Supplementary Document: Bayesian Influence Measures of Joint Models for Longitudinal and Survival Data

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1 Examples of Bayesian Influence Measures

We focus on assessing the influence of a perturbation scheme $\boldsymbol{\omega}$ to the posterior distribution based on three objective functions, these being the $\phi-$divergence, the posterior mean distance, and the Bayes factor as follows.

Example 1 (Bayes factor). The logarithm of the Bayes factor for comparing $\boldsymbol{\omega}$ with $\boldsymbol{\omega}^0$ is

$$\begin{aligned}\mathrm{BF}(\boldsymbol{\omega},\boldsymbol{\omega}^0) &= \log(p(D_o;\boldsymbol{\omega})) - \log(p(D_o;\boldsymbol{\omega}^0)) \\ &= \log(\textstyle\int p(D_o,\mathbf{b};\boldsymbol{\theta},\boldsymbol{\omega})p(\boldsymbol{\theta};\boldsymbol{\omega})d\mathbf{b}d\boldsymbol{\theta}) - \log(\textstyle\int p(D_o,\mathbf{b};\boldsymbol{\theta},\boldsymbol{\omega}^0)p(\boldsymbol{\theta};\boldsymbol{\omega}^0)d\mathbf{b}d\boldsymbol{\theta}).\end{aligned}$$

The value of $\mathrm{BF}(\boldsymbol{\omega},\boldsymbol{\omega}^0)$ can be regarded as a statistic for testing hypotheses of $\boldsymbol{\omega}$ against $\boldsymbol{\omega}^0$ (Kass and Raftery, 1995). Under some smoothness conditions, $\mathrm{BF}(\boldsymbol{\omega},\boldsymbol{\omega}^0)$ is a continuous map from $\mathcal{M}$ to R.

We set $f(\boldsymbol{\omega}) = \mathrm{BF}(\boldsymbol{\omega},\boldsymbol{\omega}^0)$ and consider a smooth curve $\boldsymbol{\omega}(t)$ on $\mathcal{M}$ with $\boldsymbol{\omega}(0) = \boldsymbol{\omega}_0$ and $d_t\boldsymbol{\omega}(t)|_{t=0} = \mathbf{h}$, where $d_t = d/dt$. Thus, we have

$$d_t \log p(D_o;\boldsymbol{\omega}(t))|_{t=0} = \boldsymbol{\nabla} f(\boldsymbol{\omega}(0))^T\mathbf{h},$$

where $\boldsymbol{\nabla} f(\boldsymbol{\omega}(0)) = E\{\partial_\omega \log p(D_o,\mathbf{b},\boldsymbol{\theta};\boldsymbol{\omega}^0)|D_o,\boldsymbol{\omega}^0\}$, in which the conditional expectation is taken with respect to $p(\mathbf{b},\boldsymbol{\theta};D_o,\boldsymbol{\omega}^0)$. We can use MCMC methods to draw samples $\{(\boldsymbol{\theta}^{(s)},\mathbf{b}^{(s)}) : s = 1,\ldots,S_0\}$ from $p(\mathbf{b},\boldsymbol{\theta};D_o)$ and then approximate $\boldsymbol{\nabla} f(\boldsymbol{\omega}(0))$ by using $S_0^{-1}\sum_{s=1}^{S_0}\partial_\omega \log p(D_o,\mathbf{b}^{(s)},\boldsymbol{\theta}^{(s)};\boldsymbol{\omega}^0)$. For instance, for the perturbation to the prior given by $p(\boldsymbol{\theta};t) = p(\boldsymbol{\theta}) + \omega\{g(\boldsymbol{\theta}) - p(\boldsymbol{\theta})\}$, it can be shown that

$$\mathrm{FI}_{BF,\mathbf{h}}(\boldsymbol{\omega}(0)) = \frac{\mathrm{E}\{g(\boldsymbol{\theta})/p(\boldsymbol{\theta})|D_o\}^2}{\mathrm{var}_P\{g(\boldsymbol{\theta})/p(\boldsymbol{\theta})\}} = \frac{\{p_g(D_o)/p(D_o)\}^2}{\mathrm{var}_P\{g(\boldsymbol{\theta})/p(\boldsymbol{\theta})\}},$$

where var_P is taken with respect to $p(\boldsymbol{\theta})$, $p(D_o) = \int p(D_o,\mathbf{b};\boldsymbol{\theta})p(\boldsymbol{\theta})d\mathbf{b}d\boldsymbol{\theta}$, and $p_g(D_o) = \int p(D_o,\mathbf{b};\boldsymbol{\theta})g(\boldsymbol{\theta})d\mathbf{b}d\boldsymbol{\theta}$. Since the ratio of $p_g(D_o)$ to $p(D_o)$ is the Bayes factor in favor of $g(\boldsymbol{\theta})$ against $p(\boldsymbol{\theta})$, the first-order local influence measure is the square of the normalized Bayes factor of $g(\boldsymbol{\theta})$ against $p(\boldsymbol{\theta})$.

Example 2 ($\phi-$divergence). The $\phi-$divergence between two posterior distributions for $\boldsymbol{\omega}_0$ and $\boldsymbol{\omega}$ is defined as

$$\Phi_{RI}(\boldsymbol{\omega}, \boldsymbol{\omega}^0) = \int \phi(R(\mathbf{b}, \boldsymbol{\theta}; \boldsymbol{\omega}, \boldsymbol{\omega}^0))p(\mathbf{b}, \boldsymbol{\theta}; D_o, \boldsymbol{\omega}^0)d\mathbf{b}d\boldsymbol{\theta},$$

where $R(\mathbf{b}, \boldsymbol{\theta}; \boldsymbol{\omega}, \boldsymbol{\omega}^0) = p(\mathbf{b}, \boldsymbol{\theta}; D_o, \boldsymbol{\omega})/p(\mathbf{b}, \boldsymbol{\theta}; D_o, \boldsymbol{\omega}^0)$ and $\phi(\cdot)$ is a convex function with $\phi(1) = 0$, such as the Kullback-Leibler divergence or the χ^2-divergence (Kass et al., 1989).

We set $f(\boldsymbol{\omega}) = \Phi_{RI}(\boldsymbol{\omega}, \boldsymbol{\omega}^0)$ and consider a smooth curve $\boldsymbol{\omega}(t)$ on $\mathcal{M}$ such that $\boldsymbol{\omega}(0) = \boldsymbol{\omega}_0$ and $d_t\boldsymbol{\omega}(t)|_{t=0} = \mathbf{h}$. It can be shown that $\partial_\omega f(\boldsymbol{\omega}(0)) = \mathbf{0}$ and

$$\boldsymbol{H}_f = \ddot{\phi}(1) \int [\partial_\omega \log p(\mathbf{b}, \boldsymbol{\theta}; D_o, \boldsymbol{\omega}^0)]^{\otimes 2} p(\mathbf{b}, \boldsymbol{\theta}; D_o, \boldsymbol{\omega}^0)d\mathbf{b}d\boldsymbol{\theta},$$

where $\mathbf{a}^{\otimes 2} = \mathbf{a}\mathbf{a}^T$ for any vector $\mathbf{a}$ and $\ddot{\phi}(t) = d^2\phi(t)/dt^2$. We need to develop a computational formula for computing $\boldsymbol{H}_f$. Note that $\partial_\omega \log p(\mathbf{b}, \boldsymbol{\theta}; D_o, \boldsymbol{\omega}^0)$ equals

$$\partial_\omega \log p(D_o, \mathbf{b}, \boldsymbol{\theta}; \boldsymbol{\omega}^0) - \int [\partial_\omega \log p(D_o, \mathbf{b}, \boldsymbol{\theta}; \boldsymbol{\omega}^0)] p(\mathbf{b}, \boldsymbol{\theta}; D_o, \boldsymbol{\omega}^0)d\mathbf{b}d\boldsymbol{\theta}.$$

