Supplementary online appendix

Discordancy Measures: A heuristic illustration

Case study: A hypothetical population

To elucidate the concepts of and differences between the different discordancy measures described in our article, we present below a hypothetical example for the exact calculation of all discordancy measures in addition to alternative approximation methods for some of the measures. The approximations are introduced either because the DHS data do not include sufficient information to calculate the measures per their exact definitions, or because the approximations minimize the effects of survey non-response rates and incompleteness of HIV testing data. The approximate expressions are valid provided there are limited variations in the practice of concurrent stable partnerships across specific sub-populations that pertain to these measures. For practical purposes these approximations provide satisfactorily precise estimates for these measures.

Let's consider a hypothetical case study where in a population of 1000 sexually active individuals:

- 160 individuals are HIV infected.
- 550 individuals are engaged in 330 stable sexual partnerships.

Among those individuals in stable sexual partnerships:

- 82 are HIV infected.
- 28 are engaged in HIV sero-discordant partnerships (in 35 stable sero-discordant partnerships with 30 HIV sero-negative persons).
- 54 are engaged in HIV sero-concordant positive partnerships (in 34 HIV sero-concordant positive partnerships)

Discordancy measures: Numerical calculations

1- Exact calculation of P_{all} :

$$P_{all} = \frac{\text{number of stable HIV discordant partnerships}}{\text{total number of stable partnerships}}$$

$$= \frac{35}{330} = 10.6\%$$

2- Exact calculation of $P_{discord}$:

$$P_{discord} = \frac{\text{number of stable HIV discordant partnerships}}{\text{total number of stable partnerships with at least one HIV infected individual}}$$

$$= \frac{35}{35+34} = 50.7\%$$

3- Exact calculation of I_{all} :

$$I_{all} = \frac{\text{number of individuals in stable HIV discordant partnerships}}{\text{total number of individuals in the sexually active population}}$$

$$= \frac{28+30}{1000} = 5.8\%$$

Approximation for I_{all} :

$I_{all} \cong$ fraction of the sexually active population in stable partnerships $\times$ P_{all}

$$= \frac{550}{1000} \times 10.6\% = 5.8\%$$

Both the exact and approximate calculations for this measure yielded similar results.

4- Exact calculation of I_{pos} :

$$I_{pos} = \frac{\text{number of HIV infected individuals in stable HIV discordant partnerships}}{\text{total number of HIV infected individuals}}$$

$$= \frac{28}{160} = 17.5\%$$

Approximation for I_{pos} :

$I_{pos} \cong$ fraction of infected persons in stable partnerships $\times$

$$\frac{\text{number of stable HIV discordant partnerships}}{\text{number of stable HIV discordant partnerships} + 2 \times \text{number of stable HIV concordant positive partnerships}}$$

$$= \frac{82}{160} \times \frac{35}{35 + (2 \times 34)} = 17.4\%$$

Alternatively this same approximation can be calculated using

$$I_{pos} \cong \text{ fraction of infected persons in stable partnerships } \times \frac{P_{all}}{P_{all} + 2 \times \text{prevalence of stable HIV concordant positive partnerships}}$$

$$= \frac{82}{160} \times \frac{\frac{35}{330}}{\frac{35}{330} + \left(2 \times \frac{34}{330}\right)} = 17.4\%$$

Both the exact and approximate calculations for this measure yielded similar results.

5- Exact calculation of the abundance of HIV infected individuals who can transmit the infection to their uninfected partners:

The abundance of HIV infected individuals who can transmit the infection to their uninfected partners is given by

$$= \frac{\text{number of HIV infected individuals in stable HIV discordant partnerships}}{\text{total number of individuals in the sexually active population}}$$

$$= \frac{28}{1000} = 2.8\%$$

Approximation for the abundance of HIV infected individuals who can transmit the infection to their uninfected partners:

This measure can be approximately given by $\frac{1}{2} I_{all}$. This can be done either using the exact definition for I_{all}:

$$\cong \frac{1}{2} I_{all}$$

$$= \frac{1}{2} \times \frac{\text{number of individuals in stable HIV discordant partnerships}}{\text{total number of individuals in the sexually active population}}$$

$$= \frac{1}{2} \times \frac{28 + 30}{1000} = 2.9\%$$

Or using the approximation for I_{all}:

$$\cong \frac{1}{2} I_{all}$$

$$\cong \frac{1}{2} \times \text{fraction of the sexually active population in stable partnerships } \times P_{all}$$

$$= \frac{1}{2} \times \frac{550}{1000} \times \frac{35}{330} = 2.9\%$$

6- Exact calculation of P_{pos} :

$$P_{pos} = \frac{\text{number of stable partnerships affected by HIV}}{\text{number of stable partnerships}}$$
$$= \frac{34+35}{330} = 20.9\%$$

Alternatively this measure can be expressed as

$$P_{pos} = \frac{P_{all}}{P_{discord}}$$
$$= \frac{\left(\frac{35}{330}\right)}{\left(\frac{35}{34+35}\right)} = 20.9\%$$