Supporting Information

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SI Text

Data. Descriptive statistics and official sources of the election results are shown in Table S1. The raw data will be made available for download at http://www.complex-systems.meduniwien.ac.at/elections/election.html. Data report election results of parliamentary (Austria, Canada, Czech Republic, Finland, Russia, Spain, and Switzerland), European (Poland), or presidential (France, Romania, Russia, and Uganda) elections on at least one aggregation level. In the rare circumstances where electoral districts report more valid ballots than registered voters, we work with a turnout of 100%. Territorial units with an electorate less than 100 are omitted at each point of the analysis to avoid extreme vote and turnout rates as spurious results because of small communities. The countries included in this work were chosen based on data availability. A country was included if the voting results were available in electronic form on an aggregation level where a number of $\bar{n}_{pop} \leq 5,000$ vote-eligible persons comprises one territorial unit. Required data are the number of vote-eligible persons N_i, the number of valid votes V_i, and the number of votes for the winning party/candidate W_i for each unit.

Model. A country is separated into n electoral units i, and each unit has an electorate of N_i people and a total of V_i valid votes. The fraction of valid votes for the winning party in unit i is denoted v_i. The average turnout over all units, $\bar{a}$, is given by $\bar{a} = 1/n\sum_i(V_i/N_i)$ with SD s_a, and the mean fraction of votes $\bar{v}$ for the winning party is $\bar{v} = 1/n\sum_i v_i$ with SD s_v. The mean values $\bar{a}$ and $\bar{v}$ are typically close but not identical to the values that maximize the empirical distribution function of turnout and votes over all units. Let v be the number of votes where the empirical distribution function assumes its (first local) maximum (rounded to entire percent values) (Fig. S1). Similarly, a is the turnout where the empirical distribution function of turnouts a_i takes its (first local) maximum. The distributions for turnout and votes are extremely skewed to the right for Uganda and Russia, which also inflates the SDs in these countries (Table S2). To account for this skewness, a left-sided (right-sided) mean deviation σ_v^L (σ_v^R) from v is introduced; σ_v^R can be regarded as the incremental fraud width, a measurable parameter quantifying the intensity of vote stuffing. This result contributes to the smearing out of the main peaks in the election fingerprints (Fig. 1). The larger the σ_v^R value, the more inflated the vote results because of urn stuffing in contrast to σ_v^L, which quantifies the scatter of the voters' actual preferences. They can be estimated from the data by (Eq. **S1**)

$$\sigma_v^L = \sqrt{\left\langle (v_i - v)^2 \right\rangle_{v_i < v}} \qquad \textbf{[S1]}$$

and (Eq. **S2**)

$$\sigma_v^R = \sqrt{\left\langle (v_i - v)^2 \right\rangle_{v_i > v}}. \qquad \textbf{[S2]}$$

Similarly, the extreme fraud width σ_x can be estimated (i.e., the width of the peak around 100% votes). We found that $\sigma_x = 0.075$ describes all encountered vote distributions reasonably well. Visualization of σ_v^L, σ_v^R, and σ_x is shown in Fig. S1.

Although f_i and f_e measure the number of units where incremental and extreme frauds occur, σ_v^R and σ_x quantify the intensity of these activities if they occur. To get an estimate for the width of the distribution of turnouts over territorial unit, which is free of possible fraudulent influences, the turnout distribution width σ_a is calculated from electoral districts i, which have both $v_i < v$ and $a_i < a$ (that is, $\sigma_a = \sqrt{\langle (a_i - a)^2 \rangle_{(a_i < a)\wedge(v_i < v)}}$.

Incremental fraud is a combination of two processes: stuffing ballots for one party into the urn and recasting or deliberately wrong-counting ballots from other parties (e.g., erasing the cross). Which one of these two processes dominates is quantified by the deliberate wrong-counting parameter $\alpha > 0$. For $0 < \alpha < 1$, the wrong-counting process dominates; for $\alpha > 1$, the urn stuffing mechanism is prevalent. In the following $\mathcal{N}(\mu, \sigma)$ denotes a normally distributed random variable with mean μ and SD σ. The model is specified by the following protocol, which is applied to each district.

i) Pick a unit i with electorate N_i taken from the data.

ii) The model turnout of unit i, $a_i^{(m)}$, is $\mathcal{N}(a, \sigma_a)$.

iii) A fraction of $v_i^{(m)} \in \mathcal{N}(v, \sqrt{(2)}\sigma_v^L)$ people vote for the winning party.