In practice, we use MCMC methods to draw samples $\{(\boldsymbol{\theta}^{(s)}, \mathbf{b}^{(s)}) : s = 1, \ldots, S_0\}$ from $p(\boldsymbol{\theta}, \mathbf{b}; D_o, \boldsymbol{\omega}^0)$ and then approximate $\boldsymbol{H}_f$ using

$$\ddot{\phi}(1)S_0^{-1} \sum_{s=1}^{S_0} [\partial_\omega \log p(\mathbf{b}^{(s)}, D_o, \boldsymbol{\theta}^{(s)}; \boldsymbol{\omega}(0)) - S_0^{-1} \sum_{s'=1}^{S_0} \partial_\omega \log p(\mathbf{b}^{(s')}, D_o, \boldsymbol{\theta}^{(s')}; \boldsymbol{\omega}(0))]^{\otimes 2}.$$

For perturbation schemes to the prior distribution, it can be shown that

$$G(\boldsymbol{\omega}^0) = \int [\partial_\omega \log p(\boldsymbol{\theta}; \boldsymbol{\omega}^0)]^{\otimes 2} p(\boldsymbol{\theta}; \boldsymbol{\omega}^0)d\boldsymbol{\theta}$$

and $\boldsymbol{H}_f = \ddot{\phi}(1)\text{var}[\partial_\omega \log p(\boldsymbol{\theta}; \boldsymbol{\omega}^0)|D_o, \boldsymbol{\omega}^0]$, which are, respectively, the Fisher information matrices of $\boldsymbol{\omega}(t)$ based on the prior and posterior distributions, where $\text{var}(\cdot|D_o, \boldsymbol{\omega}^0)$ denotes the posterior variance. For instance, for $p(\boldsymbol{\theta}; \boldsymbol{\omega}(\boldsymbol{\theta})) = p(\boldsymbol{\theta}) + \omega\{g(\boldsymbol{\theta}) - p(\boldsymbol{\theta})\}$, we can show that

$$\text{SI}_{\Phi_{RI},\mathbf{h}} = \frac{\ddot{\phi}(1)\text{var}\{g(\boldsymbol{\theta})/p(\boldsymbol{\theta})|D_o\}}{\text{var}_P\{g(\boldsymbol{\theta})/p(\boldsymbol{\theta})\}},$$

where $\text{var}_P(\cdot)$ denotes the prior variance.

Example 3 (Posterior mean distance). We measure the distance between the posterior means of $g(\boldsymbol{\theta})$ for $\boldsymbol{\omega}_0$ and $\boldsymbol{\omega}$ (Kass et al., 1989; Gustafson, 1996). Specifically, we define the posterior mean of $g(\boldsymbol{\theta})$ after introducing $\boldsymbol{\omega}$ as

$$M_g(\boldsymbol{\omega}) = \int g(\boldsymbol{\theta}) p(\mathbf{b}, \boldsymbol{\theta}; D_o, \boldsymbol{\omega}) d\mathbf{b} d\boldsymbol{\theta}.$$

Cook's posterior mean distance for characterizing the influence of $\boldsymbol{\omega}$ can be defined as follows:

$$\mathrm{CM}_g(\boldsymbol{\omega}, \boldsymbol{\omega}^0) = \{M_g(\boldsymbol{\omega}) - M_g(\boldsymbol{\omega}^0)\}^T W_g \{M_g(\boldsymbol{\omega}) - M_g(\boldsymbol{\omega}^0)\}, \tag{1}$$

where W_g is chosen to be a positive definite matrix. From here onwards, W_g is chosen to be the inverse of the posterior covariance matrix of $g(\boldsymbol{\theta})$ based on $p(\boldsymbol{\theta}; D_o, \boldsymbol{\omega}^0)$.

We set $f(\boldsymbol{\omega}) = \mathrm{CM}_g(\boldsymbol{\omega}, \boldsymbol{\omega}^0)$ and consider a smooth curve $\boldsymbol{\omega}(t)$ on $\mathcal{M}$ such that $\boldsymbol{\omega}(0) = \boldsymbol{\omega}_0$ and $d_t \boldsymbol{\omega}(t)|_{t=0} = \mathbf{h}$. It can be shown that $\partial_\omega f(\boldsymbol{\omega}(0)) = \mathbf{0}$ and $\boldsymbol{H}_f = \dot{M}_g^T W_g \dot{M}_g$, where

$$\dot{M}_g = \mathrm{Cov}\{h(\boldsymbol{\theta}), \partial_\omega \log p(D_o, \mathbf{b}, \boldsymbol{\theta}; \boldsymbol{\omega}) | D_o, \boldsymbol{\omega}^0\}.$$

We can use MCMC methods to approximate $\dot{M}_g$ and G_g.

2 Simulation studies

2.1 Model Setup

In this simulation, we consider an example with two longitudinal markers and bivariate survival times. The two longitudinal markers are monitored over time and are predictive of the bivariate survival times. In this case, $K = M = 2$. Specifically, each longitudinal response was given by

$$y_{ik}(t_{ijk}) = \eta_{ik}(t_{ijk}, \boldsymbol{b}_{ik}) + \varepsilon_{ijk} = \beta_{k0} + \beta_{k1} t_{ijk} + \beta_{k2} r_i + b_{ik0} + b_{ik1} t_{ijk} + \varepsilon_{ijk} \tag{2}$$

for $i = 1, \ldots, 100$, $k = 1, 2$ and $j = 1, \ldots, n_i$, where the r_i's represent a baseline covariate in the longitudinal model. Moreover, it is assumed that $t_{ij1} = t_{ij2}$ for all i and j, $\boldsymbol{\varepsilon}_{ij} = (\varepsilon_{ij1}, \varepsilon_{ij2})^T$ are independently and identically distributed as $N_2(\mathbf{0}, \boldsymbol{\Sigma})$, and the random effects $\boldsymbol{b}_i = (\boldsymbol{b}_{i1}^T, \boldsymbol{b}_{i2}^T)^T$ are distributed as $N_4(\mathbf{0}, \boldsymbol{\Phi})$, where $\boldsymbol{b}_{ik} = (b_{ik0}, b_{ik1})^T$ for $k = 1, 2$. Here $\boldsymbol{\Sigma}$ and $\boldsymbol{\Phi}$ are covariance matrices. Conditional on $\boldsymbol{b}_i$, the two events and censoring times are assumed to be independent and their marginal hazard functions are given by

$$\lambda_m(t|\boldsymbol{b}_i, \boldsymbol{z}_i) = \lambda_{m0}(t) \exp\{\alpha_{m1}\eta_{i1}(t, \boldsymbol{b}_{i1}) + \alpha_{m2}\eta_{i2}(t, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\} \tag{3}$$

for $m = 1$ and 2, where $\boldsymbol{z}_i = (z_{i1}, z_{i2})^T$ is a vector of time-independent covariates. Let $\boldsymbol{Y}_i(t) = (y_{i1}(t), y_{i2}(t))^T$ and $\boldsymbol{\eta}_i(t, \boldsymbol{b}_i) = (\eta_{i1}(t, \boldsymbol{b}_{i1}), \eta_{i2}(t, \boldsymbol{b}_{i2}))^T$. In this case, the density of $(\boldsymbol{Y}_i, \mathbf{T}_i, \delta_i, \boldsymbol{b}_i)$ given $\boldsymbol{\theta}$ for the i-th subject is given by

