

## Text S4

### 1 Simulation of the full ordinary differential equation model

The equations of Modules 1-3 and the following conservation relations are taken together to perform the bifurcation analysis and the numerical integration of the ODE model.

The conservation relations are as follows:

$$MK3 = MK3_{total} - (HHMK3 + HH_pMK3 + HH_{pp}MK3 + HH_{ppp}MK3) \quad (1)$$

$$PP5 = PP5_{total} - (HH_{pppp}PP5 + HH_{ppp}PP5S + HH_{ppp}PP5 + HH_{pp}PP5 + HH_{pp}PP5S + HH_pPP5 + HH_pPP5S + HHPP5) \quad (2)$$

$$HSF1 = HSF1_{total} - (HSF1S + HSF12S + HSF13S + HHSP90_{cl} + HH + HH_p + HH_{pp} + HH_{ppp} + HH_{pppp} + HHMK3 + HH_pMK3 + HH_{pp}MK3 + HH_{ppp}MK3 + HH_{pppp}PP5 + HH_{ppp}PP5 + HH_{pp}PP5 + HH_{pp}PP5S + HH_pPP5 + HH_pPP5S + HHPP5) \quad (3)$$

$$HSE = HSE_{TOT} - (HH + HH_p + HH_{pp} + HH_{ppp} + HH_{pppp} + HHMK3 + HH_pMK3 + HH_{pp}MK3 + HH_{ppp}MK3 + HH_{pppp}PP5 + HH_{ppp}PP5 + HH_{pppp}PP5S + HH_{pp}PP5 + HH_{pp}PP5S + HH_pPP5 + HH_pPP5S + HHPP5) \quad (4)$$