Supplementary Material

Appendix: Elements of the information matrix of the log- likelihood

The elements of the information matrix $I_e(\theta)$, denoted by $I_e(\theta)_{j_1 j_2}$, are the negative second partial derivatives of the log likelihood function (Eq. (5)) with respect to $\mu_1, \mu_2, \cdots, \mu_k$, and σ^2, which can be directly obtained as follows:

$$I_e(\theta)_{jj} = -\frac{\partial^2 \ln L(\theta)}{\partial \mu_j^2} = -\frac{1}{\sigma^4}\sum_{i=1}^{n}\left[p_{ij}^*(1-p_{ij}^*)(y_i-\mu_j)^2 - \sigma^2 p_{ij}^* \right], \text{ for } j = 1, 2, \cdots, k$$

$$I_e(\theta)_{j_1 j_2} = -\frac{\partial^2 \ln L(\theta)}{\partial \mu_{j_1}\partial \mu_{j_2}} = \frac{1}{\sigma^4}\sum_{i=1}^{n} p_{ij_1}^* p_{ij_2}^* (y_i-\mu_{j_1})(y_i-\mu_{j_2}), \text{ for } j_1, j_2 = 1, 2, \cdots, k \text{ and } j_1 \neq j_2$$

$$I_e(\theta)_{k+1,j} = I_e(\theta)_{j,k+1} = -\frac{\partial^2 \ln L(\theta)}{\partial \sigma^2 \partial \mu_j} = \frac{1}{2\sigma^6}\sum_{i=1}^{n} p_{ij}^*(y_i-\mu_j)\left[\sum_{l=1}^{k} p_{il}^*(y_i-\mu_l)^2 - (y_i-\mu_j)^2 \right], \text{ for } j = 1, 2, \cdots, k$$

$$I_e(\theta)_{k+1,k+1} = -\frac{\partial^2 \ln L(\theta)}{\partial (\sigma^2)^2} = \frac{n}{2\sigma^4} - \frac{1}{4\sigma^8}\sum_{i=1}^{n}\left[\sum_{l=1}^{k} p_{il}^*(y_i-\mu_l)^4 - \left(\sum_{l=1}^{k} p_{il}^*(y_i-\mu_l)^2 \right)^2 \right]$$

where

$$p_{ij}^* = \frac{p_{ij} f(y_i; \mu_j, \sigma^2)}{\sum_{j=1}^{k} p_{ij} f(y_i; \mu_j, \sigma^2)}$$