$$\begin{aligned} p(\boldsymbol{Y}_i, \mathbf{T}_i, \delta_i, \boldsymbol{b}_i; \boldsymbol{\theta}) = C \prod_{m=1}^{2} \lambda_m(T_{im}|\boldsymbol{b}_i, \boldsymbol{z}_i)^{\delta_{im}} \exp\{-\int_0^{T_{im}} \lambda_m(u|\boldsymbol{b}_i, \boldsymbol{z}_i) du\} \times \\ \prod_{j=1}^{n_i} \left(|\boldsymbol{\Sigma}|^{-1/2} \exp[-\frac{1}{2}\{\boldsymbol{Y}_i(t_{ij}) - \boldsymbol{\eta}_i(t_{ij}, \boldsymbol{b}_i)\}^T \boldsymbol{\Sigma}^{-1} \{\boldsymbol{Y}_i(t_{ij}) - \boldsymbol{\eta}_i(t_{ij}, \boldsymbol{b}_i)\}] \right) \times \\ \left(|\boldsymbol{\Phi}|^{-1/2} \exp(-\frac{1}{2} \boldsymbol{b}_i^T \boldsymbol{\Phi}^{-1} \boldsymbol{b}_i) \right), \end{aligned} \tag{4}$$

where C is a constant independent of $\boldsymbol{\theta}$.

To carry out a Bayesian analysis, we take a joint prior distribution for $\boldsymbol{\theta}$ as follows:

$$\begin{aligned} &\boldsymbol{\alpha}_m = (\alpha_{m1}, \alpha_{m2})^T \sim N(\boldsymbol{\alpha}_m^0, \boldsymbol{H}_\alpha^0),\ \boldsymbol{\gamma}_m \sim N(\boldsymbol{\gamma}_m^0, \boldsymbol{H}_\gamma^0), \boldsymbol{\Sigma}^{-1} \sim \text{Wishart}_2(\boldsymbol{R}^0, \rho^0), \\ &\boldsymbol{\beta}_k = (\beta_{k0}, \beta_{k1}, \beta_{k2})^T \sim N(\boldsymbol{\beta}_k^0, \boldsymbol{H}_\beta^0), \boldsymbol{\Phi}^{-1} \sim \text{Wishart}_4(\boldsymbol{R}_\phi^0, \rho_\phi^0) \end{aligned} \tag{5}$$

for $k, m = 1$ and 2, where $\boldsymbol{\alpha}_m^0$, $\boldsymbol{H}_\alpha^0$, $\boldsymbol{\gamma}_m^0$, $\boldsymbol{H}_\gamma^0$, $\boldsymbol{R}^0$, ρ^0, $\boldsymbol{R}_\phi^0$, and ρ_ϕ^0 are pre-specified hyper-parameters. For the baseline hazard $\lambda_m(\cdot)$, we take a piecewise constant hazards model with 250 subintervals with equal lengths such that $\lambda_{m0}(t) = \sum_{l=1}^{L} h_{ml} \mathbf{1}(t \in (c_{l-1}, c_l])$, where the c_l's are prespecified constants. Furthermore, we take $h_{ml} \sim \Gamma(\tau_{0l}, \tau_{1l})$ for $l = 1, \ldots, L = 250$ and $m = 1$ and 2. Finally, we can use MCMC methods (Ibrahim et al., 2001) to conduct Bayesian influence analysis on $\boldsymbol{\theta}$ and $\boldsymbol{b}$.

2.2 MCMC Algorithm

We need to introduce some notation. Let $\boldsymbol{t}_{ij} = (t_{ij1}, t_{ij2})^T$, $\boldsymbol{Y}_i(\boldsymbol{t}_{ij}) = (y_{i1}(t_{ij1}), y_{i2}(t_{ij2}))^T$, and $\boldsymbol{\eta}_i(\boldsymbol{t}_{ij}, \boldsymbol{b}_i) = (\eta_{i1}(t_{ij1}, \boldsymbol{b}_{i1}), \eta_{i2}(t_{ij2}, \boldsymbol{b}_{i2}))^T$. We define

$$
\begin{aligned}
\boldsymbol{Y} &= \{\boldsymbol{Y}_i(\boldsymbol{t}_{ij}) : i = 1, \ldots, 100, j = 1, \ldots, n_i\},\ \boldsymbol{T} = \{T_{im} : i = 1, \ldots, 100, m = 1, 2\}, \\
\boldsymbol{\Delta} &= \{\delta_{im} : i = 1, \ldots, 100, m = 1, 2\},\ \boldsymbol{b} = \{\boldsymbol{b}_i : i = 1, \ldots, 100\}, \\
\boldsymbol{t} &= \{t_{ijk} : i = 1, \ldots, 100, j = 1, \ldots, n_i, k = 1, 2\}, \\
\boldsymbol{r} &= \{r_1, \ldots, r_{100}\},\ \boldsymbol{Z} = \{\boldsymbol{z}_i : i = 1, \ldots, 100\}, \\
\boldsymbol{h} &= \{h_{ml} : m = 1, 2, l = 1, \ldots, L\}, \\
\boldsymbol{\theta} &= \{\boldsymbol{\Phi}, \boldsymbol{\Sigma}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{h}\}.
\end{aligned}
$$

Then, the joint probability density function of $(\boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{\theta})$ given $(\boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t})$ is proportional to

$$
\begin{aligned}
&|\boldsymbol{\Sigma}|^{-\frac{N+\rho_0-3}{2}} |\boldsymbol{\Phi}|^{-\frac{100+\rho_\phi^0-5}{2}} \exp\Bigg[-\frac{1}{2}\sum_{i=1}^{100}\sum_{j=1}^{n_i}(\boldsymbol{Y}_i(t_{ij}) - \boldsymbol{\eta}_i(t_{ij}, \boldsymbol{b}_i))^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{Y}_i(t_{ij}) - \boldsymbol{\eta}_i(t_{ij}, \boldsymbol{b}_i)) \\
&+ \sum_{m=1}^{2}\sum_{l=1}^{L}\sum_{i=1}^{100}\left\{\delta_{im}\left(\alpha_{m1}\eta_{i1}(t^*_{mil}, \boldsymbol{b}_{i1}) + \alpha_{m2}\eta_{i2}(t^*_{mil}, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\right) - h_{ml}B_{mil}\right\} \\
&+ \sum_{m=1}^{2}\sum_{l=1}^{L} d_{ml}\log(h_{ml}) - \frac{1}{2}\sum_{i=1}^{100}\boldsymbol{b}_i^T\boldsymbol{\Phi}^{-1}\boldsymbol{b}_i - \frac{1}{2}\sum_{m=1}^{2}(\boldsymbol{\alpha}_m - \boldsymbol{\alpha}_m^0)^T(\boldsymbol{H}_\alpha^0)^{-1}(\boldsymbol{\alpha}_m - \boldsymbol{\alpha}_m^0) \qquad (6) \\
&- \frac{1}{2}\mathrm{tr}(\boldsymbol{\Sigma}^{-1}\boldsymbol{R}_0^{-1}) - \frac{1}{2}\sum_{m=1}^{2}(\boldsymbol{\gamma}_m - \boldsymbol{\gamma}_m^0)^T(\boldsymbol{H}_\gamma^0)^{-1}(\boldsymbol{\gamma}_m - \boldsymbol{\gamma}_m^0) - \sum_{m=1}^{2}\sum_{l=1}^{L}\tau_{1l}h_{ml} \\
&- \frac{1}{2}\sum_{k=1}^{2}(\boldsymbol{\beta}_k - \boldsymbol{\beta}_k^0)^T(\boldsymbol{H}_\beta^0)^{-1}(\boldsymbol{\beta}_k - \boldsymbol{\beta}_k^0) - \frac{1}{2}\mathrm{tr}(\boldsymbol{\Phi}^{-1}(\boldsymbol{R}_\phi^0)^{-1})\Bigg]\left(\prod_{m=1}^{2}\prod_{l=1}^{L}h_{ml}^{\tau_{0l}-1}\right),
\end{aligned}
$$

where $N = \sum_{i=1}^{n} n_i$, d_{ml} is the number of failures in the lth time interval $I_{ml} = (c_{m,l-1}, c_{m,l}]$, and t^*_{mil} denotes the nearest past time point where responses are taken. Moreover, B_{mil} is defined as follows:

