

Supporting Information: Integrating chemical footprinting data into RNA secondary structure prediction

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Increasing β decreases average distance to normalized SHAPE data

In this section, we generalize the theorem from the text to show that as RNAsc parameter β increases, the expected distance to normalized SHAPE data decreases. The proof, which generalizes the proof of the theorem in the main text, is given below.

First, we begin with some notation. Throughout this section, S denotes a secondary structure of an arbitrary, but fixed RNA sequence a_1, \dots, a_n . As in the text, each secondary structure S is associated with a binary sequence b_1, \dots, b_n such that $b_i = 1$ if the nucleotide a_i is unpaired and $b_i = 0$ if a_i is base-paired. Given experimental SHAPE data yielding probabilities $\mathbf{q}^s = (q_1^s, \dots, q_n^s)$, where q_i^s is the probability that nucleotide i is *unpaired*, the distance of S to \mathbf{q}^s is defined by:

$$d_{\mathbf{q}^s}(S) = \sum_{i=1}^n |b_i - q_i^s|. \quad (1)$$

For secondary structure S and parameter $\beta \geq 0$ in the algorithm RNAsc, SHAPE *weight* for (S, β) is defined to be

$$\begin{aligned} \omega_{\mathbf{q}^s}(S, \beta) &= \prod_{i=1}^n \exp((- \beta |b_i - q_i^s|) / RT) \\ &= \exp((- \beta d_{\mathbf{q}^s}(S)) / RT). \end{aligned} \quad (2)$$

The β -weighted partition function then becomes

$$Z_\beta = \sum_S \omega_{\mathbf{q}^s}(S, \beta) \exp(-E(S) / RT). \quad (3)$$

The β -weighted Boltzmann probability $P(S)$ of secondary structure S is defined by

$$P_\beta(S) = \frac{\omega_{\mathbf{q}^s}(S, \beta) \exp(-E(S) / RT)}{Z_\beta} \quad (4)$$

Let $0 \leq \beta_1 \leq \beta_2$ be arbitrary, but fixed values for the parameter β in RNAsc. Define the *critical distance* $d_c(\beta_1, \beta_2)$ by

$$d_c(\beta_1, \beta_2) = \frac{-RT \ln \left(\frac{Z_{\beta_2}}{Z_{\beta_1}} \right)}{\beta_2 - \beta_1}. \quad (5)$$

Note that $d_c(\beta_1, \beta_2)$, which has the form of a change in ensemble free energy, does not depend on any particular secondary structure S , although it does depend on $n, T, \beta_1, \beta_2, \mathbf{q}^s$ and of course the input RNA sequence a_1, \dots, a_n .

CLAIM 1: If $d_1 \leq d_2$ and $\beta \geq 0$, then

$$\exp(-\beta d_1 / RT) \geq \exp(-\beta d_2 / RT).$$

Claim 1 is obvious. We now prove the following principal claim.

CLAIM 2: For any secondary structure S ,

$$d_{\mathbf{q}^s}(S) \leq d_c(\beta_1, \beta_2) \iff P_{\beta_1}(S) \geq P_{\beta_2}(S) \quad (6)$$

and strict inequalities hold as well.

For notational ease, let d_c abbreviate $d_c(\beta_1, \beta_2)$. By Equation (5) and Claim 1, for any S ,

$$d_{\mathbf{q}^s}(S) \leq d_c \iff d_{\mathbf{q}^s}(S) \leq \frac{-RT \ln(Z_{\beta_2}/Z_{\beta_1})}{\beta_2 - \beta_1},$$

hence

$$\exp\left(\frac{-(\beta_2 - \beta_1)d_{\mathbf{q}^s}(S)}{RT}\right) \geq \exp\left(\frac{-(\beta_2 - \beta_1)d_c}{RT}\right) = \frac{Z_{\beta_2}}{Z_{\beta_1}}.$$

Multiply both sides of the last line by $P(S) = \frac{\exp(-E(S)/RT)}{Z}$ to obtain

$$P_{\beta_2}(S) = P(S) \cdot \frac{\exp\left(\frac{-\beta_2 d_{\mathbf{q}^s}(S)}{RT}\right)}{Z_{\beta_2}} \geq P(S) \cdot \frac{\exp\left(\frac{-\beta_1 d_{\mathbf{q}^s}(S)}{RT}\right)}{Z_{\beta_1}} = P_{\beta_1}(S).$$

This establishes Claim 2.

