Supporting Information

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SI Materials and Methods

Multitaper Spectral Analysis. The pieces were converted to point-process data as described in *Materials and Methods* (i.e., a vector consisting of the locations in time of all note onsets in each piece). From the raster representation of a piece i with onsets at $\{t^i_j\}$, $j = \{1,2,3,\ldots,N_i\}$, and $t^i_N = T_i$, we compute the multitaper spectra using the K taper functions $\{h_k(t)\}$ as,

$$\hat{S}_i(f) = \sum_{k=1}^{K} \hat{S}_{i,k}(f)$$

where,

$$\hat{S}_{i,k}(f) = \frac{1}{N_i} J^*_{i,k}(f) J_{i,k}(f) = \frac{1}{N_i} |J_{i,k}(f)|^2$$

and,

$$J_{i,k}(f) = \sum_{j=1}^{N_i} h_k(t^i_j) e^{-i2\pi f t^i_j} - \frac{N_i H_k(f)}{T_i}$$

Here, $H_k(f)$ is the Fourier transform of $h_k(t)$. Here, we use three Slepian sequences (K = 3) to compute the spectra.

Compared with conventional spectral analysis, which uses a single type of analysis window, multitaper spectral analysis computes the spectrum several times with different windows; the final estimate is given by taking the average. In conventional spectral analysis, the window controls the trade-off between spectral resolution and spectral leakage between analysis channels. In multitaper spectral analysis, Slepian functions are used as windows, which are orthogonal and are designed to reduce spectral leakage. This approach leads to reduced estimate variance, compared with calculating the power spectrum using a single taper in a Fourier transform computed on the entire signal or using Welch's method.

Detrended Fluctuation Analysis and Hurst Exponent Calculation. Detrended fluctuation analysis (DFA) and Hurst exponent calculation were computed as additional tests of $1/f$ structure; these are often used to confirm $1/f$ structure after it has been revealed by spectral analysis. It has been shown that the slope of the log power spectrum (i.e., β in $[1/f]^\beta$) for time series can be estimated using these techniques (1–6).

For each song, as with the multitaper analysis, all voices were collapsed into a single time series representing the onset times of each note. Next, this time series was converted to durations by measuring the time between successive onsets. This duration time series was used for the DFA and Hurst analysis.

DFA is computed by converting the time series into a cumulative time series. This new series is then divided into windows of varying length (L); for each, a first-order polynomial was fit. The root-mean-square deviation from the trend of the cumulative series gives the fluctuation [F(L)]. A scatter plot was made of the log[F(L)] vs. log(L), for various values of L. The slope of the regression curve gives an estimate of α. An α-value of 0.5 indicates white noise, and an α-value of 1 indicates $1/f$ structure with a β-value of 1. Our estimate of β, averaged across all pieces, using this technique was 0.84, very similar to the estimate 0.85 derived using multitaper spectral analysis.

The Hurst exponent is based on calculating the rescaled range of the time series for all partial time series of length n. First, the mean is subtracted from the time series. Next, the cumulative time series is computed. The range and SD for the time series is calculated for all n less than the total length. The ratio of the range to the SD is referred to as the rescaled range and is averaged for all partial time series of length n. The curve of n versus the rescaled range is then fit using a power law curve, with the exponent giving the Hurst exponent (H), where $\beta = 2H - 1$. Using this procedure on the duration time series of each piece, the average β was 0.88, nearly identical to the DFA and multitaper estimates.

Statistics, Bootstrapping, and Control Experiment. In the experiments reported here, we were interested in (*i*) computing the statistical significance of each power exponent β and (*ii*) computing the confidence intervals for β by genre and composer. To accomplish this, we used a bootstrapping approach (4) to compute the null distribution and estimate the statistical significance of obtaining the estimated β-parameter estimates for each genre and composer. Shuffled versions of each musical composition were created by randomly permuting the note onsets and rasterizing the resulting sequence. The shuffled versions of the compositions control for the possibility that a random collection of durations would give rise to $1/f$, or even that a randomly ordered collection of the durations used in the composition would give rise to $1/f$. The shuffling also allows for a tighter experimental design in which each stimulus can serve as its own control. Instead, we find that only this particular arrangement of durations gives rise to $1/f$, strongly supporting the point that $1/f$ characterizes not all sound sequences, but those that are considered to be well-formed. This is a rigorous and widely used method that requires minimal assumptions about the underlying distribution and is the recommended procedure when the testing the significance of individual samples and when the theoretical distribution of a measure is unknown (4).