(i) if $T_{im} < c_{m,l-1}$, $B_{mil} = 0$;

(ii) if $T_{im} > c_{m,l}$, letting $(h_{mi1}, s_{mi1}) = \max\{(h, s) : A^*_{ihs} \leq c_{m,l-1}\}$ and $(h_{mi2}, s_{mi2}) =$

$\max\{(h,s): A^*_{ihs} \le c_{m,l}\}$, where A^*_{ihs} is the rescaled t_{ihs} so that A^*_{ihs} has the same unit as T_{im}, then if $h_{mi1} = h_{mi2}$ and $s_{mi1} = s_{mi2}$,

$$B_{mil} = (c_{m,l} - c_{m,l-1}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi1},s_{mi1}}, \boldsymbol{b}_{i1}) + \alpha_{m2}\eta_{i2}(t_{i,h_{mi1},s_{mi1}}, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\};$$

if $h_{mi1} = h_{mi2}$ and $s_{mi1} < s_{mi2}$,

$$\begin{aligned} B_{mil} = \;& (A^*_{i,h_{mi1},s_{mi1}+1} - c_{m,l-1}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi1},s_{mi1}+1}, \boldsymbol{b}_{i1}) + \alpha_{m2}\eta_{i2}(t_{i,h_{mi1},s_{mi1}+1}, \boldsymbol{b}_{i2}) \\ & + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\} + \sum_{l=s_{mi1}+1}^{s_{mi2}} (A^*_{i,h_{mi1},l+1} - A^*_{i,h_{mi1},l}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi1},l}, \boldsymbol{b}_{i1}) \\ & + \alpha_{m2}\eta_{i2}(t_{i,h_{mi1},l}, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\} + (c_{m,l} - A^*_{i,h_{mi1},s_{mi2}}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi1},s_{mi2}}, \boldsymbol{b}_{i1}) \\ & + \alpha_{m2}\eta_{i2}(t_{i,h_{mi1},s_{mi2}}, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\}; \end{aligned}$$

if $s_{mi1} = s_{mi2}$ and $h_{mi1} < h_{mi2}$,

$$\begin{aligned} B_{mil} = \;& (A^*_{i,h_{mi1}+1,s_{mi1}} - c_{m,l-1}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi1}+1,s_{mi1}}, \boldsymbol{b}_{i1}) + \alpha_{m2}\eta_{i2}(t_{i,h_{mi1}+1,s_{mi1}}, \boldsymbol{b}_{i2}) \\ & + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\} + \sum_{l=h_{mi1}+1}^{h_{mi2}} (A^*_{i,l+1,s_{mi1}} - A^*_{i,l,s_{mi1}}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,l,s_{mi1}}, \boldsymbol{b}_{i1}) \\ & + \alpha_{m2}\eta_{i2}(t_{i,l,s_{mi1}}, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\} + (c_{m,l} - A^*_{i,h_{mi2},s_{mi1}}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi2},s_{mi1}}, \boldsymbol{b}_{i1}) \\ & + \alpha_{m2}\eta_{i2}(t_{i,h_{mi2},s_{mi1}}, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\}; \end{aligned}$$

if $h_{mi1} < h_{mi2}$ and $s_{mi1} < s_{mi2}$,

$$\begin{aligned} B_{mil} = \;& (A^*_{i,h_{mi1}+1,s_{mi1}+1} - c_{m,l-1}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi1}+1,s_{mi1}+1}, \boldsymbol{b}_{i1}) + \alpha_{m2}\eta_{i2}(t_{i,h_{mi1}+1,s_{mi1}+1}, \boldsymbol{b}_{i2}) \\ & + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\} + \sum_{\kappa=h_{mi1}+1}^{h_{mi2}} \sum_{l=s_{mi1}+1}^{s_{mi2}} (A^*_{i,\kappa+1,l+1} - A^*_{i\kappa l}) \exp\{\alpha_{m1}\eta_{i1}(t_{i\kappa l}, \boldsymbol{b}_{i1}) \\ & + \alpha_{m2}\eta_{i2}(t_{i\kappa l}, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\} + (c_{m,l} - A^*_{i,h_{mi2},s_{mi2}}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi2},s_{mi2}}, \boldsymbol{b}_{i1}) \\ & + \alpha_{m2}\eta_{i2}(t_{i,h_{mi2},s_{mi2}}, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\}; \end{aligned}$$

(iii) if $c_{m,l-1} < T_{im} \le c_{m,l}$, using $h_{mi1}, h_{mi2}, s_{mi1}$ and s_{mi2} given in (ii), then if $(h_{mi1}, s_{mi1}) = (h_{mi2}, s_{mi2})$ or $T_{im} \le A^*_{i,h_{mi1}+1,s_{mi1}}$ when $h_{mi1} < h_{mi2}$ and $s_{mi1} = s_{mi2}$ or $T_{im} \le A^*_{i,h_{mi1},s_{mi1}+1}$ when $h_{mi1} = h_{mi2}$ and $s_{mi1} < s_{mi2}$ or $T_{im} \le A^*_{i,h_{mi1}+1,s_{mi1}+1}$ when $h_{mi1} < h_{mi2}$ and $s_{mi1} < s_{mi2}$,

$$B_{mil} = (T_{im} - c_{m,l-1}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi1},s_{mi1}}, \boldsymbol{b}_{i1}) + \alpha_{m2}\eta_{i2}(t_{i,h_{mi1},s_{mi1}}, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\},$$

and otherwise, we define

$$\begin{aligned} B_{mil} = \; & (A^*_{i,h_{mi1}+1,s_{mi1}+1} - c_{m,l-1}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi1}+1,s_{mi1}+1}, \boldsymbol{b}_{i1}) + \alpha_{m2}\eta_{i2}(t_{i,h_{mi1}+1,s_{mi1}+1}, \boldsymbol{b}_{i2}) \\ & + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\} + \sum_{\kappa=h_{mi1}+1}^{\iota_{i1}} \sum_{l=s_{mi1}+1}^{\iota_{i2}} (A^*_{i,\kappa+1,l+1} - A^*_{i\kappa l}) \exp\{\alpha_{m1}\eta_{i1}(t_{i\kappa l}, \boldsymbol{b}_{i1}) \\ & + \alpha_{m2}\eta_{i2}(t_{i\kappa l}, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\} + (T_{im} - A^*_{i,\iota_{i1},\iota_{i2}}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,\iota_{i1},\iota_{i2}}, \boldsymbol{b}_{i1}) \\ & + \alpha_{m2}\eta_{i2}(t_{i,\iota_{i1},\iota_{i2}}, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\}, \end{aligned}$$

where $h_{mi1} + 1 \leq \iota_{i1} \leq h_{mi2}$ and $s_{mi1} \leq \iota_{i2} \leq s_{mi2}$ are chosen so that $A^*_{i,\iota_{i1},\iota_{i2}} < T_{im} \leq A^*_{i,\iota_{i1}+1,\iota_{i2}+1}$.