For $0 \leq \beta$, define the β -expected distance $\langle D_\beta \rangle$ between \mathbf{q}^s , obtained by normalizing SHAPE data, and the ensemble of low energy structures by

$$\langle D_\beta \rangle = \sum_S P_\beta(S) d_{\mathbf{q}^s}(S). \quad (7)$$

When $\beta = 0$, we write $\langle D \rangle$, instead of $\langle D_0 \rangle$.

Define disjoint sets A, B of secondary structures S of a_1, \dots, a_n by

$$\begin{aligned} A &= \{S : d_{\mathbf{q}^s}(S) \leq d_c(\beta_1, \beta_2)\} \\ B &= \{S : d_{\mathbf{q}^s}(S) > d_c(\beta_1, \beta_2)\}. \end{aligned}$$

It follows by definition that for all $S \in A$, $d_{\mathbf{q}^s}(S) \leq d_c$, and for all $S \in B$, $d_{\mathbf{q}^s}(S) > d_c$.

THEOREM: For any given RNA sequence a_1, \dots, a_n , normalized SHAPE data \mathbf{q}^* and $0 \leq \beta_1 \leq \beta_2$, $\langle D_{\beta_1} \rangle \geq \langle D_{\beta_2} \rangle$; moreover, strict inequalities hold as well.

PROOF:

$$\begin{aligned} \langle D_{\beta_1} \rangle - \langle D_{\beta_2} \rangle &= \sum_{S \in A} d_{\mathbf{q}^s}(S) (P_{\beta_1}(S) - P_{\beta_2}(S)) + \sum_{S \in B} d_{\mathbf{q}^s}(S) (P_{\beta_1}(S) - P_{\beta_2}(S)) \\ &> \sum_{S \in A} d_c (P_{\beta_1}(S) - P_{\beta_2}(S)) + \sum_{S \in B} d_c (P_{\beta_1}(S) - P_{\beta_2}(S)) \\ &= d_c \cdot \sum_S (P_{\beta_1}(S) - P_{\beta_2}(S)) = d_c \cdot \left(\sum_S P_{\beta_1}(S) - \sum_S P_{\beta_2}(S) \right) = d_c \cdot 0 = 0. \end{aligned}$$

To justify the inequality, note that for $S \in A$, $P_{\beta_1}(S) - P_{\beta_2}(S) \leq 0$, hence for $S \in A$, we have $d_{\mathbf{q}^s}(S) \cdot (P_{\beta_1}(S) - P_{\beta_2}(S)) \geq d_c \cdot (P_{\beta_1}(S) - P_{\beta_2}(S))$. On the other hand, for $S \in B$, $P_{\beta_1}(S) - P_{\beta_2}(S) > 0$, hence for $S \in B$, we also have $d_{\mathbf{q}^s}(S) \cdot (P_{\beta_1}(S) - P_{\beta_2}(S)) \geq d_c \cdot (P_{\beta_1}(S) - P_{\beta_2}(S))$. Finally, the last line follows from the fact that P_{β_1} and P_{β_2} are both probability distributions, hence $\sum_S P_{\beta_1}(S) = 1 = \sum_S P_{\beta_2}(S)$. This completes the proof that $\langle D_{\beta_1} \rangle \geq \langle D_{\beta_2} \rangle$. The proof for inequality is similar.

SHAPE discrepancies

In order to directly characterize how well SHAPE data reflects RNA secondary structure, we compared normalized SHAPE data with base pairing status, as determined from crystallographic or NMR structures. We define SHAPE *distance* to equal the difference between *normalized* SHAPE reactivity (see Methods), scaled from 0 to 1, as just defined, and binary base-pairing status, with 0 for paired, 1 for unpaired, as derived from NMR or crystal structure. Using SHAPE data for *S. cerevisiae* apartyI-tRNA [1], HCV IRES [2], bI3 group I intron p456 [3], *E. coli* phenylalanine-tRNA [4], *E. coli* 5S RNA [4], and *Fusobacterium nucleatum* glycine riboswitch [4], we computed SHAPE distance at each nucleotide. We observed that at many positions the SHAPE distance has an absolute value greater than 0.5, thus indicating a significant difference between SHAPE reactivity and the actual secondary structure. We refer to these positions as discrepancies. Over the the set of RNAs we examined, between 24 – 35% of the total data corresponded to such discrepancies (see Fig. 1).