iv) With probability f_i, incremental fraud takes place. In this case, the unit is assigned a fraud intensity $x_i \in |\mathcal{N}(0, \sqrt{\sigma_v^R})|$. Values for x_i are only accepted if they lie in the range $0 < x_i < 1$. This result is the fraction of votes not cast, $(1 - a_i^{(m)})N_i$, which is added to the winning party. Votes for the opposition are wrong-counted for the winning party with a rate x_i^α (where α is an exponent). To summarize, if incremental fraud takes place, the winning party receives $N_i(a_i^{(m)}v_i^{(m)} + x_i(1 - a_i^{(m)}) + x_i^\alpha(1 - v_i^{(m)})a_i^{(m)})$ votes.

v) With probability f_e, extreme fraud takes place. In this case, opposition votes are canceled and added to the winning party with probability $y_i \in 1 - |\mathcal{N}(1,\sigma_x)|$ (i.e., the same change in the winning party's votes as for incremental fraud with y_i replacing x_i). Acceptable values for y_i are again from the range $0 < y_i < 1$.

Fitting the Model. The parameters for incremental and extreme fraud, f_i and f_e, as well as the deliberate wrong-counting parameter α are estimated by a goodness-of-fit test. Let $pdf(v_i)$ be the empirical distribution function of votes for the winning party (the data are binned with one bin corresponding to 1%) over all territorial units. The distribution function for the model units $pdf(v_i^{(m)})$ is calculated for each set of (f_i, f_e, α) values, where f_i, $f_e \in \{0, 0.01, 0.02, \dots 1\}$, $\alpha \in \{0, 0.1 \dots 5\}$. We report values for the fraud parameters, where the statistic (Eq. **S3**)

$$S(f_i, f_e, \alpha) = \sum_{i=1}^{n} \left(\frac{pdf(v_i) - pdf(v_i^{(m)})}{pdf(v_i)} \right)^2 \qquad \textbf{[S3]}$$

assumes its minimum averaged over 100 realizations over the parameter space (Table S3 shows f_i and f_e).

The extreme fraud parameter f_e is zero (within 1 SD) for almost all elections except elections in Russia (2003, 2007, 2011, and 2012) and Uganda. For very small n ($n < 100$), estimates for f_e become less robust. These elections are also the only elections where the incremental fraud parameter f_i is not close to zero. Values for α for the Russian elections are $\alpha_{Ru03} = 2.5(1)$ (2003), $\alpha_{Ru07} = 2.2(2)$ (2007), $\alpha_{Ru11} = 2.3(3)$ (2011), and $\alpha_{Ru12} = 1.5(2)$ (2012); $\alpha_{\mathrm{Uganda}} = 0.31(3)$ for Uganda. Results for α from countries where f_i is close to zero cannot be detected in a robust way and are irrelevant because there are (almost) no deviations from the fair election case.

Special care is needed in the interpretation of f_i and f_e values in countries where election units contain several polling stations. It may be the case that extreme fraud takes place only in a subset of the polling stations within an electoral unit. In that case, extreme fraud would be indistinguishable from the incremental fraud mechanism.

On Alternative Explanations for Election Irregularities. It is hard to construct other plausible mechanisms leading to a large number of territorial units having 100% turnout and votes for a single party than urn stuffing. The case is not so clear for the smeared out main cluster. In some cases, namely Canada and Finland, this cluster also takes on a slightly different form. This effect clearly does not inflate the turnout as much as the case in Russia and Uganda, but it is nevertheless present.

In Canada, the distribution of vote preferences is bimodal, with one peak around 50% and one peak around 10% (of the vote-eligible population), but with similar turnout levels. This finding is the result of large-scale heterogeneity in the data: English and French Canada. Votes are shown for the winning Conservatives. Looking at their results by province, they tallied 16.5% of votes cast in Quebec but more than 40% of votes cast in 8 of the remaining 12 other provinces. As a consequence, the logarithmic vote rate kurtosis becomes inflated. However, these statistical deviations are perfectly distinguishable from the traces of ballot stuffing, resulting in vanishing fraud parameters on all aggregation levels.

Another possible mechanism leading to irregularities in the voting results is successful voter mobilization. This result may lead to a correlation between turnout and a party's votes. The Finland elections, for example, were marked by radical campaigns by the True Finns. They managed to evenly spread out across the country, with the exception of the Helsinki region, where the winning National Coalition Party performed significantly better than in the rest of the country.