When (h_{mi1}, s_{mi1}) does not exist, we define $h_{mi1} = s_{mi1} = 1$, and the calculation of B_{mil} needs a minor adjustment.

The Gibbs sampler is used to sample a sequence of random observations from the above joint posterior distribution given in (6). Specifically, $\{\boldsymbol{\Phi}, \boldsymbol{\Sigma}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{h}, \boldsymbol{b}\}$ are iteratively drawn from the following full conditional distributions:

$$\begin{aligned} & p(\boldsymbol{\Phi}^{-1}|\boldsymbol{b}), \\ & p(\boldsymbol{\Sigma}^{-1}|\boldsymbol{Y}, \boldsymbol{t}, \boldsymbol{b}), \\ & p(\boldsymbol{\alpha}_m, \boldsymbol{\gamma}_m|\boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{h}), \\ & p(\boldsymbol{\beta}_k|\boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{h}), \\ & p(\boldsymbol{b}|\boldsymbol{\Delta}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{h}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}), \\ & p(\boldsymbol{h}|\boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2). \end{aligned}$$

The above mentioned full conditional distributions are briefly discussed as follows. Let $\mathbf{a}^{\otimes 2} =$

$\mathbf{a}\mathbf{a}^T$ for any vector or matrix $\mathbf{a}$. It is easily shown from (6) and (5) that

$$p(\boldsymbol{\Phi}^{-1}|\boldsymbol{b}) \sim \text{Wishart}_4(\rho_\phi^0 + 100, (\boldsymbol{R}_\phi^0)^{-1} + \sum_{i=1}^{n} \boldsymbol{b}_i \boldsymbol{b}_i^T),$$

$$p(\boldsymbol{\Sigma}^{-1}|\boldsymbol{Y}, \boldsymbol{t}, \boldsymbol{b}) \sim \text{Wishart}_2(\rho^0 + \sum_{i=1}^{100} n_i, ((\boldsymbol{R}^0)^{-1} + \sum_{i=1}^{100}\sum_{j=1}^{n_i}[\boldsymbol{Y}_i(\boldsymbol{t}_{ij}) - \boldsymbol{\eta}_i(\boldsymbol{t}_{ij}, \boldsymbol{b}_i)]^{\otimes 2})^{-1}),$$

$$p(h_{ml}|\boldsymbol{b}, \boldsymbol{t}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2) \sim \Gamma(\tau_{0l} + d_{ml}, \tau_{1l} + \sum_{i=1}^{100} B_{mil}).$$

It follows from (6) and (5) that $p(\boldsymbol{\alpha}_m, \boldsymbol{\gamma}_m|\boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{h})$ is proportional to

$$\exp\left[\sum_{l=1}^{L}\sum_{i=1}^{100}\left\{\delta_{im}\left(\alpha_{m1}\eta_{i1}(t^*_{mil}, \boldsymbol{b}_{i1}) + \alpha_{m2}\eta_{i2}(t^*_{mil}, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\right) - h_{ml}B_{mil}\right\}\right. \tag{7}$$
$$\left.-\frac{1}{2}(\boldsymbol{\alpha}_m - \boldsymbol{\alpha}_m^0)^T(\boldsymbol{H}_\alpha^0)^{-1}(\boldsymbol{\alpha}_m - \boldsymbol{\alpha}_m^0) - \frac{1}{2}(\boldsymbol{\gamma}_m - \boldsymbol{\gamma}_m^0)^T(\boldsymbol{H}_\gamma^0)^{-1}(\boldsymbol{\gamma}_m - \boldsymbol{\gamma}_m^0)\right].$$

Moreover, $p(\boldsymbol{\beta}_k|\boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{h})$ is proportional to

$$\exp\left[-\frac{1}{2}\sum_{i=1}^{100}\sum_{j=1}^{n_i}(\boldsymbol{Y}_i(t_{ij}) - \boldsymbol{\eta}_i(t_{ij}, \boldsymbol{b}_i))^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{Y}_i(t_{ij}) - \boldsymbol{\eta}_i(t_{ij}, \boldsymbol{b}_i)) + \right.$$
$$\left.\sum_{m=1}^{2}\sum_{l=1}^{L}\sum_{i=1}^{100}(\delta_{im}\alpha_{mk}\eta_{ik}(t^*_{mil}, \boldsymbol{b}_{ik}) - h_{ml}B_{mil}) - \frac{1}{2}(\boldsymbol{\beta}_k - \boldsymbol{\beta}_k^0)^T(\boldsymbol{H}_\beta^0)^{-1}(\boldsymbol{\beta}_k - \boldsymbol{\beta}_k^0)\right] \tag{8}$$

and $p(\boldsymbol{b}_i|\boldsymbol{\Delta}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{h})$ is proportional to

$$\exp\left[-\frac{1}{2}\sum_{j=1}^{n_i}(\boldsymbol{Y}_i(t_{ij}) - \boldsymbol{\eta}_i(t_{ij}, \boldsymbol{b}_i))^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{Y}_i(t_{ij}) - \boldsymbol{\eta}_i(t_{ij}, \boldsymbol{b}_i))\right. \tag{9}$$
$$\left.+\sum_{m=1}^{2}\sum_{l=1}^{L}\left\{\delta_{im}\left(\alpha_{m1}\eta_{i1}(t^*_{mil}, \boldsymbol{b}_{i1}) + \alpha_{m2}\eta_{i2}(t^*_{mil}, \boldsymbol{b}_{i2})\right) - h_{ml}B_{mil}\right\} - \frac{1}{2}\boldsymbol{b}_i^T\boldsymbol{\Phi}^{-1}\boldsymbol{b}_i\right].$$

Let $\boldsymbol{\Omega}_u^{-1} = \text{diag}((\boldsymbol{H}_\alpha^0)^{-1}, (\boldsymbol{H}_\gamma^0)^{-1}) + \sum_{i=1}^{100}\sum_{l=1}^{L} h_{ml}\mathcal{B}_{mil}$, where $\mathcal{B}_{mil} = \partial^2 B_{mil}/\partial\boldsymbol{u}_m\partial\boldsymbol{u}_m^T|_{\boldsymbol{u}_m=\boldsymbol{0}}$ with $\boldsymbol{u}_m = (\boldsymbol{\alpha}_m^T, \boldsymbol{\gamma}_m^T)^T$. The Metropolis-Hasting (MH) algorithm is implemented to draw random samples from the conditional distribution (7) as follows. At the $(d+1)$st iteration with a $\boldsymbol{u}_m^{(d)}$, a new candidate $\boldsymbol{u}_m$ is generated from $N(\boldsymbol{u}_m^{(d)}, \sigma_u^2\boldsymbol{\Omega}_u)$ and is accepted with probability

$$\min\left\{1, \frac{p(\boldsymbol{\alpha}_m, \boldsymbol{\gamma}_m|\boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{h})}{p(\boldsymbol{\alpha}_m^{(d)}, \boldsymbol{\gamma}_m^{(d)}|\boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{h})}\right\},$$

in which σ_u^2 is selected such that the average acceptance rate is about 25% or more.