Cumulative distributions and Histograms of SHAPE reactivity

Nucleotides with SHAPE reactivities ≥ 0.7 or $0.3 - 0.7$ are considered highly and moderately reactive, respectively [2]. Hence it is reasonable to normalize the SHAPE reactivities in a piecewise linear fashion, where 0.3 will be roughly mapped to 0.5. However, very low SHAPE reactivities should not be mapped close to 0.5 either. For this reason the SHAPE reactivity values < 0.25 are linearly mapped to the interval $[0.0, 0.35]$, the reactivity values in $[0.25, 0.3)$ are linearly mapped to the interval $[0.35, 0.55]$, the reactivity values in $[0.3, 0.7)$ are linearly mapped to the interval $[0.55, 0.85]$, and lastly, the reactivities ≥ 0.7 are linearly mapped to the interval $[0.85, 1.0]$. See Fig. 2 for the distribution of SHAPE reactivities.

Integrating pseudo-energy terms into the Partition function recursions

In this section, we provide the full recursive definitions necessary to compute the partition function, where pseudo-energy factors have been included for every nucleotide position. The recursions are adaptations of recursions from [5]. Corresponding recursions for the minimum free energy (MFE) secondary structure are obtained by replacing the Boltzmann factors of energy by the energy, replacing addition by minimization, and replacing multiplication by addition– hence will not be given explicitly.

Definition 1 *Define:*

- $V(i, j)$: The partition function for the fragment from nucleotides i to j , inclusive, with i paired to j .
- $W(i, j)$: The partition functions for the fragment from nucleotides i to j , inclusive, such that this fragment will be incorporated in a multibranch loop and it has one single helical branch.
- $WL(i, j)$: The partition functions for the fragment from nucleotides i to j , inclusive, such that this fragment will be incorporated in a multibranch loop and it has one single helical branch. Also j is required to terminate the helical branch as either a paired nucleotide, a 3' dangling end, or a nucleotide in a terminal mismatch.
- $WMB(i, j)$: The partition function from nucleotides i to j , inclusive, such that this fragment will be incorporated into a multibranch loop and it contains two or more branches.