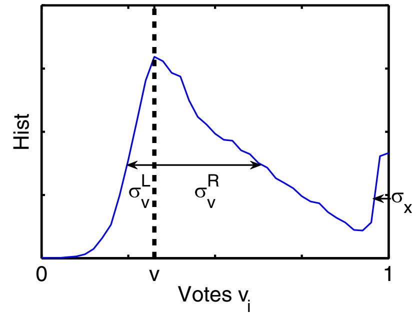

Fig. S1. A stylized version of an empirical vote distribution function shows how v, σ_v^L, σ_v^R, and σ_x are estimated from the election results. v is the maximum of the distribution function. σ_v^L measures the distribution width of values to the left of v (i.e., smaller than v). The incremental fraud with σ_v^R measures the distribution width of values to the right of v (i.e., larger than v). The extreme fraud width σ_x is the width of the peak at 100% votes.

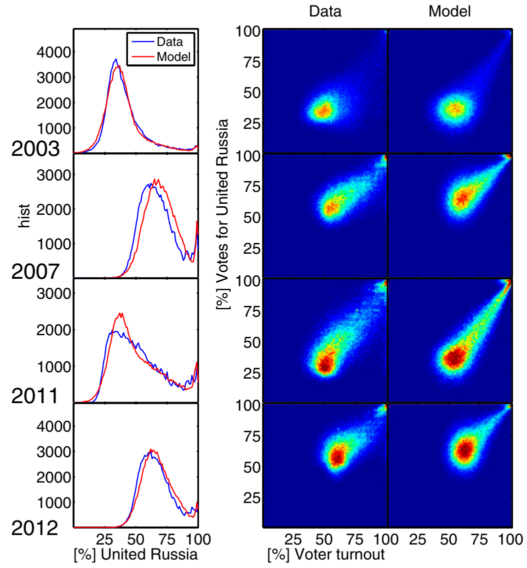

Fig. S2. Comparison of results from the 2003, 2007, 2011, and 2012 Russian elections and the fraud model. In *Left*, the distributions of the number of districts with a given percentage of votes for United Russia are shown for the data (blue) and fraud model (red). *Center* shows the observed vote turnout distributions. The data from 2007, 2011, and 2012 show the same pattern, although the main cluster for United Russia is at a higher percentage of votes. For 2003, there is a smaller number of districts with 100% turnout and votes, and the main cluster is spread out to a smaller extent. *Right* shows fits for the data with the fraud model using parameters f_i = 0.31, f_e = 0.009 (2003); f_i = 0.636, f_e = 0.038 (2007); f_i = 0.64, f_e = 0.033 (2011); and f_i = 0.39, f_e = 0.021 (2011).

