Supporting Material for "Actin network growth under load"

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Derivation of the average lifetime τ and average resistive force f_a of a filament-nucleator link

We showed in the main text that the growth of an actin network from a surface imposes a dynamic loading, $f_\ell(t)$, on the filament-nucleator attachment points (links) at the surface. Kramers rate theory [1] relates the instantaneous dissociation rate, $k_{id}(t)$, of the filament and the nucleator to the time-dependent load applied on the filament-nucleator link, so that

$$k_{id}(t) = k_d^0 \exp\left(\frac{f_\ell(t) b}{K_B T}\right) . \tag{1}$$

For an arbitrary instantaneous dissociation rate $k_{id}(t)$, we showed in the main text that the probability distribution $p(t)$ for the lifetime of a filament-nucleator link reads

$$p(t) = k_{id}(t) \exp\left(-\int_0^t dt'\, k_{id}(t')\right) , \tag{2}$$

which can be conveniently rewritten as

$$p(t) = -\frac{d}{dt}\left[\exp\left(-\int_0^t dt'\, k_{id}(t')\right)\right] . \tag{3}$$

In order to evaluate the average resistive force per attached filament, we show in the main text that it is convenient to evaluate the probability distribution $p(\delta_\ell)$ that a filament-nucleator link is stretched a length δ_ℓ which, for a general instantaneous dissociation rate $k_{id}(t)$, reads

$$p(\delta_\ell) = \frac{\exp\left(-\int_0^{t(\delta_\ell)} dt'\, k_{id}(t')\right)}{\int_0^\infty d\delta_\ell \, \exp\left(-\int_0^{t(\delta_\ell)} dt'\, k_{id}(t')\right)} , \tag{4}$$

where $t(\delta_\ell)$ is the inverse function of $\delta_\ell(t)$.

Using the general expressions above we now derive explicit expressions for the lifetime probability distribution, the average lifetime of a filament-nucleator and its average resistive force, in all regimes derived in the main text (see Fig. 2 of the main text).

Link-dominated uncoupled regime

In this regime, the deformations of different filament-nucleator links are not coupled and the network is rigid enough so that the filament-nucleator links assume all the deformation. In this case, the instantaneous force on each filament-nucleator link is $f_\ell(t) = \kappa v_p t$ (see main text). Using the explicit form for $f_\ell(t)$ and Eqs. 1, 2 we obtain:

$$p(t) = k_d^0 \exp\left[k_\ell t + \frac{k_d^0}{k_\ell}\left(1 - \exp\left(k_\ell t\right)\right)\right] \;, \tag{5}$$

where k_ℓ corresponds to the rate at which attached filaments are loaded by network growth (loading rate) and is given by

$$k_\ell = \frac{\kappa v_p b}{K_B T} \;. \tag{6}$$

The characteristic scale for the network growth velocity, v_p^0, defines the natural loading rate as $k_\ell^0 = \frac{\kappa v_p^0 b}{K_B T}$.

Once the probability distribution $p(t)$ is known, we can evaluate the average lifetime τ of an attached filament:

$$\tau = \int_0^\infty dt\, t\, p(t) = \frac{1}{k_\ell} \exp\left(\frac{k_d^0}{k_\ell}\right) \mathrm{E}_1\left(\frac{k_d^0}{k_\ell}\right) \;, \tag{7}$$

where $\mathrm{E}_n(x)$ ($n \in \mathcal{N}$) is the exponential integral function, defined as $\mathrm{E}_n(x) \equiv \int_1^\infty dz\, e^{-xz}/z^n$. The average detachment rate of attached filaments is simply given by the inverse of the average lifetime τ, so that

$$k_d = \frac{1}{\tau} = k_\ell \frac{\exp\left(-\frac{k_d^0}{k_\ell}\right)}{\mathrm{E}_1\left(\frac{k_d^0}{k_\ell}\right)} \;. \tag{8}$$

In order to calculate the average resistive force performed by a single attached filament, f_a, we first need to know the probability distribution, $p(\delta_\ell)$, for an attached filament-nucleator link to be stretched a length δ_ℓ. In the regime under consideration, the stretch of a filament-nucleator link is given by $\delta_\ell(t) = v_p t$ (see main text) and, using Eq. 4, we obtain

$$p(\delta_\ell) = \frac{1}{\delta_\ell^0} \frac{\exp\left(-\frac{k_d^0}{k_\ell}\exp\left(\frac{\delta_\ell}{\delta_\ell^0}\right)\right)}{\mathrm{E}_1\left(\frac{k_d^0}{k_\ell}\right)} \;, \tag{9}$$

where $\delta_\ell^0 \equiv K_B T/\kappa b$ is the characteristic deformation of a filament-nucleator link. Using the probability distribution $p(\delta_\ell)$, the average value of the resistive force per attached filament reads

$$\begin{aligned} f_a &= \int_0^\infty d\delta_\ell\; f_\ell(\delta_\ell)\, p(\delta_\ell) = \kappa \int_0^\infty d\delta_\ell\, \delta_\ell\, p(\delta_\ell) \\ &= \frac{K_B T}{b} \frac{\int_1^\infty dz\, \frac{\ln z}{z} \exp\left(-\frac{k_d^0}{k_\ell} z\right)}{\mathrm{E}_1\left(\frac{k_d^0}{k_\ell}\right)} \;. \end{aligned} \tag{10}$$

Elasticity-dominated uncoupled regime

In a similar way as in the *link-dominated uncoupled regime*, in the present case the deformations induced by filaments attached to nucleators at the surface are not coupled to each other. In contrast to the *link-dominated uncoupled regime* though, in this case the filament-nucleator link is very rigid compared to the actin network, meaning that the deformation induced by the attachment of a filament at the surface is essentially assumed by the network. The instantaneous force on each filament-nucleator link is then $f_\ell(t) = E(v_p t)^2$ (see main text).