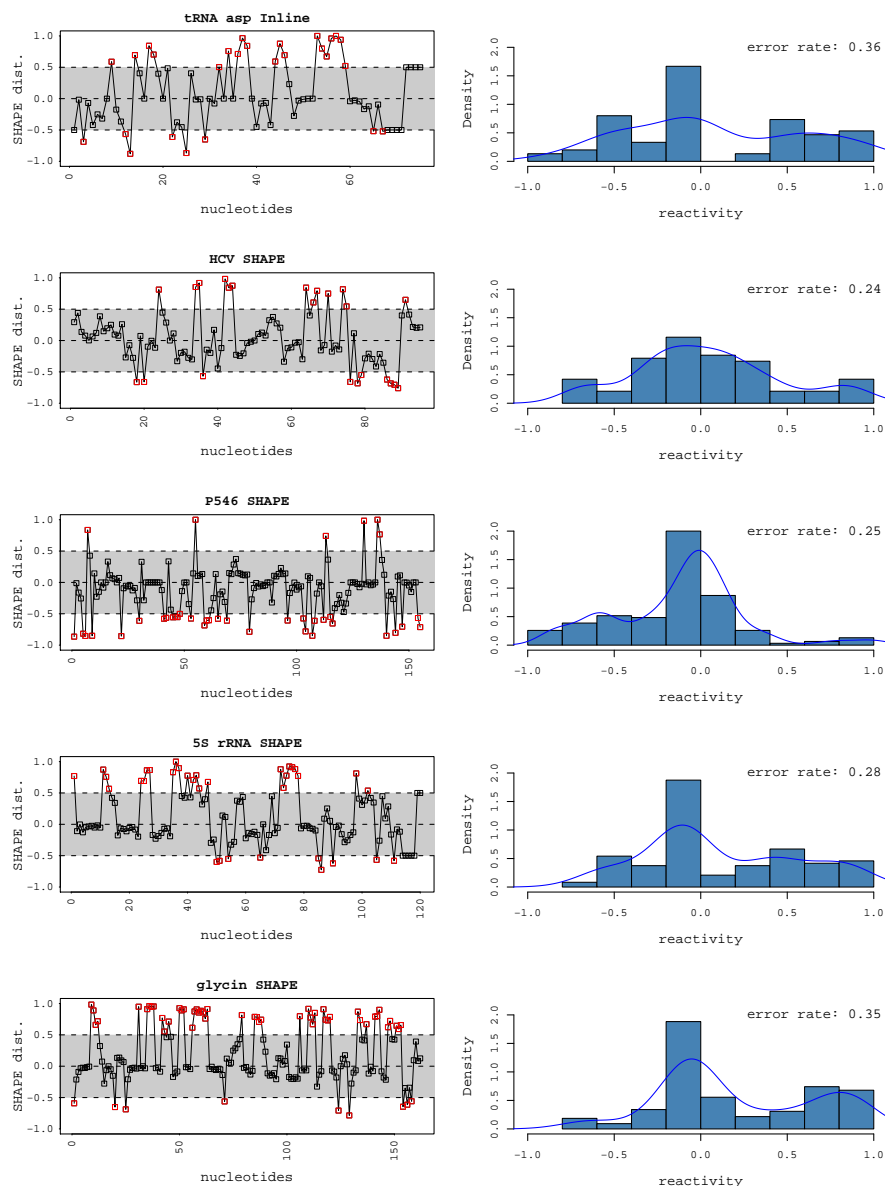


Figure 1. SHAPE discrepancies. Distribution of SHAPE discrepancies for east tRNA-Asp, HCV IRES [2], bI3 group I intron p456 [3], *E. coli* 5S RNA [4], and *Fusobacterium nucleatum* glycine riboswitch [4]. Using crystal structure as ‘gold standard’, red squares indicate locations, where the absolute value of the difference of SHAPE data and crystal structure (1 unpaired, 0 paired) exceeds 0.5.

- $WMBL(i, j)$: The partition function from nucleotides i to j , inclusive, such that this fragment will be incorporated into a multibranch loop and it contains two or more branches. Also j is required to be paired or associated with a helix as either a 3' dangling end, a nucleotide in a terminal mismatch, or a nucleotide in a mismatch between two coaxially stacked helices.
- $WC(i, j)$: The partition function from nucleotides from i to j , inclusive, such that there are two

