Cell Mechanics, Structure, and Function Are Regulated by the Stiffness of the Three-Dimensional Microenvironment

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Supplementary information

For the neo-Hookean model, the energy function is given by,

$$U = C_1\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3\right) \quad \text{(Eq. A1)}$$

The Cauchy stress at direction 1 is given by,

$$\sigma_1 = \lambda_1 \frac{dU}{d\lambda_1} - p = 2C_1\lambda_1^2 - p \quad \text{(Eq. A2)}$$

For uni-axial test, the stretch ratios λ_i is given by,

$$\lambda_1 = \lambda \ , \lambda_2 = \lambda_3 = \lambda^{-\frac{1}{2}}, \ \sigma_2 = \sigma_3 = 0 \quad \text{(Eq. A3)}$$

$$\sigma_1 = 2C_1(\lambda_1^2 - \lambda_3^2) \quad \text{(Eq. A4)}$$

$$\sigma_1 = 2C_1\left(\lambda^2 - \lambda^{-1}\right) = 2C_1\left(\lambda^2 - \lambda^{-1}\right) \quad \text{(Eq. A5)}$$

$\lambda^2 = 1 + 2e$, where e is the Green strain. (Eq. A6)

In the case of infinitesimal strain, it gives,

$$\sigma_1 = 2C_1\left\{1 + 2e - (1+2e)^{-\frac{1}{2}}\right\} = 2C_1\left\{1 + 2e - \left\{1 - \frac{1}{2}2e + \frac{1}{2} \times \frac{1}{2} \times \frac{3}{2}(2e)^2 + \cdots \ \right\}\right\} = 6C_1 e \quad \text{(Eq. A7)}$$

Therefore, $E = 6C_1$ (Eq. A8)

For the Ogden model, the energy function is given by,

$$U = \sum_{i=1}^{n} \frac{2\mu_i}{\alpha_i^2}\left(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3\right) \quad \text{(Eq. A9)}$$

$$\sigma_1 = \sum_{i=1}^{n} \lambda_1 \frac{dU}{d\lambda_1} - p = \sum_{i=1}^{n} \frac{2\mu_i}{\alpha_i}\lambda_1^{\alpha_i} - p \quad \text{(Eq. A10)}$$

For the uniaxial test,

$$\sigma_1 = \sum_{i=1}^{n}\left(\frac{2\mu_i}{\alpha_i}\lambda_1^{\alpha_i} - \frac{2\mu_i}{\alpha_i}\lambda_3^{\alpha_i}\right) = \sum_{i=1}^{n}\left(\frac{2\mu_i}{\alpha_i}\lambda^{\alpha_i} - \frac{2\mu_i}{\alpha_i}\lambda_i^{-\frac{\alpha_i}{2}}\right) \quad \text{(Eq. A11)}$$

In the case of infinitesimal strain, it gives,

$$\sigma_1 = \sum_{i=1}^{n} \frac{2\mu_i}{\alpha_i}\left\{\left[1 + \alpha_i e + \frac{\alpha_i}{4}\left(\frac{\alpha_i}{2} - 1\right) \times 4e^2 + \cdots\right] - \left[1 - \frac{\alpha_i}{2}e + \frac{\alpha_i}{8}\left(\frac{\alpha_i}{4} + 1\right) \times 4e^2 + \cdots\right]\right\} \approx \sum_{i=1}^{n} \frac{2\mu_i}{\alpha_i}\left(\frac{3}{2}\alpha_i e\right) = 3\sum_{i=1}^{n} \mu_i e \quad \text{(Eq. A12)}$$

Therefore,

$$E = 3\sum \mu_i \quad \text{(Eq. A13)}$$