Table S1. Descriptive statistics of the election result datasets

Country	Election	n	$\bar{n}_{pop}$	$\bar{a}$	$\bar{v}$	Source
Austria-I	Parliament (2008)	2,371	2,528	0.74 ± 0.06	0.26 ± 0.11	http://www.bmi.gv.at/cms/BMI_wahlen/nationalrat/start.aspx
Austria-II	Parliament (2008)	335	52,340	0.77 ± 0.06	0.29 ± 0.08	
Canada-I	Parliament (2011)	65,372	364	0.52 ± 0.13	0.40 ± 0.21	http://www.elections.ca/scripts/resval/ovr_41ge.asp?prov=&lang=e
Canada-II	Parliament (2011)	307	78,547	0.51 ± 0.04	0.40 ± 0.19	
Czech Republic	Parliament (2003)	5,586	1,298	0.59 ± 0.09	0.74 ± 0.07	http://www.volby.cz/index_en.htm
Finland-I	Parliament (2011)	2,349	1,887	0.70 ± 0.09	0.17 ± 0.10	http://pxweb2.stat.fi/database/StatFin/vaa/evaa/evaa_2011/evaa_2011_fi.asp
Finland-II	Parliament (2011)	335	13,894	0.65 ± 0.06	0.14 ± 0.07	
France-I	President (2007)	31,599	1,157	0.86 ± 0.04	0.31 ± 0.08	http://www.interieur.gouv.fr/sections/a_votre_service/elections
France-II	President 2007	95	440,874	0.84 ± 0.02	0.30 ± 0.03	
Poland-I	European (2004)	24,168	1,216	0.20 ± 0.10	0.20 ± 0.13	http://pkw.gov.pl/
Poland-II	European (2004)	2,481	12,086	0.17 ± 0.05	0.17 ± 0.10	
Poland-III	European (2004)	382	78,498	0.18 ± 0.06	0.21 ± 0.10	
Poland-IV	European (2004)	13	2,306,623	0.20 ± 0.04	0.23 ± 0.07	
Romania	President (2009)	17,808	843	0.52 ± 0.10	0.32 ± 0.12	http://www.bec2009p.ro/rezultate.html
Russia-I	Parliament (2003)	90,182	1,146	0.62 ± 0.17	0.40 ± 0.17	http://cikrf.ru/
Russia-II	Parliament (2003)	2,754	39,532	0.60 ± 0.13	0.40 ± 0.14	
Russia-III	Parliament (2003)	89	1,223,708	0.56 ± 0.09	0.40 ± 0.12	
Russia-I	Parliament (2007)	91,205	1,135	0.70 ± 0.17	0.68 ± 0.14	http://cikrf.ru/
Russia-II	Parliament (2007)	2,750	39,690	0.67 ± 0.13	0.68 ± 0.12	
Russia-III	Parliament (2007)	86	1,269,134	0.65 ± 0.12	0.66 ± 0.11	
Russia-I	Parliament (2011)	90,836	1,147	0.65 ± 0.18	0.52 ± 0.20	http://cikrf.ru/
Russia-II	Parliament (2011)	2,719	39,629	0.63 ± 0.14	0.52 ± 0.18	
Russia-III	Parliament (2011)	84	1,297,729	0.61 ± 0.13	0.50 ± 0.17	
Russia-I	President (2012)	91,197	1,152	0.69 ± 0.15	0.67 ± 0.14	http://cikrf.ru/
Russia-II	President (2012)	2,725	40,149	0.67 ± 0.12	0.67 ± 0.12	
Russia-III	President (2012)	85	1,295,870	0.66 ± 0.10	0.65 ± 0.10	
Spain-I	Parliament (2008)	6,908	4,192	0.77 ± 0.08	0.40 ± 0.14	http://www.infoelectoral.mir.es
Spain-II	Parliament (2008)	52	651,423	0.75 ± 0.05	0.44 ± 0.06	
Spain-III	Parliament (2008)	19	1,782,842	0.74 ± 0.06	0.43 ± 0.05	
Switzerland-I	Parliament (2007)	2,542	1,826	0.50 ± 0.09	0.32 ± 0.15	http://www.portal-stat.admin.ch/nrw/files/de/01b2.xml
Switzerland-II	Parliament (2007)	29	193,680	0.44 ± 0.12	0.24 ± 0.13	
Uganda-I	President (2011)	23,891	582	0.59 ± 0.15	0.68 ± 0.20	http://www.ec.or.ug/eresults.html
Uganda-II	President (2011)	6,597	2,116	0.60 ± 0.12	0.70 ± 0.18	
Uganda-III	President (2011)	1,406	9,911	0.60 ± 0.10	0.70 ± 0.17	
Uganda-IV	President (2011)	394	35,416	0.58 ± 0.10	0.68 ± 0.17	

Each row in the table corresponds to one election of the given type in the respective country (with Roman numbers labeling different aggregation levels). The number of electoral units n, their average electorate $\bar{n}_{pop}$, mean turnout $\bar{a}$, and votes for winning party $\bar{v}$ are shown together with the sources where the data can be downloaded (as of 2012). Units with an electorate smaller than 100 are not considered.

Table S2. Normality tests using standardized moments for the raw vote W_i/N_i and turnout distributions V_i/N_i rescaled to zero mean and unit variance