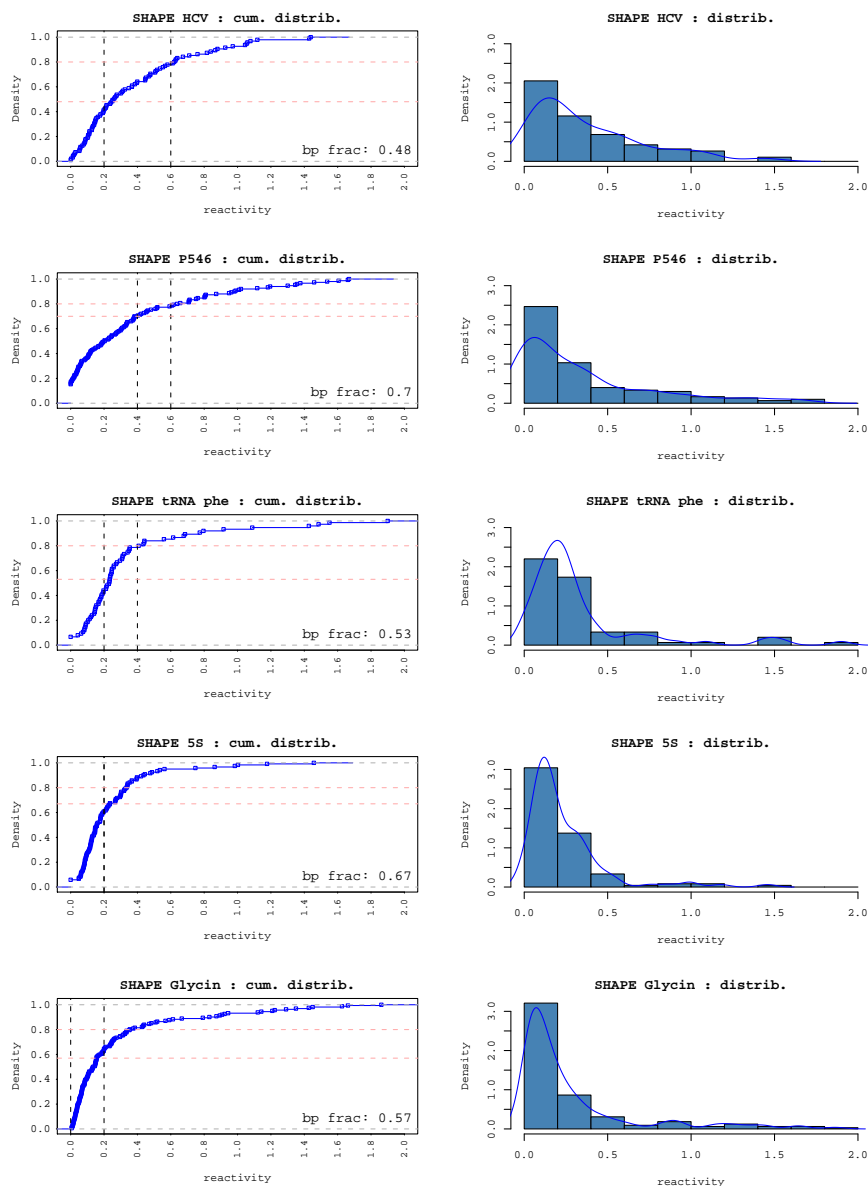


Figure 2. SHAPE reactivity distributions. Distribution of reactivities of data for HCV IRES [2], bI3 group I intron p456 [3], *E. coli* phenylalanine-tRNA [4], *E. coli* 5S RNA [4], and *Fusobacterium nucleatum* glycine riboswitch [4]. The fraction of base-pairs could be used to estimate the threshold SHAPE moderate reactivity.

coaxially stacked branches. Nucleotides i and j must either be paired, e.g. (i, k) , $(k + 1, j)$, or be in a single mismatch separating the two helices, e.g. $(i + 1, k - 1)$, $(k + 1, j)$ or (i, k) , $(k + 2, j - 1)$.

- $W5(i)$: The partition function for the nucleotide fragment from the 5' end of the sequence to and including nucleotide i

- $W3(i)$: The partition function from and including nucleotide i to the 3' end of the sequence.

See [5,6] for details on the storage and calculation of the arrays. V is the sum of terms involving a pair between i and j , base pair stacking, hairpin loop closure, internal loop closure, and multibranch loop closure. For $j \leq N$, we have:

$$V(i, j) = \{Vh(i, j) + Vs(i, j) + Vi(i, j) + Vm(i, j)\} \times \omega(0, i) \times \omega(0, j),$$

and for $j > N$ we have:

$$V(i, j) = \{Vs(i, j) + Vi(i, j) + Vm(i, j) + Ve(i, j)\} \times \omega(0, i) \times \omega(0, j).$$

Note that V is not defined for $i = N$ and for $j = N + 1$ (i.e., the ends of the sequence.) Vh is the energy contribution of the hairpin and is defined by

$$Vh(i, j) = \begin{cases} e^{-\Delta G(\text{hairpin})/RT} \times F(i + 1, j - 1) & \text{for } j \leq N, \\ 0 & \text{else.} \end{cases}$$

Vs is the stacking contribution (of a base pair on a previous pair) and is defined by:

$$Vs(i, j) = e^{-\Delta G(\text{stack})/RT} \times V(i + 1, j - 1).$$

Vi is the energy contribution of internal loops which also considers bulge loops. Vi is defined by,

$$Vi(i, j) = \sum_{i', j'} V(i', j') F(i + 1, i' - 1) F(j' + 1, j - 1) \times e^{-\Delta G(\text{stack})/RT}.$$

Computation of Vi requires a search over i' and j' where $i < i' < j' < j$ except where $i = i + 1$ and $j = j - 1$ simultaneously, that is, the base pair stacking case. The search is limited to internal loops of size 30 nt that is, $i' - i + j - j' - 2 \leq 30$, to limit the algorithm to $O(N^3)$. In what follows 3' d refers to 3' dangles, 5' d refers to 5' dangles, TM refers to *Terminal mismatch* and CS refers to *Coaxial Stacking*. Vm is the multibranch contribution and is define by