Similarly, the MH algorithm for sampling $\boldsymbol{\beta}_k$ from $p(\boldsymbol{\beta}_k|\boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{h})$ given in (8) is implemented as follows. At the $(d+1)$th iteration with a $\boldsymbol{\beta}_k^{(d)}$, a new candidate $\boldsymbol{\beta}_k$ is simulated from the proposal distribution $N(\boldsymbol{\beta}_k^{(d)}, \sigma_\beta^2 \boldsymbol{\Omega}_\beta)$, where

$$\boldsymbol{\Omega}_\beta^{-1} = \sum_{i=1}^{100} \sum_{j=1}^{n_i} \sigma^{kk} \boldsymbol{q}_{ijk} \boldsymbol{q}_{ijk}^T + \sum_{m=1}^{2} \sum_{l=1}^{L} \sum_{i=1}^{100} h_{ml} \mathcal{C}_{mil} + (\boldsymbol{H}_\beta^0)^{-1}$$

with $\boldsymbol{q}_{ijk}^T = (1, t_{ijk}, r_i)$ and $\mathcal{C}_{mil} = \partial^2 B_{mil} / \partial \boldsymbol{\beta}_k \partial \boldsymbol{\beta}_k^T |_{\boldsymbol{\beta}_k = \mathbf{0}}$, in which σ^{kk} is the (k,k)th component of $\boldsymbol{\Sigma}^{-1}$. The acceptance probability is

$$\min \left\{ 1, \frac{p(\boldsymbol{\beta}_k|\boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{h})}{p(\boldsymbol{\beta}_k^{(d)}|\boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{h})} \right\}.$$

Sampling $\boldsymbol{b}_i$ from $p(\boldsymbol{b}_i|\boldsymbol{\Delta}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{h})$ as given in (9) can be implemented as follows. At the $(d+1)$st iteration with a current value $\boldsymbol{b}_i^{(d)}$, a new candidate $\boldsymbol{b}_i$ is simulated from the proposal distribution $N(\boldsymbol{b}_i^{(d)}, \sigma_b^2 \boldsymbol{\Omega}_b)$ and is accepted with probability

$$\min \left\{ 1, \frac{p(\boldsymbol{b}_i|\boldsymbol{\Delta}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{h})}{p(\boldsymbol{b}_i^{(d)}|\boldsymbol{\Delta}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{h})} \right\},$$

where $\boldsymbol{\Omega}_b = \boldsymbol{\Phi}^{-1} + \sum_{j=1}^{n_i} \boldsymbol{A}_{ij}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{A}_{ij} + \sum_{m=1}^{2} \sum_{l=1}^{L} h_{ml} \partial^2 B_{mil} / \partial \boldsymbol{b}_i \partial \boldsymbol{b}_i^T |_{\boldsymbol{b}_i = \mathbf{0}}$ with

$$\boldsymbol{A}_{ij} = \begin{pmatrix} 1 & t_{ij1} & 0 & 0 \\ 0 & 0 & 1 & t_{ij2} \end{pmatrix}.$$

2.3 Influence analysis

In the first simulation study, we considered the JMLS specified in (2)-(4) with the following additional specifications. In model (2), the longitudinal measurements were observed at time points $t_{ij1} = t_{ij2} = 0.0, 0.5, 1.0, 1.5, 2.0$ and r_i was generated from $N(0, 0.8)$ for $i = 1, \ldots, 100$ and other true parameter values were given by $(\beta_{10}, \beta_{11}, \beta_{12}) = (0.4, 0.3, 0.5)$, $(\beta_{20}, \beta_{21}, \beta_{22}) =$

$(0.2, 0.4, 0.3)$, and

$$\boldsymbol{\Sigma} = \begin{pmatrix} 0.4 & 0.1 \\ 0.1 & 0.4 \end{pmatrix}, \boldsymbol{\Phi}_1 = \begin{pmatrix} 0.4 & 0.2 \\ 0.2 & 0.4 \end{pmatrix}, \boldsymbol{\Phi}_2 = \begin{pmatrix} 0.3 & 0.1 \\ 0.1 & 0.3 \end{pmatrix}.$$

In model (3), the two components of $\boldsymbol{z}_i = (z_{i1}, z_{i2})^T$ were, respectively, generated from $z_{i1} \sim N(0.0, 0.15)$ and $z_{i2} \sim N(0.5z_{i1}, 0.20)$. Moreover, the survival times T_{1i} and T_{2i} were independently generated from $\lambda_m(t|\boldsymbol{b}_i, \boldsymbol{z}_i)$ with the following specifications:

$$\lambda_{10}(t) = 0.85, \lambda_{20}(t) = 0.70, \alpha_{11} = -\alpha_{21} = 0.3, \alpha_{12} = -\alpha_{22} = 0.4, \text{and } \boldsymbol{\gamma}_1 = \boldsymbol{\gamma}_2 = (0.25, 0.35)^T.$$

The censoring times C_{mi} for $m = 1$ and 2 were independently generated via $C_{mi} = 1.2$ when $1.2 > 4.5u_{mi}$ and $4.5u_{mi}$ otherwise, where u_{mi} was independently generated from a uniform distribution $U(0, 1)$.

To carry out the Bayesian analysis, we specify the prior distribution of $\boldsymbol{\theta}$ according to (5). The hyperparameters of $p(\boldsymbol{\theta})$ were set as $\boldsymbol{\alpha}_1^0 = (0.3, 0.4)^T$, $\boldsymbol{\alpha}_2^0 = (-0.3, -0.4)^T$, $\boldsymbol{H}_\alpha^0 = \boldsymbol{I}_2$, $\boldsymbol{\gamma}_m^0 = (0.25, 0.35)^T$ for $m = 1$ and 2, $\boldsymbol{H}_\gamma^0 = \boldsymbol{I}_2$, $\boldsymbol{\beta}_1^0 = (0.4, 0.3, 0.5)^T$, $\boldsymbol{\beta}_2^0 = (0.2, 0.4, 0.3)^T$, $\boldsymbol{H}_\beta^0 = \boldsymbol{I}_3$, $\rho^0 = \rho_1^0 = \rho_2^0 = 10$, $\boldsymbol{R}^0 = 5\boldsymbol{\Sigma}$, $\boldsymbol{R}_{\phi k}^0 = 5\boldsymbol{\Phi}_k$ for $k = 1$ and 2, and $\tau_{0l} = 10.0$ and $\tau_{1l} = 8.0$ for $l = 1, \ldots, 250$.

We simultaneously perturbed the mean of the longitudinal measures $\boldsymbol{Y}_i(t)$, the distribution of $\boldsymbol{b}_i$, the prior distributions of $\boldsymbol{\alpha}_m$ and $\boldsymbol{\gamma}_m$ and the marginal hazard function $\lambda_m(t|\boldsymbol{b}_i, \boldsymbol{z}_i)$ as follows:

$$\begin{aligned}
&\boldsymbol{Y}_i(t_{ij}) = \boldsymbol{\eta}_i(t_{ij}, \boldsymbol{b}_i) + \omega_{y,i}\mathbf{1}_2 + \boldsymbol{\varepsilon}_{ij},\ p(\boldsymbol{b}_i; \boldsymbol{\theta}_b, \boldsymbol{\omega}_{b,i}) \sim N_4(\mathbf{0}, \omega_{b,i}^{-1}\Phi), \\
&\lambda_m(t|\boldsymbol{b}_i, \boldsymbol{z}_i, \omega_\lambda) = \lambda_{m0}(t)\exp\{\alpha_{m1}\eta_{i1}(t, \boldsymbol{b}_{i1}) + \alpha_{m2}\eta_{i2}(t, \boldsymbol{b}_{i2}) + \omega_{\lambda,m}\boldsymbol{z}_i^T\boldsymbol{\gamma}_m\}, \\
&p(\boldsymbol{\alpha}_m|\omega_{\alpha1}, \omega_{\alpha2}) \sim N(\boldsymbol{\alpha}_m^0 + \omega_{\alpha2}\mathbf{1}_2, \boldsymbol{H}_\alpha^0/\omega_{\alpha1}), \\
&p(\boldsymbol{\gamma}_m|\omega_{\gamma1}, \omega_{\gamma2}) \sim N(\boldsymbol{\gamma}_m^0 + \omega_{\gamma2}\mathbf{1}_2, \boldsymbol{H}_\gamma^0/\omega_{\gamma1}).
\end{aligned} \tag{10}$$