Country	Election	Turnout skewness	Turnout kurtosis	Vote skewness	Vote kurtosis
Austria-I	Parliament (2008)	−0.98*	4.4*	0.50*	3.2*
Austria-II	Parliament (2008)	−0.51†	2.3†	0.11	2.1
Canada-I	Parliament (2011)	0.25*	7.4*	0.10*	2.2*
Canada-II	Parliament (2011)	−0.40†	3.2†	−0.046†	2.2†
Czech Republic	Parliament (2003)	0.021*	3.3*	−0.58*	3.8*
Finland-I	Parliament (2011)	−1.5*	8.4*	0.71*	3.6*
Finland-II	Parliament (2011)	−1.8*	8.14*	0.56*	3.0*
France-I	President (2007)	−0.45*	3.6*	0.34*	3.7*
France-II	President 2007	−0.38	2.3	0.74*	4.6*
Poland-I	European (2004)	1.7*	10*	0.47*	2.8*
Poland-II	European (2004)	2.2*	20*	0.96*	3.8*
Poland-III	European (2004)	4.9*	41*	0.7*	3.1*
Poland-IV	European (2004)	2.0*	6.7*	0.45	2.4
Romania	President (2009)	0.44*	4.5*	0.48*	3.7*
Russia-I	Parliament (2003)	0.65*	2.8*	1.5*	5.5*
Russia-II	Parliament (2003)	1.0*	4.0*	1.7*	6.3*
Russia-III	Parliament (2003)	1.6*	5.6*	1.6*	5.6*
Russia-I	Parliament (2007)	0.36*	2.1*	0.47*	2.7*
Russia-II	Parliament (2007)	0.77*	2.8*	0.78*	3.4*
Russia-III	Parliament (2007)	1.3*	4.0*	1.4*	4.3*
Russia-I	Parliament (2011)	0.5*	2.3*	0.64*	2.5*
Russia-II	Parliament (2011)	0.90*	3.0*	0.85*	3.1*
Russia-III	Parliament (2011)	1.3*	3.8*	1.2*	3.6*
Russia-I	President (2012)	0.62*	2.6*	0.56*	2.9*
Russia-II	President (2012)	1.1*	3.4*	0.81*	3.7*
Russia-III	President (2012)	1.5*	4.4*	1.4*	4.9*
Spain-I	Parliament (2008)	−1.8*	9.6*	−0.16*	3.2*
Spain-II	Parliament (2008)	−0.9†	3.4†	0.56	3.0
Spain-III	Parliament (2008)	−0.47	1.8	−0.0059	2.74
Switzerland-I	Parliament (2007)	0.30*	4.7*	0.18*	2.9*
Switzerland-II	Parliament (2007)	−0.96†	3.9†	−0.45	2.4
Uganda-I	President (2011)	−0.14*	3.6*	−0.48*	2.4*
Uganda-II	President (2011)	−0.026*	4.1*	−0.59*	2.8*
Uganda-III	President (2011)	−0.094*	4.3*	−0.55*	2.65*
Uganda-IV	President (2011)	−0.19	2.6	−0.35*	2.3*

It becomes apparent that the largest deviations from normality are not observed for Russia and Uganda but for turnout in Poland and Spain. This finding can be attributed to statistical outliers with higher or lower turnouts than the average units. We test for normality using the Jarque–Bera test. If the hypothesis that turnout or vote is normally distributed can be rejected with a P value up to $P \leq 0.005$ or $P \leq 0.05$, then the corresponding skewness and kurtosis values are marked with an asterisk or dagger, respectively.

Table S3. Overview of the results of statistical anomaly tests in the election data discussed in this work