$$\begin{aligned} Vm(i, j) &= WMB(i + 1, j - 1) \times e^{-(a+c)/RT} + e^{-\Delta G(3' \text{ d})/RT} \\ &\times WMB(i + 2, j - 1) \times e^{-(a+b+c)/RT} \times \omega(1, i + 1) + e^{-\Delta G(5' \text{ d})/RT} \\ &\times WMB(i + 1, j - 2) \times e^{-(a+b+c)/RT} \times \omega(1, j - 1) + e^{-\Delta G(\text{TM})/RT} \\ &\times WMB(i + 2, j - 2) \times e^{-(a+2b+c)/RT} \times \omega(1, i + 1) \times \omega(1, j - 1) \\ &+ \sum_k e^{-\Delta G(\text{CS})/RT} \times V(i + 1, k) \times \{W(k + 1, j - 1) + WMB(k + 1, j - 1)\} \times e^{-(a+2c)/RT} \\ &+ \sum_k e^{-\Delta G(\text{CS})/RT} \times V(k, j - 1) \times \{W(i + 1, k - 1) + WMB(i + 1, k - 1)\} \times e^{-(a+2c)/RT} \\ &+ \sum_k e^{-\Delta G(\text{CS})/RT} \times V(i + 2, k) \times \{W(k + 2, j - 1) + WMB(k + 2, j - 1)\} \times e^{-(a+2b+2c)/RT} \\ &\times \omega(1, i + 1) \times \omega(1, k + 1) + \sum_k e^{-\Delta G(\text{CS})/RT} \times V(k, j - 2) \\ &\times \{W(i + 1, k - 2) + WMB(i + 1, k - 2)\} \times e^{-(a+2b+2c)/RT} \times \omega(1, k - 1) \times \omega(1, j - 1). \end{aligned}$$

Ve is the energy contribution from the exterior loops. It is defined only when $j > N$:

$$\begin{aligned}
Ve(i, j) = & W3(i+1) \times W5(j-1-N) + W3(i+2) \times W5(j-1-N) \times e^{-\Delta G(3' \text{ d})/RT} \\
& \times \omega(1, i+1) + W3(i+1) \times W5(j-2-N) \times e^{-\Delta G(5' \text{ d})/RT} \times \omega(1, j-1-N) \\
& + W3(i+2) \times W5(j-2-N) \times e^{-\Delta G(\text{TM})/RT} \times \omega(1, i+1) \times \omega(1, j-1-N) \\
& + \sum_k e^{-\Delta G(\text{CS})/RT} \times V(i+1, k) \times W3(k+1) \times W5(j-1-N) \\
& + \sum_k e^{-\Delta G(\text{CS})/RT} \times V(k, j-1-N) \times W3(i+1) \times W5(k-1) \\
& + \sum_k e^{-\Delta G(\text{CS})/RT} \times V(i+2, k-2) \times W3(k) \times W5(j-1-N) \\
& \times \omega(1, i+1) \times \omega(1, k-1) + \sum_k e^{-\Delta G(\text{CS})/RT} \times V(i+2, k) \times W3(k+1) \\
& \times W5(j-2-N) \times \omega(1, i+1) \times \omega(1, j-1-N) + \sum_k e^{-\Delta G(\text{CS})/RT} \\
& \times V(k+1, j-2-N) \times W3(i+1) \times W5(k-1) \times \omega(1, j-1-N) \times \omega(1, k) \\
& + \sum_k e^{-\Delta G(\text{CS})/RT} \times V(k, j-2-N) \times W3(i+2) \times W5(k-1) \\
& \times \omega(1, j-1-N) \times \omega(1, i+1).
\end{aligned}$$

$W5(i)$ is the fragment from 1 to i and is computed by:

$$\begin{aligned}
W5(i) = & W5(i-1) \times \omega(1, i) + \sum_k W5(k) \times V(k+1, i) + \sum_k W5(k) \times e^{-\Delta G(3' \text{ d})/RT} \\
& \times V(k+1, i-1) \times \omega(1, i) + \sum_k W5(k) \times e^{-\Delta G(5' \text{ d})/RT} \times V(k+2, i) \\
& \times \omega(1, k+1) + \sum_k W5(k) \times e^{-\Delta G(\text{TM})/RT} \times V(k+2, i-1) \\
& \times \omega(1, k+1) \times \omega(1, i) + \sum_{k,m} W5(k) \times e^{-\Delta G(\text{CS})/RT} \times V(k+1, m) \\
& \times V(m+1, i) + \sum_{k,m} W5(k) \times e^{-\Delta G(\text{CS})/RT} \times V(k+1, m) \times V(m+2, i-1) \\
& \times \omega(1, m+1) \times \omega(1, i) + \sum_{k,m} W5(k) \times e^{-\Delta G(\text{CS})/RT} \times V(k+2, m-1) \\
& \times V(m+1, i) \times \omega(1, k+1) \times \omega(1, m).
\end{aligned}$$

$W5(0)$ is initialized to 1.