Then, we calculated $G(\boldsymbol{\omega}^0)$, and then chose the perturbation scheme $\tilde{\boldsymbol{\omega}} = \boldsymbol{\omega}^0 + G(\boldsymbol{\omega}^0)^{1/2}(\boldsymbol{\omega} - \boldsymbol{\omega}^0)$.

Subsequently, we used the logarithm of the Bayes factor to calculate the associated local influence measures $\mathbf{v}_{\max}^{f} = \text{argmax}\{\text{FI}_f[\mathbf{v}](\tilde{\boldsymbol{\omega}}(0))\}$ using the aforementioned MCMC methods. To introduce some outliers, we changed the longitudinal measurements of the last two individuals, $\boldsymbol{Y}_i(t_{ij1})$, to $\boldsymbol{Y}_i(t_{ij1}) + 2.0\mathbf{1}_2$ for $j = 1, \ldots, n_i$ and $i = 99, 100$. Posterior estimates of the parameters under the perturbations are obtained by the Gibbs sampler, and a total of 5000 iterations after 5000 burn-in are used to compute the local influence measures with respect to a given perturbation scheme. Cases 99 and 100 were detected to be influential by our local influence measures (Fig. 1(a)). Furthermore, we used the same setup except that we employed a perturbed prior distribution for $\boldsymbol{\gamma}_m$, namely $p(\boldsymbol{\gamma}_m) = N(\mathbf{1}_2, 0.08\boldsymbol{I}_2)$, and then we applied the same MCMC methods, perturbation scheme, and local influence measures. Cases 99 and 100 and the perturbed prior distribution of $\boldsymbol{\gamma}_m$ were identified to be influential (Fig. 1(b)). Figures 1(c) and 1(d) show that the metric tensor $g_{ii}(\boldsymbol{\omega}^0)$ for perturbation (10) has little change for all the individuals, and there is a large change for the metric tensor $g_{ii}(\boldsymbol{\omega}^0)$ corresponding to the perturbation in the hazard function.

In the second simulation study, we used almost the same setup except that we employed a single-case perturbation to the variances of the errors in model (2). In this case, $g_{y,ii} = n_i$ for $i = 1, \ldots, n$ in the metric tensor $\boldsymbol{G}(\boldsymbol{\omega}^0)$. We considered the local influence measures based on both the logarithm of the Bayes factor and the Kullback-Leibler divergence. As expected, cases 99 and 100 were detected to be influential by our local influence measures $\mathbf{v}_{\max}^{B}$ and $\text{SI}_{\text{D}_{\phi,\mathbf{e}_j}}$ with the two priors of $\boldsymbol{\gamma}_m$, namely $p(\boldsymbol{\gamma}_m) = N(\boldsymbol{\gamma}_m^0, \boldsymbol{H}_\gamma^0)$ (Figs. 2(a) and 2(c)) and $p(\boldsymbol{\gamma}_m) = N(\mathbf{1}_2, 0.08\boldsymbol{I}_2)$ (Figs. 2(b) and 2(d)). Also, the perturbed prior distribution of $\boldsymbol{\gamma}_m$ was identified to have a big effect (Figs. 2(b) and 2(d)). Figures 2(e) and 2(f) show that the metric tensor $g_{ii}(\boldsymbol{\omega}^0)$ for the variance perturbation has little change for all the individuals, and there is a large change for the metric tensor $g_{ii}(\boldsymbol{\omega}^0)$ corresponding to the perturbation in the hazard function.

In the third simulation study, we examined whether our local influence measures can detect the misspecified relationship between the survival time and the covariates of interest. We generated the data using the same setting as specified in the above simulation studies, but the hazard functions were taken as

$$\lambda_m(t|\boldsymbol{b}_i, \boldsymbol{z}_i) = \lambda_{m0} \exp\{\alpha_{m1}\eta_{i1}(t, \boldsymbol{b}_{i1}) + \alpha_{m2}\eta_{i2}(t, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m + atz_{i2}\} \tag{11}$$

for $m = 1$ and 2 with the values of parameters $(\lambda_{m0}, \alpha_{m1}, \alpha_{m2}, \boldsymbol{\gamma}_m)$ being pre-specified for a given value of a. However, we fitted the model specified in (2) and

$$\lambda_m(t|\boldsymbol{b}_i, \boldsymbol{z}_i) = \lambda_{m0}(t) \exp\{\alpha_{m1}\eta_{i1}(t, \boldsymbol{b}_{i1}) + \alpha_{m2}\eta_{i2}(t, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\}$$

using the same priors as given in the first simulation study. The fitted model would be misspecified if $a \neq 0$. We considered a global perturbation as follows:

$$\lambda_m(t|\boldsymbol{b}_i, \boldsymbol{z}_i, \omega) = \lambda_{m0} \exp\{\alpha_{m1}\eta_{i1}(t, \boldsymbol{b}_{i1}) + \alpha_{m2}\eta_{i2}(t, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m + \omega t z_{i2}\}, \tag{12}$$

which was used to check whether the local influence measures can detect the missing interaction term between time and z_{i2}. In this case, $\omega = 0$ represents no perturbation. The local influence measure of the logarithm of the Bayes factor was calculated and denoted as t_B. Then, 1000 bootstrap datasets were generated from the fitted model to calculate the local influence measures of the logarithm of the Bayes factor, denoted as t_B^k $(k = 1, \ldots, 1000)$, which led to the associated p-values. The p-values were calculated to be 3%, 2% and 1% when a was taken to be $0.28, 0.30$ and 0.35, respectively. These results indicate that the survival model is misspecified at the 5% significance level for these values of a. Therefore, the local influence method was useful for detecting the model misspecification in this example.

In the forth simulation study, we examined whether our local influence measure can assess the misspecified relationship between the longitudinal measurements and the survival times.

We generated the data according to almost the same setting except that the hazard functions were taken as

$$\lambda_m(t|\boldsymbol{b}_i, \boldsymbol{z}_i) = \lambda_{m0} \exp\{\alpha_{m1}\eta_{i1}(t, \boldsymbol{b}_{i1}) + a\eta_{i2}(t, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\} \text{ for } m = 1, 2. \tag{13}$$

However, we fitted the model specified in (2)-(4), in which $\lambda_m(t|\boldsymbol{b}_i, \boldsymbol{z}_i) = \lambda_{m0} \exp\{\alpha_{m1}\eta_{i1}(t, \boldsymbol{b}_{i1}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\}$ under the same prior distribution as given in the first simulation study. The fitted model would be misspecified if $a \neq 0$. We considered the following global perturbation

$$\lambda_m(t|\boldsymbol{b}_i, \boldsymbol{z}_i) = \lambda_{m0} \exp\{\alpha_{m1}\eta_{i1}(t, \boldsymbol{b}_{i1}) + \omega\eta_{i2}(t, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\}$$

to check model misspecification. In this case, $\omega = 0$ represents no perturbation. Again, by using the wild bootstrap method, the corresponding p-values were given by 4% and 5% when a was taken to be 0.36 and 0.32, respectively. These results indicate that the longitudinal model is misspecified at the 5% significance level for $b = 0.36$ and 0.32.