Country	Election	Skewness (ν_i)	Kurtosis (ν_i)	f_i	f_e
Austria-I	Parliament (2008)	0.89*	5.2*	0	0
Austria-II	Parliament (2008)	0.47	2.8	0.0006 ± 0.0008	0.011 ± 0.011
Canada-I	Parliament (2011)	−0.71*	3.4*	0	0
Canada-II	Parliament (2011)	0.82*	2.9*	0.0006 ± 0.0007	0.0096 ± 0.010
Czech Republic	Parliament (2003)	−0.00047*	3.6*	0	0
Finland-I	Parliament (2011)	1.0*	4.8*	0	0
Finland-II	Parliament (2011)	0.97*	4.5*	0.0008 ± 0.0009	0.019 ± 0.015
France-I	President (2007)	0.28*	4.2*	0	0
France-II	President 2007	0.45†	3.7†	0.0003 ± 0.0005	0.0054 ± 0.0080
Poland-I	European (2004)	0.59*	2.9*	0	0
Poland-II	European (2004)	−0.046	2.8	0.003 ± 0.005	0.0053 ± 0.0073
Poland-III	European (2004)	−0.48*	3.8*	0	0
Poland-IV	European (2004)	−0.31	2.8	0.0011 ± 0.0011	0.023 ± 0.016
Romania	President (2009)	0.86*	6.6*	0	0
Russia-I	**Parliament (2003)**	**−2.2***	**11***	**0.31 ± 0.01**	**0.009 ± 0.002**
Russia-II	**Parliament (2003)**	**−2.2***	**9.7***	**0.292 ± 0.008**	**0.002 ± 0.002**
Russia-III	**Parliament (2003)**	**−1.8***	**6.9***	**0.33 ± 0.02**	**0.006 ± 0.003**
Russia-I	**Parliament (2007)**	**−1.9***	**7.6***	**0.636 ± 0.003**	**0.038 ± 0.003**
Russia-II	**Parliament (2007)**	**−2.3***	**9.3***	**0.581 ± 0.002**	**0.028 ± 0.003**
Russia-III	**Parliament (2007)**	**−2.8***	**12***	**0.64 ± 0.03**	**0.03 ± 0.01**
Russia-I	**Parliament (2011)**	**−1.6***	**6.3***	**0.64 ± 0.01**	**0.033 ± 0.004**
Russia-II	**Parliament (2011)**	**−2.0***	**83***	**0.60 ± 0.01**	**0.025 ± 0.003**
Russia-III	**Parliament (2011)**	**−2.4***	**11***	**0.62 ± 0.04**	**0.028 ± 0.010**
Russia-I	**President (2012)**	**−2.3***	**9.7***	**0.39 ± 0.01**	**0.021 ± 0.03**
Russia-II	**President (2012)**	**−2.7***	**13***	**0.33 ± 0.01**	**0.011 ± 0.002**
Russia-III	**President (2012)**	**−3.7***	**21***	**0.38 ± 0.04**	**0.017 ± 0.006**
Spain-I	Parliament (2008)	1.6*	9.1*	0	0
Spain-II	Parliament (2008)	0.052	2.8	0.0012 ± 0.0012	0.019 ± 0.017
Spain-III	Parliament (2008)	0.066	2.3	0.0015 ± 0.0015	0.026 ± 0.021
Switzerland-I	Parliament (2007)	0.93*	5.0*	0	0
Switzerland-II	Parliament (2007)	0.39	2.6	0.0010 ± 0.0010	0.019 ± 0.016
Uganda-I	**President (2011)**	**−0.58***	**7.4***	**0.49 ± 0.01**	**0.011 ± 0.003**
Uganda-II	**President (2011)**	**−0.75***	**8.4***	**0.62 ± 0.01**	**0.022 ± 0.008**
Uganda-III	**President (2011)**	**−0.76***	**7.7***	**0.83 ± 0.01**	**0.011 ± 0.006**
Uganda-IV	**President (2011)**	**0.068**	**3.2**	**0.68 ± 0.02**	**0.009 ± 0.006**

Skewness and kurtosis for the logarithmic vote rate distributions in all Russian elections (at the highest aggregation level) hint at strong anomalies. However, these deviations are less pronounced on levels with higher data resolution. In stark contrast to this finding, the estimates for fraud parameters f_i and f_e show no change when the data are aggregated. For example, if the Russian data are aggregated to 80–90 federal units, the f_e values are still greater than zero by a factor of 2–3 SDs, and the f_i values are greater than zero by a factor of more than 10 SDs. This finding is a robust signal for systematic election irregularities, which can be explained by ballot stuffing. If the hypothesis that ν_i is normally distributed can be rejected by a Jarque–Bera test with a P value up to $P \leq 0.005$ and $P \leq 0.05$, then the corresponding skewness and kurtosis values are marked with an asterisk or dagger, respectively. Results for Russia and Uganda are shown in bold.

Table S4. Sample size, average electorate per unit, skewness and kurtosis for turnout, vote and logarithmic vote rate, and fraud parameters are shown for parliamentary elections in Sweden (2010) and the United Kingdom (2010) and presidential elections in the United States (2000 and 2008)

Country	Election	n	$\bar{n}_{pop}$	Turnout skewness	Turnout kurtosis	Vote skewness	Vote kurtosis	Skewness (ν_i)	Kurtosis (ν_i)	f_i	f_e
Sweden	2010	290	24,564	−0.30*	4.5*	0.06	2.9	0.70*	4.7*	0.0005 ± 0.0004	0
United Kingdom	2010	650	70,290	−0.46*	3.0*	−0.37*	3.3*	0.81*	2.9*	0.0005 ± 0.0003	0
United States	2000	3,096	59,282	−0.58*	5.5*	0.23	3.4	0.62*	12.3*	0.0006 ± 0.0003	0
United States	2008	3,104	73,756	−0.39*	4.3*	0.38	3.1	0.20*	3.3*	0.0005 ± 0.0003	0

The application of the parametric model yields no significant signals of systematic election irregularities. If the hypothesis that turnout, votes, or ν_i is normally distributed can be rejected by a Jarque–Bera test with a P value up to $P \leq 0.005$, then the corresponding skewness and kurtosis values are marked with an asterisk.