Why the bigger live longer and travel farther:

animals, vehicles, rivers and the winds

Prof. Adrian Bejan
Duke University

Supplementary Document

Supplementary Information A

The scales of a two-dimensional turbulent jet are [12]

$$\overline{u}_c = U\left(\frac{x}{x_0}\right)^{-1/2} \quad \text{(A1)}$$

$$\gamma \frac{D}{x_0} = \frac{4}{3} \quad \text{(A2)}$$

where $\gamma = 7.67$ is an empirical constant. In the limit $x_0 << L$ (or $\varepsilon << 1$), the travel time from $x = x_0$ to x = L along the jet axis is

$$t \sim 3.83 \frac{D}{\varepsilon^3 U} \quad \text{(A3)}$$

The corresponding length of travel is

$$L \sim 5.75 \frac{D}{\varepsilon^2} \quad \text{(A4)}$$

Supplementary Information B

The turbulent plume [12] rising above a concentrated heat source q[W] has a centerline speed $(\overline{v}_c)$ that decreases with altitude (y),

$$\overline{v}_c = \left(K \frac{q}{y}\right)^{1/3} \quad \text{(B1)}$$

where K is a constant of order $g\beta\alpha/k$, where g is the gravitational acceleration, β is the coefficient of volumetric thermal expansion of the fluid, α is the thermal diffusivity, and k is the thermal conductivity. If the length scale of the heat source is D, then $q \sim q'''D^3$, where q''' is the volumetric heat generation rate. The highest speed occurs in the vicinity of the source, $\overline{v}_c\ (y = D)$, and the lowest is at y = L, which marks the travel of the flow. As in Eq. (2), we define the plume travel as $\overline{v}_c(y = L)/\overline{v}_c(y = D) \sim \varepsilon << 1$, therefore

$$L \sim \frac{D}{\varepsilon^3} \tag{B2}$$

The life time of the fluid packet is the integral of $dt = dy/\overline{v}_c$, from y = D to y = L,

$$t \sim \frac{3D^{1/3}}{4\varepsilon^4 (q'''K)^{1/3}} \tag{B3}$$

Equations (B2) and (B3) show that larger plumes (with larger D) travel farther and have longer life spans.

Supplementary Information C

The inefficiency of flow systems is due to finite sizes, which means finite flow resistances (fluid flow, heat transfer, mass transfer, etc.). Consider a power plant as a system (open or closed) in steady state. Inside the system there are components (e.g., heat exchangers) of surface area A, which have the function of transferring the heat current q from a hot fluid (T_H) to a cold fluid (T_L). In Fig. C1 the system is closed and executes cycles. The thermodynamic imperfection due to the surface A is measurable as the rate of entropy generation associated with the component [14]

$$\dot{S}_{gen} = \frac{q}{T_L} - \frac{q}{T_H} \cong \frac{q\Delta T}{T_L^2} \tag{C1}$$

where $\Delta T = T_H - T_L$, and $\Delta T << (T_H, T_L)$. The heat current is proportional to the size of the surface [12]

$$q = h\, A\, \Delta T \tag{C2}$$

where h is the overall convective heat transfer or mass transfer coefficient. Combining Eqs. (C1) and C2), and recognizing $T_L\dot{S}_{gen}$ as the useful power destroyed because of the irreversibility of heat transfer components (see the Gouy Stodola theorem [16]), we obtain

$$T_L \dot{S}_{gen} \cong \frac{q^2}{T_L h A} \tag{C3}$$

The power output of the plant is

$$\dot{W} = \dot{W}_{rev} - T_L \dot{S}_{gen} \tag{C4}$$

where $\dot{W}_{rev}$ is the power output in the limit of reversible operation. The second law efficiency of the power plant (η_{II}) is a number between 0 and 1 [14, 16], which after combining Eqs. (C3) and (C4) becomes

$$\eta_{II} = \frac{\dot{W}}{\dot{W}_{rev}} = 1 - \frac{q^2}{T_L \dot{W}_{rev} h A} \tag{C5}$$

Next, if we use $A^{1/2}$ as an indicator of the length scale of the power plant of mass M, then $A^{1/2}$ is proportional to $M^{1/3}$, and A scales as $M^{2/3}$. In conclusion, Eq. (C5) reduces to

$$\eta_{II} = 1 - C' M^{-k} \tag{C6}$$

where C' is a constant and k = 2/3. Equation (C6) shows that η_{II} increases monotonically with the size of the power plant system. We express the same trend analytically if we approximate Eq. (C6) locally [at a point (η_{II}, M) on the curve] with a power-law expression

$$\eta_{II} = C'' M^{\alpha} \tag{C7}$$

where C'' is a constant, and $\eta = \eta_{II}\, \eta_C$, where η_C is the Carnot efficiency of the power plant. Note that Eq. (C7) is the same as Eq. (22), where $C_1 \eta_C = C''$. If we require that Eqs. (C6) and (C7) match in value (η_{II}) and slope $(d\eta_{II} / dM)$ at the point (η_{II}, M), then

$$\frac{k}{\alpha} = \frac{1 - \eta_{II}}{\eta_{II}} \tag{C8}$$

The η_{II} data for the efficiency of modern power generation technology show that $\eta_{II} << 1$ [14, 16]. This means that α is comparable with k but smaller than k, approximately $\alpha \cong \eta_{II} k$. Note that the factors C' and C'' do not appear in Eq. (C8).

