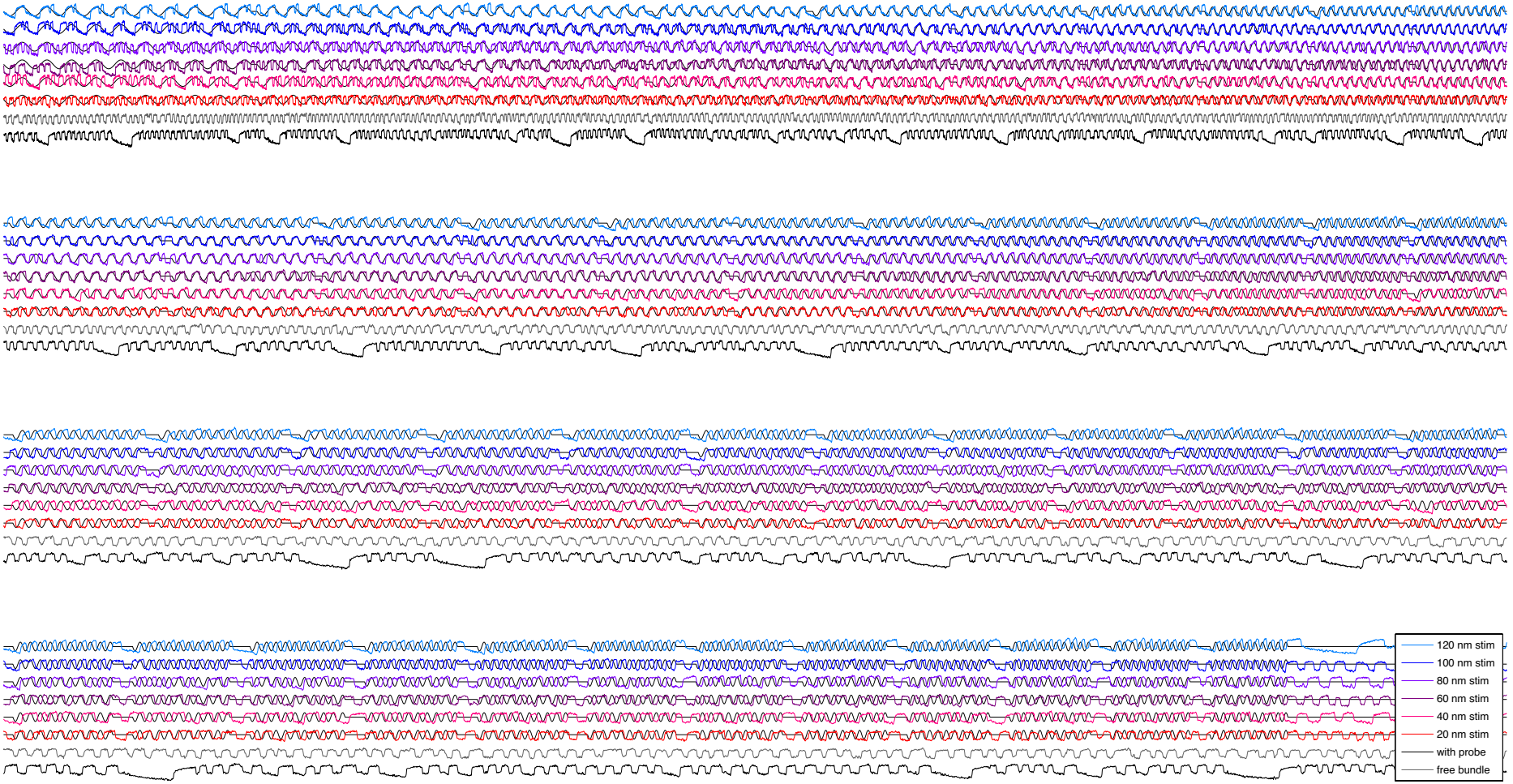

FIG. 1: Figure S1. Traces of hair bundle displacement in response to sinusoidal forcing via an attached probe of stiffness 100 μN/m. (The traces shown are an evenly spaced (in stimulus amplitude) subset of the entire set of traces used to generate Figure 3, as they are sufficient to highlight the qualitative changes in the behavior of the response.) The bottom trace (black) shows the spontaneous oscillation of the hair bundle in the absence of the attached probe, while the trace above it (grey) shows the innate oscillations with the probe attached. The subsequent colored traces show how the bundle responds to sinusoidal forcing by the probe, at distinct amplitudes applied to the probe's base, as indicated in the legend. An indication of the stimulus waveform is overlaid on black, scaled to an amplitude of 25 nm for visual clarity. (The stimulus was comprised of sections of sinusoidal waveforms from 5 Hz (top left) to 50 Hz (bottom right) in 1 Hz increments. Each frequency was applied for 10 periods followed by a 50 ms section of no stimulus.)

