

Supplemental Information

1. Angular relations necessary to generate point group symmetry related projection directions for orthoaxial projections.

For C_n symmetry, in order to generate necessary all symmetry-related projection directions from an orthoaxial projection direction $(\varphi, \theta = 90^\circ, \psi)$ within the unique range, the transformations are (Fig.S2):

1. for n even, unique range $\varphi \in [0, 360/n)$: $\varphi_k = k \frac{360}{n} + \varphi$, $k = 0, \dots, \frac{n}{2} - 1$,
2. for n odd, unique range $\varphi \in [0, 360/2n)$: $\varphi_k = k \frac{360}{n} + \varphi$, $k = 0, \dots, n-1$.

For D_n symmetry, in order to generate necessary all symmetry-related projection directions from an orthoaxial projection direction $(\varphi, \theta = 90^\circ, \psi)$ within the unique range, the transformations are (Fig.S3):

1. for n even, unique range $\varphi \in [0, 360/2n)$, we have $k = 0, \dots, n-1$, and

- 1.1. for k even, $\varphi_k = \frac{k}{2} \frac{360}{n} + \varphi$

- 1.2. for k odd, additional change of in-plane rotation,

$$\begin{cases} \varphi_k = \left[\frac{k}{2} \right] \frac{360}{n} - \varphi + 180 \\ \psi_k = \psi + 180 \end{cases}, \text{ where } [] \text{ means integer part of the division,}$$

2. for n odd, unique range $\varphi \in [0, 360/4n)$, we have $k = 0, \dots, 2n-1$, and

- 2.1. for $k \bmod 4 = 0$, $\varphi_k = \left[\frac{k}{4} \right] \frac{360}{n} + \varphi$,

- 2.2. for $k \bmod 4 = 1$, $\begin{cases} \varphi_k = \left[\frac{k}{4} \right] \frac{360}{n} + \frac{360}{2n} + 180 - \varphi \\ \psi_k = \psi + 180 \end{cases}$,

- 2.3. for $k \bmod 4 = 2$, $\varphi_k = \left[\frac{k}{4} \right] \frac{360}{n} + \frac{360}{2n} + 180 + \varphi$,

$$2.4. \text{ for } k \bmod 4 = 3, \begin{cases} \varphi_k = \left[\frac{k}{4} \right] \frac{360}{n} + 2 \frac{360}{2n} - \varphi \\ \psi_k = \psi + 180 \end{cases} .$$

As can be seen from the above equations, in order to generate all necessary orthoaxial projections given a subset in unique range, we need to consider three operations on projection images: mirror, rotation by 180° , and combined mirror and rotation by 180° . Based on simple representations of four possible orientations (Fig.S1), we can illustrate the equations defining relations between orthoaxial projections using simple schematics (Figs.S2 and S3).

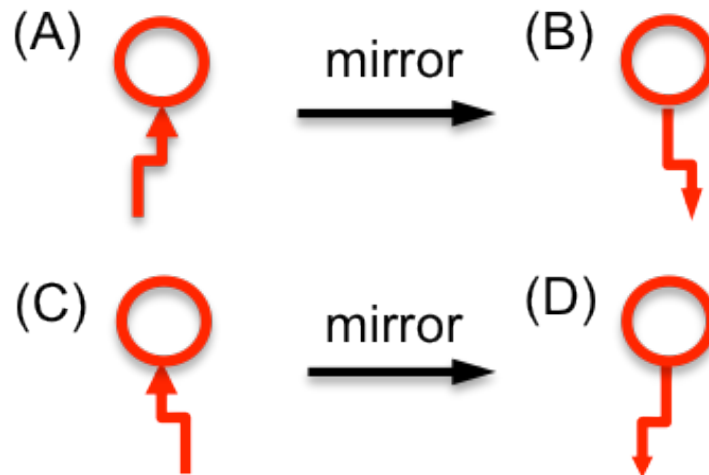


Figure S1: Overview of all four possible orientations of an orthoaxial projection within the unique range: (A) the original projection, (B) mirrored version of the original projection, (C) the 180° in-plane rotated original projection, and (D) mirrored version of the 180° in-plane rotated projection.