The Wilcoxon signed-rank test was chosen to compare distributions because it is a nonparametric (distribution independent) test, and so does not require that we make any assumptions about the underlying distribution of the data. Although not as powerful as parametric or General Linear Model-based tests, it is a more conservative test, far less prone to type I errors (7). The fact that the results reached significance at $P < 0.01$ with a conservative test, after adjusting for multiple comparisons, strengthens our claim of an effect.

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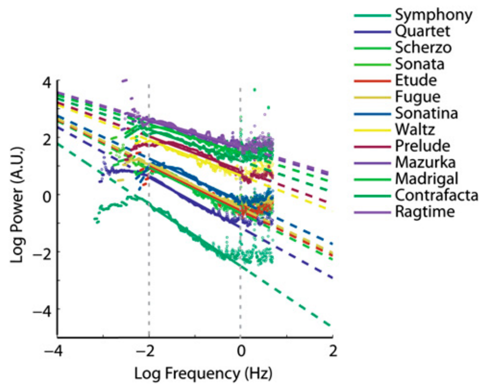

Fig. S1. Average rhythm spectra for select genres ordered by spectral exponent. Spectra are displaced by different amounts along the *y* axis (based on slope) for clarity of presentation. Other conventions are as in Fig. 2.

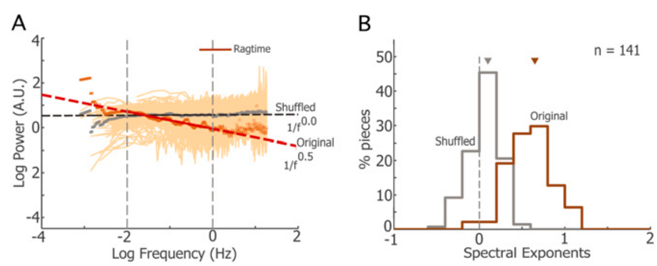

Fig. S2. The 1/*f* rhythm spectra for Ragtime (compare with Fig. 3 in main text) (*A*) Rhythm spectra for ragtime. Average spectra (brown points) and linear fit (brown) to average spectrum in the frequency range of 0.01 to 1 Hz. Faded orange lines represent spectra of individual pieces. Gray data represent spectra of shuffled rhythms. Other conventions are as in Fig. 2*B*. (*B*) Distribution of rhythm spectral exponents obtained by linear fits to individual pieces (brown), and for the corresponding shuffled rhythms (gray). Inverted triangle represents median exponents. Dashed vertical line: $\beta = 0$.

Table S1. Summary statistics for compositions of select genres ordered by decreasing mean spectral exponents

Genre	No. of movements (no. of voices)	Note durations (s)	Piece lengths (s)	β (original)	β (shuffled)	*P* value $\beta_{orig}>\beta_{shuf}$
Symphony	25 (2–21)	0.42 (0.37)	671 (417)	1.04 (0.22)	0.03 (0.13)	0.000
Quartet	723 (4–6)	0.23 (0.29)	347 (191)	0.85 (0.27)	0.00 (0.15)	0.000
Scherzo	11 (2–5)	0.12 (0.17)	211 (169)	0.81 (0.18)	0.10 (0.24)	0.001
Sonata	586 (1–8)	0.20 (0.25)	163 (93)	0.78 (0.37)	0.05 (0.22)	0.000
Etude	19 (2)	0.12 (0.16)	101 (31)	0.77 (0.40)	0.07 (0.16)	0.000
Fugue	51 (2–5)	0.14 (0.17)	164 (82)	0.75 (0.21)	0.01 (0.17)	0.000
Sonatina	48 (2–3)	0.15 (0.14)	95 (40)	0.67 (0.39)	0.05 (0.16)	0.000
Waltz	10 (2–3)	0.19 (0.17)	155 (66)	0.66 (0.36)	0.03 (0.18)	0.004
Prelude	56 (1–4)	0.26 (0.30)	115 (71)	0.62 (0.38)	0.09 (0.25)	0.000
Mazurka	50 (2)	0.20 (0.21)	138 (79)	0.53 (0.37)	0.03 (0.14)	0.000
Madrigal	24 (3–7)	0.45 (0.37)	146 (50)	0.53 (0.31)	0.02 (0.17)	0.000
Contrafacta	21 (5–7)	0.52 (0.36)	149 (47)	0.51 (0.33)	0.00 (0.14)	0.000
Ragtime	141 (2–3)	0.15 (0.14)	131 (53)	0.48 (0.26)	0.05 (0.17)	0.000

Values outside parentheses indicate means. Values inside parentheses indicate SDs across compositions (except for number of voices, which indicates a range) for that row.