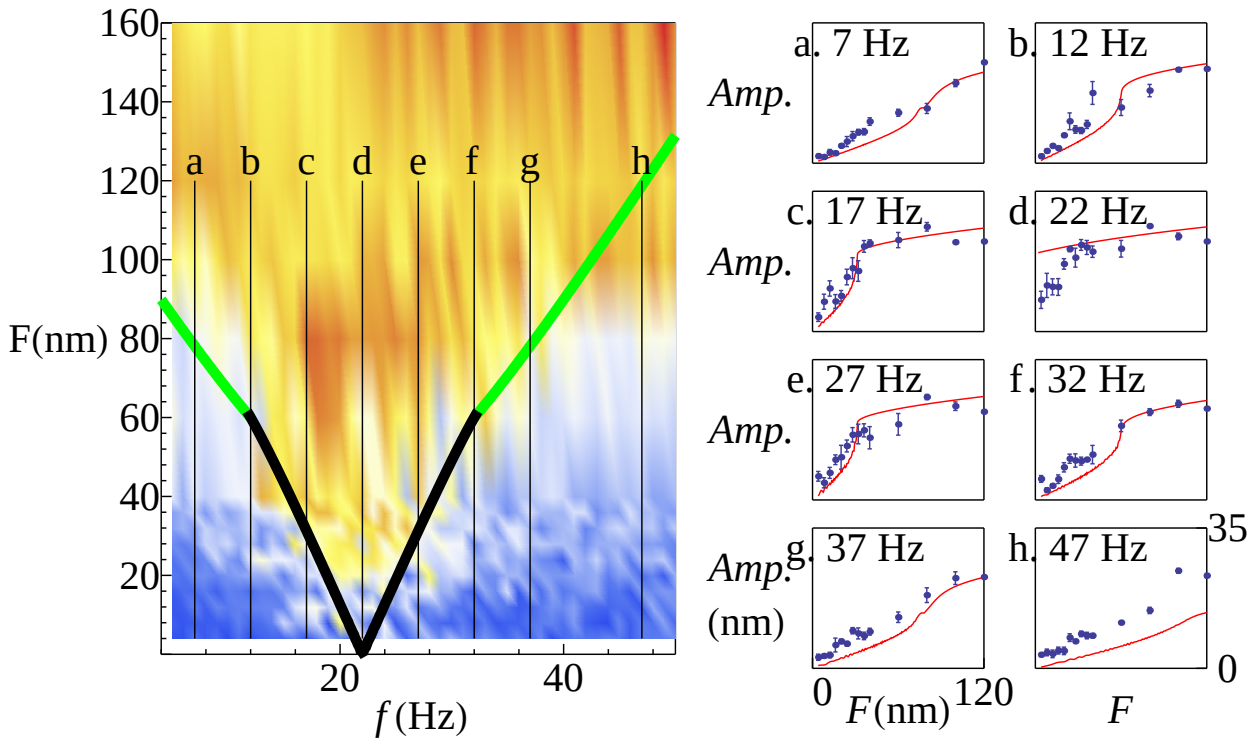

FIG. 2: Figure S2. Numerical fits to vertical slices of the Arnold Tongue, for a different choice of the Bogdanov-Takens point. As in the text, the data are shown in blue (binned into 5 Hz intervals, with blue dots indicating the mean, and the error bars showing the standard deviation), and the red lines indicate the numerical fits. For this calculation, the Bogdanov-Takens bifurcation was chosen to occur at $F_{bt} = 60$ nm, $f_{bt} = 12$ Hz for $f_0 = 22$ Hz. Scale factors were determined by fitting mu ($\mu = 2.9$, $F_{scale} = 22.9$, and $f_{scale} = 6.17$). As can be seen from the panels to the right, the quality of the fits was not substantively changed. Note: The Numerical fitting was done based on the assumption that the theoretical and experimental inflection points behave the same way. The theoretical inflection point is defined by the ratios $F_{bt}/\mu^{3/2} = 0.53$ and $(\omega_{bt} - \omega_0)/\mu = 0.559$. When the experimental inflection point is chosen, the factors F_{scale} and ω_{scale} are defined by the following relations; $F_{bt} = F_{expbt}/F_{scale} = 0.53\mu^{3/2}$, and $\omega_{bt} - \omega_0 = (f_{expbt} - f_0)/f_{scale} = 0.559\mu$. Then, all the fitting parameters are reduced to the functions of μ. Numerical fits are performed on each slice of the data and best μ value is found.

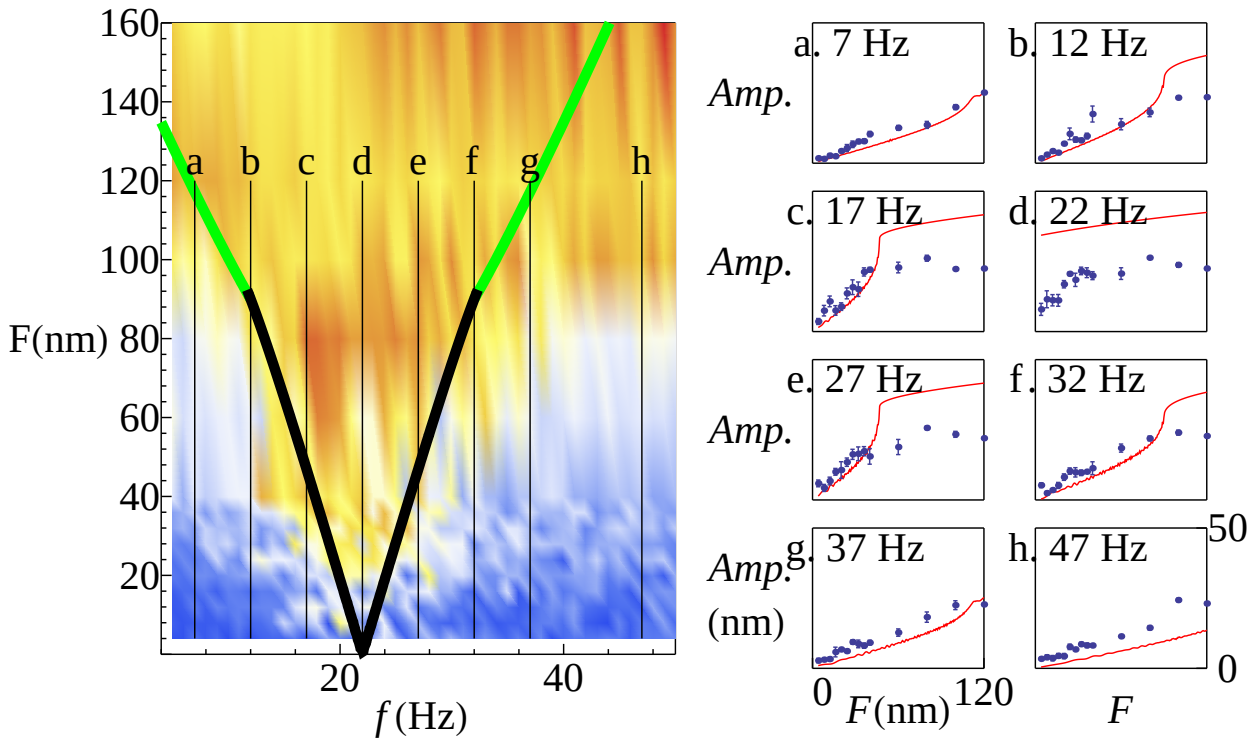

FIG. 3: Figure S3. Numerical fits to vertical slices of the Arnold Tongue, for a different choice of the Bogdanov-Takens point. For this calculation, the multi-critical bifurcation was chosen to occur at $F_{bt} = 90$ nm, $f_{bt} = 12$ Hz for $f_0 = 22$ Hz. Scale factors were determined by fitting μ ($\mu = 2.5, F_{scale} = 42.9$, and $f_{scale} = 7.16$). For choices significantly far from the ones presented in the text, the fits were degraded (right panels).