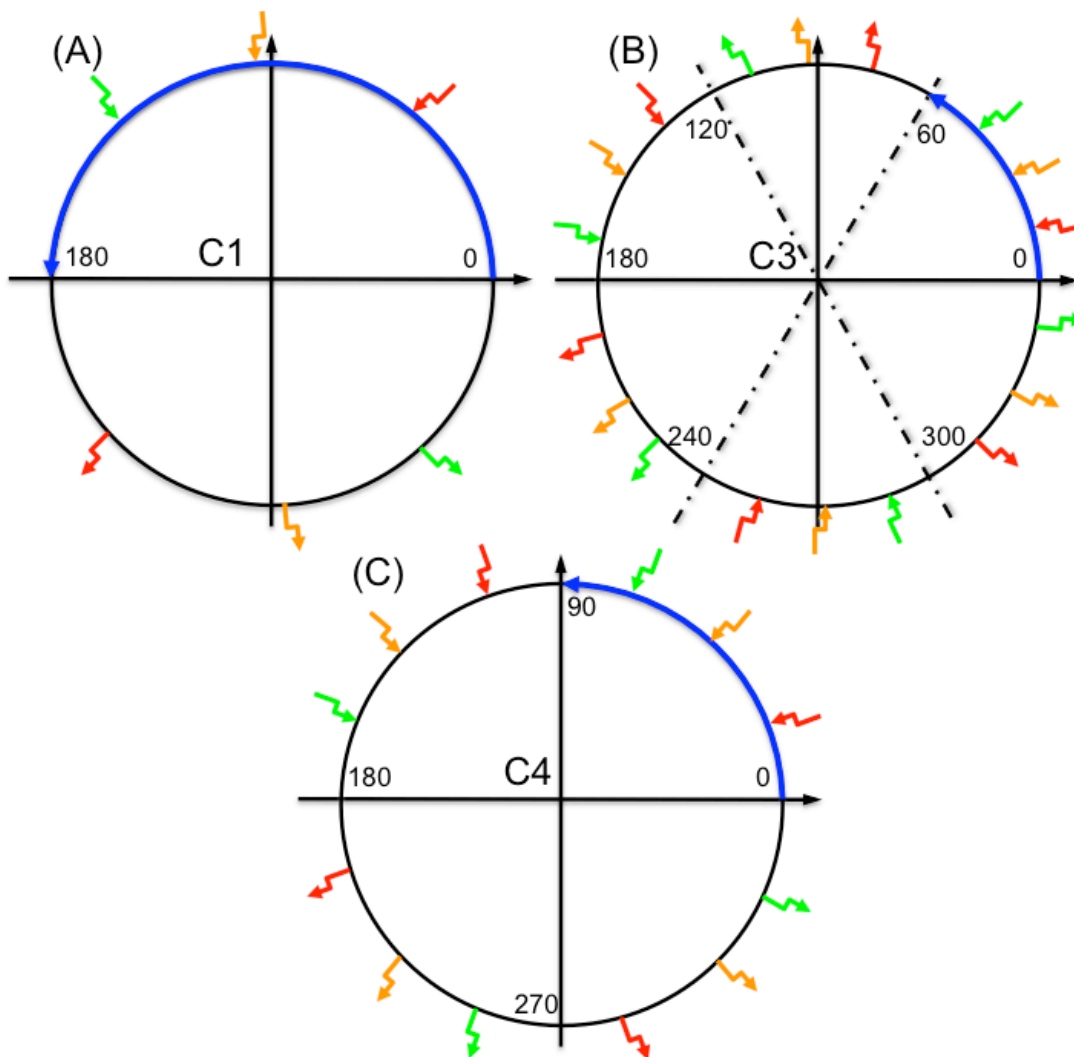


Figure S2: Schematic diagram of relations between orthoaxial projections within the full angular range ($\varphi \in [0, 360^\circ)$) for (A) C1 symmetry, (B) C3 symmetry, and (C) C4 symmetry. The blue arc represents the unique angular range. The progression of φ angle within the unique range is illustrated by three different-color arrows. Arrows outside the unique range with same color denote projections related to the same projection of the unique range.