We now derive the expressions for the lifetime of a filament-nucleator link and its average resistive force. Combining Eq. 2 and $f_\ell(t) = E(v_p t)^2$ one obtains the probability distribution of filament-nucleator lifetime in this regime:

$$p(t) = k_d^0 \exp\left[(k_\ell t)^2 - \frac{\sqrt{\pi}}{2}\frac{k_d^0}{k_\ell}\,\mathrm{Erfi}\,(k_\ell t)\right] , \tag{11}$$

where $\mathrm{Erfi}(x)$ is the imaginary error function, defined as $\mathrm{Erfi}(x) \equiv \frac{2}{\sqrt{\pi}}\int_0^x dz\, e^{z^2}$, and k_ℓ corresponds to the rate at which attached filaments are loaded by the network growth (loading rate) that, in this regime, is given by

$$k_\ell = \sqrt{\frac{Eb}{K_B T}}v_p \; . \tag{12}$$

The characteristic scale for the network growth velocity, v_p^0, defines the natural loading rate in the *elasticity-dominated uncoupled regime* as $k_\ell^0 = \sqrt{\frac{Eb}{K_BT}}v_p^0$.

Once the probability distribution $p(t)$ is known, we can evaluate the average lifetime τ of an attached filament:

$$\tau = \int_0^\infty dt\, t\, p(t) = \frac{1}{k_d^0}\int_0^\infty dt\, \exp\left[-\frac{\sqrt{\pi}}{2}\frac{k_d^0}{k_\ell}\mathrm{Erfi}\left(\frac{k_\ell}{k_d^0}t\right)\right] \; . \tag{13}$$

The average detachment rate of attached filaments is simply given by the inverse of the average lifetime, i.e. $k_d = 1/\tau$.

In order to calculate the average resistive force of an attached filament, f_a, it is first necessary to evaluate the probability distribution, $p(\delta)$, for the network to be stretched a length δ due to the attachment of a filament to a nucleator. In this regime, the stretch of the filament-nucleator link is of second order and it cannot be used to calculate the average resistive force. Using Eq. 11 from the main text in the limit considered here, the dependence of the network deformation δ on time is simply $\delta(t) = v_p t$. This is to say that the network assumes all the deformation. Using the relation $\delta(t)$ above, the probability distribution $p(\delta)$ reads

$$p(\delta) = \sqrt{\frac{Eb}{K_BT}}\frac{\exp\left[-\frac{\sqrt{\pi}}{2}\frac{k_d^0}{k_\ell}\mathrm{Erfi}\left(\sqrt{\frac{Eb}{K_BT}}\,\delta\right)\right]}{\int_0^\infty d\tilde{\delta}\, \exp\left[-\frac{\sqrt{\pi}}{2}\frac{k_d^0}{k_\ell}\mathrm{Erfi}(\tilde{\delta})\right]} , \tag{14}$$

where the typical scale of network deformation is given by $\sqrt{\frac{K_BT}{Eb}}$. Using the probability distribution $p(\delta)$, the fact that $f_\ell(t) = E(v_p t)^2$ and $\delta(t) = v_p t$, the average resistive force per link, f_a, reads

$$
\begin{aligned}
f_a &= \int_0^{\infty} d\delta\; f_\ell(\delta)\, p(\delta) = E \int_0^{\infty} d\delta\, \delta^2\, p(\delta) \\
&= \frac{K_B T}{b} \frac{\int_0^{\infty} d\tilde{\delta}\, \tilde{\delta}^2\; \exp\left[-\frac{\sqrt{\pi}}{2}\frac{k_d^0}{k_\ell}\mathrm{Erfi}(\tilde{\delta})\right]}{\int_0^{\infty} d\tilde{\delta}\; \exp\left[-\frac{\sqrt{\pi}}{2}\frac{k_d^0}{k_\ell}\mathrm{Erfi}(\tilde{\delta})\right]}\,. \qquad (15)
\end{aligned}
$$

Elasticity-dominated coupled regime

In this case, the link force f_ℓ does not depend on time and is given by $f_\ell = E/\rho_a$. This scenario is equivalent to the limit of a static network discussed in the main text, the only difference being that for a static network the force per link is $f_\ell = f_a = \sigma_{nn}/\rho_a$ and for the present case is $f_\ell = f_a = E/\rho_a$. In other words, each filament-nucleator link feels a constant average force E/ρ_a, independently of the external stress applied to the surface. The relevant parameters in this regime are, in analogy with the static network limit, k_a/k_d^0 and $Eb/\rho_f K_B T$.

References

[1] van Kampen NG (2004) *Stochastic Processes in Physics and Chemistry.* (North Holland, Amsterdam).