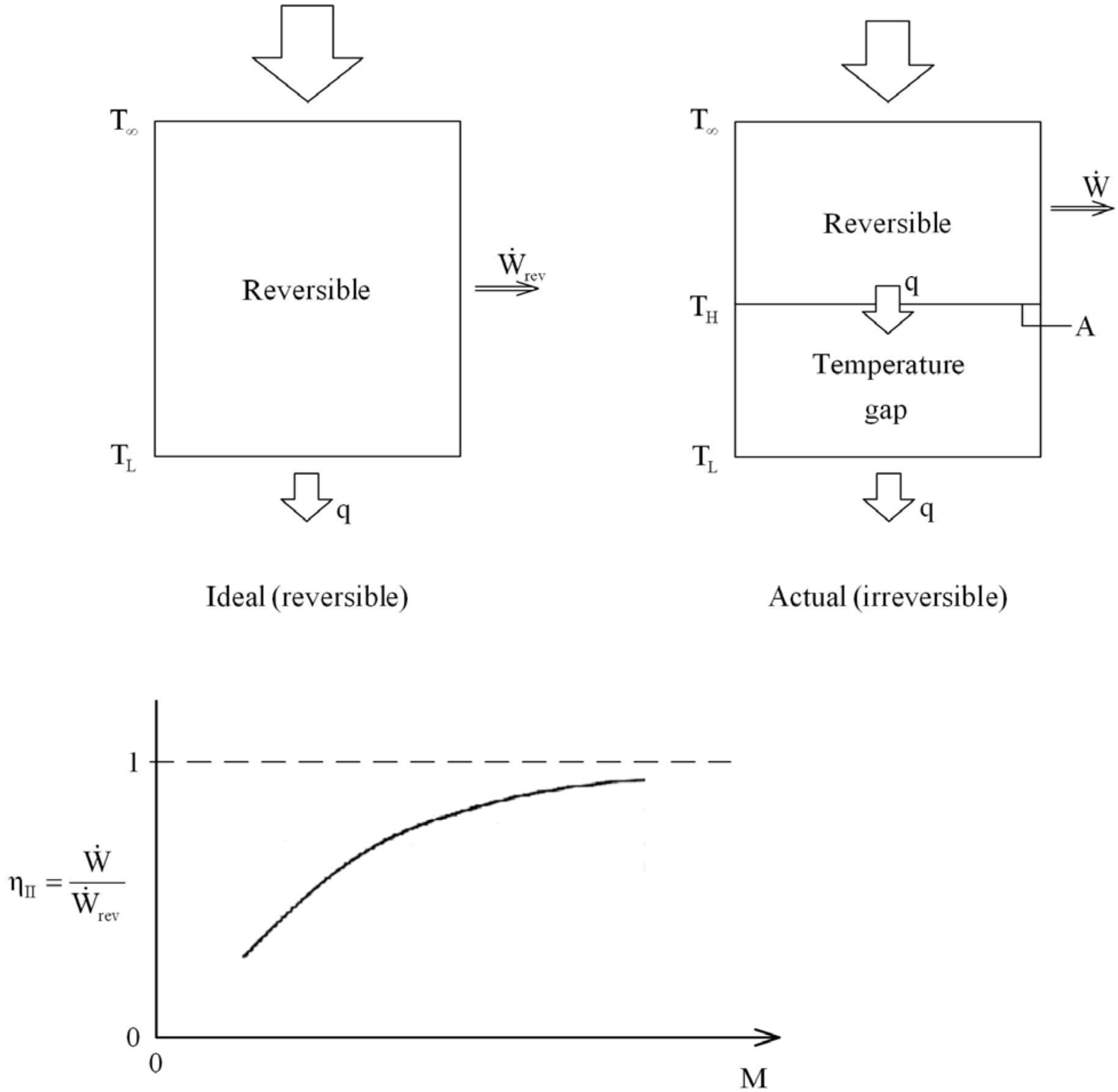
T∞
Reversible
Ẇrev
TL
q
Ideal (reversible)
T∞
Reversible
Ẇ
q
TH
A
Temperature
gap
TL
q
Actual (irreversible)
1
ηII = Ẇ / Ẇrev
0
0
M

Figure C1