$W3$ is defined similarly, as follows.

$$\begin{aligned}
W3(i) = & W3(i+1) \times \omega(1, i) + \sum_k W3(k) \times V(i, k-1) + \sum_k W3(k) \times e^{-\Delta G(5' \text{ d})/RT} \\
& \times V(i+1, k-1) \times \omega(1, i) + \sum_k W3(k) \times e^{-\Delta G(3' \text{ d})/RT} \times V(i, k-2) \\
& \times \omega(1, k-1) + \sum_k W3(k) \times e^{-\Delta G(TM)/RT} \times V(i+1, k-2) \times \\
& \omega(1, i) \times \omega(1, k-1) + \sum_{k,m} W3(k) \times e^{-\Delta G(CS)/RT} \times V(i, m) \\
& \times V(m+1, k-1) + \sum_{k,m} W5(k) \times e^{-\Delta G(CS)/RT} \times V(i, m) \times V(m+2, k-2) \\
& \times \omega(1, m+1) \times \omega(1, k-1) + \sum_{k,m} W5(k) \times e^{-\Delta G(CS)/RT} \times V(i+1, m-1) \\
& \times V(m+1, k-1) \times \omega(1, i) \times \omega(1, m).
\end{aligned}$$

$W3(N+1)$ is initialized to 1.

WC is the energy contribution of coaxial stacking inside a multiloop and is defined by:

$$\begin{aligned}
WC(i, j) = & \sum_k e^{-\Delta G(CS)/RT} \times V(i, k) \times V(k+1, j) \times e^{-2c/RT} \\
& + \sum_k e^{-\Delta G(CS)/RT} \times V(i+1, k) \times V(k+2, j) \times e^{-2b-2c/RT} \\
& \times \omega(1, i) \times \omega(1, k+1) + \sum_k e^{-\Delta G(CS)/RT} \times V(i, k) \\
& \times V(k+2, j-1) \times e^{-2b-2c/RT} \times \omega(1, k+1) \times \omega(1, j)
\end{aligned}$$

$WMBL$, the energy contribution of the last part of a multibranchloop with at least two components is defined by:

$$\begin{aligned}
WMBL(i, j) = & \sum_k \left(WL(i, k) - \omega(1, i) \times WL(i+1, k) \times e^{-b/RT} \right) \\
& + WC(i, k) \times \{ WL(k+1, j) + WMBL(k+1, j) \}
\end{aligned}$$

WL , the energy contribution of the last part of a multibranchloop with one components is defined by:

$$\begin{aligned}
WL(i, j) = & V(i, j) \times e^{-c/RT} + e^{-\Delta G(3' \text{ d})/RT} \times V(i, j-1) \times e^{-(b+c)/RT} \times \omega(1, j) \\
& + e^{-\Delta G(5' \text{ d})/RT} \times V(i+1, j) \times e^{-(b+c)/RT} \times \omega(1, i) \\
& + e^{-\Delta G(TM)/RT} \times V(i+1, j-1) \times e^{-(2b+c)/RT} \times \omega(1, i) \times \omega(1, j) \\
& + WL(i+1, j) \times e^{-b/RT} \times \omega(1, i).
\end{aligned}$$

Finally, WMB , the energy contribution of multibranch loop with at least 2 components is:

$$WMB(i, j) = WMBL(i, j) + WMB(i, j-1) \times e^{-b/RT} \times \omega(1, j)$$

The probability of a base pair involving nucleotides i and j is:

$$P_{i,j} = \frac{V(i, j) \times V(j, i+N)}{W5(N) \times \omega(0, i) \times \omega(0, j)}$$

This is because $W5(N) = Q$ and $V(i, j) \times V(j, i+N)$ represents the total contribution to Q of secondary structure conformations that contain the base pair of i to j .

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