# Flow Loading Induces Oscillatory Trajectories in a Bloodstream Parasite

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## SUPPLEMENTARY INFORMATION

As it is well known, there are two types of solutions to eq. (1), as depicted in the phase plot in fig. S1. If $H < \omega^2/2$, the swimmer oscillates around the upstream direction ($\phi \approx 0$) with angular frequency $\omega$. This corresponds to the closed orbits in the phase plot. For $H > \omega^2/2$, it keeps turning indefinitely. The case $H = \omega^2/2$ corresponds to the separatrix, $\partial_t \phi = \omega \cos\frac{\phi}{2}$, which is shown as the solid curve. It is straightforward to identify the closed orbits around $(\phi, \partial_t \phi) = (0,0)$ as the oscillating motion observed for upstream swimming, and the open orbits outside the separatrix with the tumbling motion observed for downstream swimming. However, a signicant part of a large subset of the open orbits correspond to downstream orientation ($\phi \approx \pi/2$). In fact, for orbits close to the separatrix, an oscillating swimmer would be directed downstream most of the time. This is not observed in the experiment, such that a more detailed discussion is necessary.

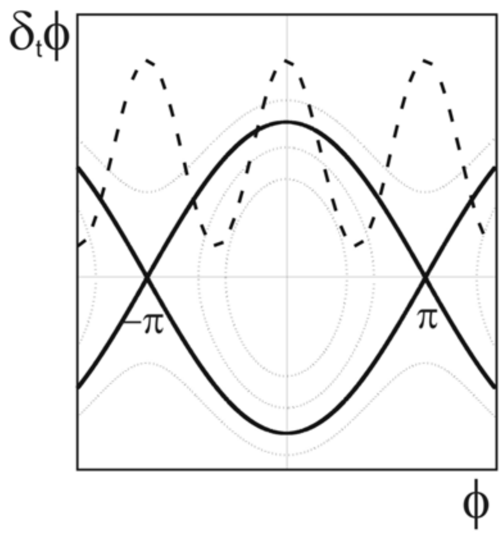


**FIG. S1**: Phase plot of trajectories of swimmer direction, $\phi(t)$. The solid curve is the separatrix, the dashed curve indicates the interaction with the channel boundary. If the channel width is of the same order the size of the swimmer, as in our case, this gives rise to strong discrimination among trajectories. We see that the dashed curve deeply dives into the family of oscillation trajectories, singling out only those which are well oriented upstream.

The wall of the channel limits the sideways motion of the swimmer. The maximum distance of its center of mass from the centerline, $r_{max}$, depends upon its orientation $\phi$, and can be written as

$$r_{\max} = \left( r_c - \frac{L}{2} \right) + \frac{1}{2}(L-l)\cos^2\phi \qquad \textbf{Equation S1}$$

This provides a boundary to the motions of the swimmer. Exploiting the linear relation between *r* and $\partial_t\phi$, we can plot this boundary in the phase plot, fig. S1. It is shown as the dashed curve, $\partial_t\phi = \delta + \Delta\cos^2\phi$. An oscillation with the maximum amplitude corresponds to the trajectory which just touches this boundary from below. We can obtain the corresponding 'energy', $E_b = H(\partial_t\phi,\phi)$ by looking for the minimum value of *H* along the boundary curve. On that curve, we have

$$H_b = (\delta + \Delta\cos^2\phi)^2 - \frac{\omega^2}{2}\cos\phi \qquad \textbf{Equation S2}$$

In order to find the extremum, we differentiate with respect to $\cos\phi$ and seek the zero. The resulting cubic equation

$$\cos^3\phi + \frac{\delta}{\Delta}\cos\phi = \frac{\omega^2}{8\Delta^2} \qquad \textbf{Equation S3}$$

has for positive $\delta$ and $\Delta$ (which we assume) one real solution, which can be written down in closed form. The corresponding amplitude, which is given by the intersection of the trajectory with the $\partial_t\phi$-axis, can be found by inserting the solution of eq. (S3) into eq. (2). In our experiments, $L \approx 2r_c$, such that $\delta \approx 0$, and thus $\cos\phi \approx \frac{1}{2}(\omega/\Delta)^{2/3}$, and we have $\Delta = G\omega_0(L-l)/r_c$. The corresponding maximal amplitude is readily found to be

$$A = \frac{1}{2}(L-l)\frac{\omega}{\Delta}\sqrt{1-\frac{3}{8}\left(\frac{\omega}{\Delta}\right)^{2/3}} \qquad \textbf{Equation S3}$$

Since $\omega$ is directly linked to the flow velocity via

$$\sqrt{\frac{GU_{\max}}{u_0}} = \frac{\Delta r_c}{\frac{\omega}{2}(L-l)} \qquad \textbf{Equation S4}$$

eq. (S3) can be used to fit our data for the oscillation amplitudes at different flow velocities. We furthermore see that the dashed boundary curve in fig. S1 separates the set of possible motions into those which tumble and hit the wall repeatedly ('open' trajectories), and those which are closed (i.e., oscillating) and limited to amplitudes which keeps them well within upstream directions. This is in accordance with the experimental observation.