Derivation of Equation for Diameter Estimation

Parabolic Flow Profile Approximation

For this approach, the equations for fiber diameter prediction are adapted from volume conservation, assuming known inlet and outlet velocities. A parabolic flow profile in the tube is approximated by

$$v_z(r) = \frac{\frac{\partial p}{\partial z}}{4\eta}(R^2 - r^2) = \frac{K}{4\eta}(R^2 - r^2) = v_0(1 - \frac{r^2}{R^2}), \quad (2)$$

where v is velocity, p is pressure, z is the direction of flow, η is viscosity, R is radius, $K=\frac{\partial p}{\partial z}$ and v_0 is the maximum velocity at the center of the tube, as defined by

$$v_0 = \frac{KR^2}{4\eta}. \quad (3)$$

Volume flow (V) in a cylindrical tube is given by

$$V = \int_0^R v_z(r) 2\pi r dr. \quad (4)$$

For this system

$$\frac{V_{silk}}{V_{outlet}} = \frac{\int_0^{r_{fiber}} v_z(r) 2\pi r dr}{\int_0^R v_z(r) 2\pi r dr} = \frac{2r_{fiber}^2}{R^2} - \frac{r_{fiber}^4}{R^4} \quad (5)$$

and

$$R = \sqrt{\frac{Q_{outlet}}{\pi}} \quad (6)$$

where Q is cross-sectional area, giving

$$\frac{V_{silk}}{V_{outlet}} = \frac{2\pi r_{fiber}{}^{2}}{Q_{outlet}} - \frac{\pi^{2} r_{fiber}{}^{4}}{Q_{outlet}{}^{2}}. \qquad (7)$$

Since $r_{fiber}{}^{2} << Q_{outlet}$ this can be simplified to

$$\frac{V_{silk}}{V_{outlet}} = \frac{2\pi r_{fiber}{}^{2}}{Q_{outlet}} \qquad (8)$$

and therefore

$$d = \sqrt{\frac{2}{\pi}\frac{V_{silk}}{V_{outlet}} Q_{outlet}}\,. \qquad (9)$$