

Text S1. Note on model identifiability and on likelihood-based inference.

Conditionally to initial inoculation frequencies, the random vectors $N^p(t_p)$, $p = 1, \dots, 40$ (8 plants x 5 sampling dates) of virus sequences obtained by HTS for each plant p at its specific time sampling t_p were independent and followed Dirichlet multinomial (DM) distributions. To write down the likelihood of our experiment, we first recall that the probability mass function

of a $DM(\lambda, N_{tot}, \theta)$ distribution has the form

$$\Pr(N^p = y | \lambda, N_{tot}, \theta) = p(y | \lambda, N_{tot}, \theta) = \frac{N_{tot}! \Gamma(\theta)}{\Gamma(N_{tot} + \theta)} \prod_{i=1}^4 \frac{\Gamma(y_i + \theta \lambda_i)}{y_i! \Gamma(\theta \lambda_i)}$$

where $y = (y_1, \dots, y_4)$, $\lambda = (\lambda_1, \dots, \lambda_4)$, and $N_{tot} = \sum_{i=1}^4 y_i$.

Next, let us describe the set of parameters underlying our statistical model. Since we had five sampling dates, i.e. $t_p \in \{6, 10, 15, 24, 35\} = \{\tau_s ; s = 1, \dots, 5\}$, the set of parameters amounted to

$\theta^s = \theta(\tau_s)$, and $\lambda^s = \lambda(\tau_s) = (\lambda_1^s, \dots, \lambda_4^s)$, $s = 1, \dots, 5$. Therefore, as $\sum_{i=1}^4 \lambda_i^s = 1$, $s = 1, \dots, 5$, the

full statistical model included 20 (=5+3x5) parameters. Actually, the 15 free parameters λ_i^s depended themselves on the 22 (=4+12+1+1+4) unknown parameters r_i , $\beta_{i,j}$, $1 \leq i \neq j \leq 4$, K , μ (recall that $\mu_{i,j} = f_{i,j}(\mu)$) and initial conditions $V_i(0)$ of ODE System 1.

Consequently, statistical identification of our statistical model would require at least 7 (=22-15) supplementary constraints on the set of parameters of ODE System 1.

Since we dealt only with virus variant frequencies, the time scale (i.e. derivatives could be defined up to a constant), the carrying capacity K and the number (or size) of virus variants were immaterial at the level of our observations and therefore could be “normalized”. K was arbitrarily set to 10^6 and $\sum_{i=1}^4 r_i = 4$. The mutation rate μ was set to 10^{-5} and, for the initial

values $V(0) = V_{tot}^{inoc} \times (\lambda_1(0), \dots, \lambda_4(0))$ of ODE System 1, V_{tot}^{inoc} was set 100 whereas $(\lambda_1(0), \dots, \lambda_4(0)) = (0.32, 0.22, 0.22, 0.24)$ corresponded to the observed frequencies of virus variants in the inoculum.

Consequently, the remaining set of 20 parameters was now formally identifiable via the following log-likelihood function (withdrawing the indices of θ^s , r_i , and $\beta_{i,j}$)

$$l(N^p(t_p), p=1, \dots, 40 | \theta, r, \beta) = \sum_{p=1}^{40} \ln(p(N^p(t_p) | \lambda(t_p | r, \beta), N_{tot}^p(t_p), \theta(t_p)))$$

Additionally, we investigated the sensitivity of the model best supported by the data (Table 1: $\mathfrak{M}_{D_1 \times C_2}$). We first analysed the model sensitivity to μ and V_{tot}^{inoc} . Sixteen cases, combining four values (10^{-6} , 10^{-5} , 10^{-4} , 10^{-3}) of μ with four values (10, 100, 1000, 10^4) of V_{tot}^{inoc} were analysed by iterating the same statistical procedure. The percentage of variations of the parameter estimates were all $< 5\%$. We next investigated the model sensitivity to a 20% random fluctuation of the initial frequencies of the variants in the inoculum $(\lambda_1(0), \dots, \lambda_4(0)) = (0.32, 0.22, 0.22, 0.24)$. The percentage of variations of the parameter estimates were also $< 5\%$ except for θ^l which reached 15%.