in plant water potential is calculated from Hooke's law to be

$$
\frac{dP}{dt} = E_m \frac{1}{V_0} \frac{dV}{dt}
$$
 (S6)

where E_m is the "bulk" elastic modulus of the plant $[Pa]$, *V* is the volume of plant numerical element $[m³]$ (all compartments including xylem conduits, bark tissue, needles etc..), and $V₀$ is the volume of plant numerical element at zero water potential [m³]. The change in *V* is composed of two terms, one arising from water movement in or out of the plant (the first term in eq. (S7), and another arising from the increase in volume due to the introduction of gas due to cavitation V_g [m³], the second term in eq. (S7)

$$
\frac{dV}{dt} = \frac{1}{\rho} \frac{dm}{dt} + \frac{dV_g}{dt}
$$
 (S7)

where ρ is water density [kg m⁻³]. The second term causes the "capacitive" effect of cavitation, and it is assumed to be linearly proportional to loss in hydraulic conductance due to cavitation

$$
dV_g = VdPLC \tag{S8}
$$

The numerical time step used in the simulations was 25 µs. The time step had to be reduced to such as small magnitude to ensure the stability of the Explicit Euler scheme, which was used to solve the equations. Note that a dynamic treatment of the problem is required as a steady-state approach would not be able to capture the tension release due to cavitation.

Fig S1. A schematic presentation of the model.

2. Scaling of transit time to plant height according to the metabolic scaling theory

According to the metabolic scaling theory (West et al. 1999), plant transpiration rate (T) scales to three-quarters of plant mass (m)

T α m^{3/4}

Similarly, plant height (L) scales to one-quarter of plant mass

L α m $^{1/4}$

The transit time (τ) , plant mass divided by transpiration rate, is then proportional to plant height.

$$
\tau = \frac{m}{T} \propto \frac{m}{m^{3/4}} = m^{1/4} \propto L
$$

Given a tree height of 50 cm and a transit time of 70 minutes in our experiment, the constant of proportionality between tree height and transit time is 1.4 min per cm of plant height for a maximum value of transpiration rate. For a 50 m tree, the expected transit time is then 7000 minutes for a maximum value of transpiration rate. In addition, assuming that the average diurnal transpiration is one third of maximum transpiration, the expected transit time is then 21000 minutes, which equals approximately 15 days. The transit time of water through the plant in the experiment was estimated to be equal to the time it took from the addition of the surfactant for the peak in the apparent hydraulic conductance to be reached.

References

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West GB, Brown JH, Enquist BJ. 1999. A general model for the structure and allometry of plant vascular systems. *Nature* 400, 664–667.