Table S2. Summary statistics for compositions of select composers ordered by decreasing mean spectral exponents

Composer (period)	No. of movements (no. of voices)	Note durations (s)	Piece lengths (s)	β (original)	β (shuffled)	*P* value $\beta_{orig}>\beta_{shuf}$
Beethoven (1770–1827)	115 (2–5)	0.11 (0.17)	313 (162)	1.05 (0.25)	0.02 (0.14)	0.000
Vivaldi (1678–1741)	127 (2–8)	0.25 (0.21)	282 (153)	0.95 (0.37)	0.10 (0.29)	0.000
Frescobaldi (1583–1643)	40 (2–5)	0.44 (0.45)	237 (71)	0.94 (0.17)	0.08 (0.14)	0.000
Haydn (1732–1809)	230 (2–21)	0.36 (0.33)	371 (239)	0.84 (0.27)	0.00 (0.14)	0.000
Corelli (1653–1713)	141 (2–8)	0.36 (0.34)	160 (73)	0.76 (0.37)	0.11 (0.24)	0.000
Schubert (1797–1828)	14 (2–4)	0.15 (0.23)	174 (204)	0.76 (0.37)	0.04 (0.23)	0.000
Scarlatti (1685–1757)	54 (2)	0.13 (0.13)	116 (43)	0.68 (0.36)	0.01 (0.15)	0.000
Bach (1685-1750)	167 (1–13)	0.13 (0.20)	161 (124)	0.66 (0.37)	0.06 (0.23)	0.000
Chopin (1810–1849)	72 (2–3)	0.17 (0.19)	132 (92)	0.65 (0.42)	−0.04 (0.20)	0.000
Monteverdi (1567–1643)	22 (5–11)	0.53 (0.37)	157 (60)	0.55 (0.34)	0.00 (0.15)	0.000
Mozart (1756–1791)	173 (2–4)	0.29 (0.29)	198 (238)	0.54 (0.40)	0.03 (0.15)	0.000
Joplin (1868–1917)	90 (2–3)	0.15 (0.14)	133 (54)	0.49 (0.26)	0.02 (0.15)	0.000

The complete list of composers analyzed is J. S. Bach, Beethoven, Brahms, Buxtehude, Byrd, Chopin, Clementi, Corelli, Dufay, Dunstable, John Field, Flecha, Foster, Frescobaldi, Giovannelli, Grieg, Haydn, Friedrich Himmel, Isaac, Joplin, Josquin, Landini, Lassus, Liszt, MacDowell, Mendelssohn, Monteverdi, Mozart, Pachelbel, Prokoviev, Ravel, Scarlatti, Schubert, Schumann, Scriabin, Sinding, Turpin, Vecchi, Victoria, and Vivaldi. Other conventions are as in Table S1.

SI Appendix 1. Rhythm spectra for musical compositions grouped by genre (16 genres). Figures in each page show a different genre. In each figure, the *Upper Left* panel shows the individual rhythm spectra (light grey), mean rhythm spectrum for that genre (dark circles), and linear fit to the mean rhythm spectrum (dark line). Spectra are plotted as power as a function of frequency in a log-log scale. Dashed lines: extrapolations of the mean fit. The *Upper Right* panel shows the distribution of rhythm spectral exponents for that genre across pieces, with the mean exponent indicated on the top. Similarly, the *Lower Left* and *Right* panels of each figure show the rhythm spectra and distribution of spectral exponents, respectively, for the pieces with shuffled note onsets. The *P* value below the *Lower Right* panel indicates the level of significance for the difference between the means of the exponent distribution of the original and shuffled onset rhythm spectra (Wilcoxon signed rank test). Only pieces with more than 200 notes (onsets) were analyzed.

SI Appendix 1

SI Appendix 2. Same as *SI Appendix 1*, except that pieces are grouped by composer (41 composers). Some genres and composers contributed very few pieces (less than 10) to the analysis: for these genres and composers, differences between original and shuffled rhythm spectral exponents did not always reach significance at the $P = 0.05$ level, although spectra for the original and shuffled pieces look clearly different (see for example, ballads, and composer, Josquin).

SI Appendix 2