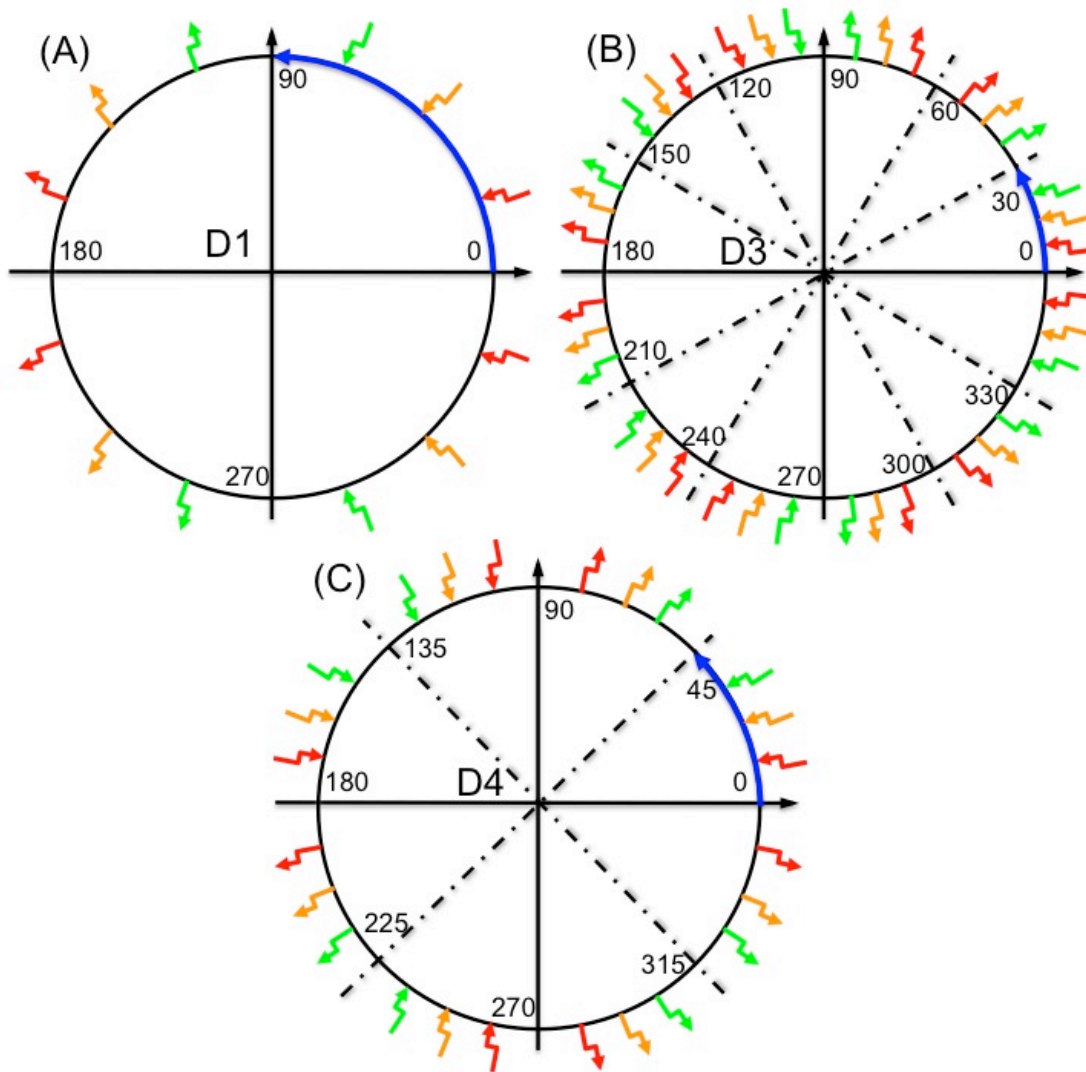


Figure S3: Schematic diagram of relations between orthoaxial projections within the full angular range ($\varphi \in [0, 360^\circ)$) for (A) D1 symmetry, (B) D3 symmetry, and (C) D4 symmetry. The blue arc represents the unique angular range. The progression of φ angle within the unique range is illustrated by three different-color arrows. Arrows outside the unique range with same color denote projections related to the same projection of the unique range.

2. A Python program that computes pixel adjustment factor given original pixel size, rise, and segment length:

```
#!/usr/bin/env python

def match_pixel_rise(dz, px, nz, rele = 0.1):
    # find pixel size closest to the given one (px)
    # such that rise (dz) is approximately equal to integer number of pixels
    # Input:
    # dz - axial rise [Angstrom]
    # px - pixel size [Angstrom]
    # nz - z length of the segment [pixel]
    # rele - relative error of the approximation
    # Output:
    # q - pixel size adjustment factor
    # error - relative error of resulting approximation

    dnz = nz*px
    # odd number of full disks in the segment
    ndisk = 2*((int(dnz/dz)-1)//2) + 1
    # However, I believe we should only use half of them,
    # because error increase counting from the center of the volume
    ndisk = (int(dnz/dz)-1)//2

    q = 1.0
    for i in xrange(900000):
        q = 1.0 - 0.000001*i
        error = ((int(ndisk*dz/q/px) - ndisk*dz/q/px)/px)**2
        if( error < rele ): return q,error
    return -1.0,-1.0

# execute program
dz = 27.6
px = 1.31
nz = 220

o = match_pixel_rise(dz, px, nz, 0.001)
print o

# The resampling is done using function resample of SPARX system:

#vn = resample( vo , 1.0/o[0] )
```