In the fifth simulation study, we examined whether our local influence measure can assess the multiple misspecifications of the longitudinal model and the survival model, as well as detect outliers. We generated the data using almost the same setting as specified in the first simulation study except for $\boldsymbol{\Sigma} = \begin{pmatrix} 1.000 & 0.485 \\ 0.485 & 0.640 \end{pmatrix}$, but the longitudinal measurement model was taken as

$$y_{ik}(t_{ijk}) = \eta_{ik}(t_{ijk}) + \varepsilon_{ijk} = \beta_{k0} + \beta_{k1}t_{ijk} + \beta_{k2}r_i + \varepsilon_{ijk},$$

and the hazard functions were taken as

$$\lambda_m(t|\boldsymbol{z}_i) = \lambda_{m0} \exp\{\alpha_{m1}\eta_{i1}(t) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\} \text{ for } m = 1, 2.$$

However, we fitted the generated data with the model specified in (2)-(4) under the same prior distribution as given in the first simulation study. The above specified longitudinal measurement model violates the model assumptions in the following two ways: (i) we don't include random effects to induce correlation between the repeated observations, (ii) the variability of the marginal error terms decreases as time increases. The above specified hazard function violates the assumption of the survival model in the following two ways: (i) we postulate that the time-to-event does not depend on the underlying random effects; (ii) we postulate that the time-to-event is not associated with $\eta_{i2}(t)$. To introduce some outliers, we changed the longitudinal measurements of the last two individuals, $\boldsymbol{Y}_i(t_{ij1})$, to $\boldsymbol{Y}_i(t_{ij1}) + 2.0\mathbf{1}_2$ for $j = 1, \ldots, n_i$ and $i = 99, 100$. We consider a simultaneous perturbation as follows:

$$p(\boldsymbol{y}_i(\boldsymbol{t}_{ij})|\omega_i) \sim N(\boldsymbol{\eta}_i(\boldsymbol{t}_{ij}), \boldsymbol{\Sigma}/\omega_i), \ p(\boldsymbol{\gamma}_m|\omega_\gamma) \sim N(\boldsymbol{\gamma}_m^0, \boldsymbol{H}_\gamma^0/\omega_\gamma),$$

$$y_{ik}(t_{ijk}) = \eta_{ik}(t_{ijk}, \boldsymbol{b}_i) + \varepsilon_{ijk} = \beta_{k0} + \beta_{k1}t_{ijk} + \beta_{k2}r_i + \omega_b(b_{ik0} + b_{ik1}t_{ijk}) + \varepsilon_{ijk},$$

$$\lambda_m(t|\boldsymbol{b}_i, \boldsymbol{z}_i, \omega_\lambda) = \lambda_{m0}\exp\{\alpha_{m1}\eta_{i1}(t, \boldsymbol{b}_{i1}) + \omega_\lambda\alpha_{m2}\eta_{i2}(t, \boldsymbol{b}_{i2}) + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\},$$

where $\boldsymbol{t}_{ij} = (t_{ij1}, t_{ij2})^T$, $\boldsymbol{y}_i(\boldsymbol{t}_{ij}) = (y_{i1}(t_{ij1}), y_{i2}(t_{ij2}))^T$ and $\boldsymbol{\eta}_i(\boldsymbol{t}_{ij}) = (\eta_{i1}(t_{ij1}), \eta_{i2}(t_{ij2}))^T$. In this case, $\omega_i = 1$, $\omega_\gamma = 1$, $\omega_b = 0$ and $\omega_\lambda = 0$ represent no perturbation. We considered the local influence measures based on both the logarithm of the Bayes factor and the Kullback-Leibler divergence. As expected, cases 99 and 100 were detected to be influential by our local influence measures $\mathbf{v}^B_{\max}$ and $\mathrm{SI}_{\mathrm{D}_{\phi,\mathbf{e}_j}}$ with the two priors of $\boldsymbol{\gamma}_m$, namely $p(\boldsymbol{\gamma}_m) = N(\boldsymbol{\gamma}_m^0, \boldsymbol{H}_\gamma^0)$ (Figs. 3(a) and 3(c)) and $p(\boldsymbol{\gamma}_m) = N(\mathbf{1}_2, 0.08\boldsymbol{I}_2)$ (Figs. 3(b) and 3(d)). Also, the perturbed prior distribution of $\boldsymbol{\gamma}_m$ was identified to have a big effect (Figs. 3(b) and 3(d)); the longitudinal measurement model and the hazard function were detected to be misspecified. Figures 3(e) and 3(f) show that the metric tensor $g_{ii}(\boldsymbol{\omega}^0)$ for the perturbation to the variance of the errors has little change for all the individuals, and there is a large change for the metric tensor $g_{ii}(\boldsymbol{\omega}^0)$ corresponding to the perturbation to the longitudinal measurement model and the hazard function.

3 Application to the IBCSG data

3.1 MCMC Algorithm

We need to introduce some notation. Let $\boldsymbol{t}_{ij} = (t_{ij1}, \ldots, t_{ij4})^T$, $\boldsymbol{Y}_i(\boldsymbol{t}_{ij}) = (y_{i1}(t_{ij1}), \ldots, y_{i4}(t_{ij4}))^T$, and $\boldsymbol{\eta}_i(\boldsymbol{t}_{ij}, \boldsymbol{b}_i) = (\eta_{i1}(t_{ij1}, \boldsymbol{b}_{i1}), \ \ldots, \eta_{i4}(t_{ij4}, \boldsymbol{b}_{i4}))^T$, where $\boldsymbol{b}_i = \{\boldsymbol{b}_{ik} : k = 1, \ldots, 4\}$ and $\boldsymbol{b}_{ik} = (b_{ik0}, b_{ik1})^T$ and $\eta_{ik}(t, \boldsymbol{b}_{ik}) = \beta_{k0} + \beta_{k1}x_{i1} + \ldots + \beta_{k6}x_{i6} + \beta_{k7}t + b_{ik0} + b_{ik1}t$. We define

$$
\begin{aligned}
\boldsymbol{Y} &= \{\boldsymbol{Y}_i(\boldsymbol{t}_{ij}) : i = 1, \ldots, 832; j = 1, 2, 3\}, \boldsymbol{T} = \{T_{im} : i = 1, \ldots, 832; m = 1, 2\}, \\
\boldsymbol{\Delta} &= \{\delta_{im} : i = 1, \ldots, 832, m = 1, 2\}, \boldsymbol{b} = \{\boldsymbol{b}_i : i = 1, \ldots, 832\}, \\
\boldsymbol{t} &= \{t_{ijk} : i = 1, \ldots, 832; j = 1, 2, 3; k = 1, \ldots, 4\}, \boldsymbol{X} = \{\boldsymbol{x}_i = (x_{i1}, \ldots, x_{i6}) : i = 1, \ldots, 832\}, \\
\boldsymbol{Z} &= \{\boldsymbol{z}_i : i = 1, \ldots, 832\}, \boldsymbol{h} = \{h_{ml} : m = 1, 2, l = 1, \ldots, L\}, \\
\boldsymbol{\alpha}_m &= (\alpha_{m1}, \ldots, \alpha_{m4})^T \text{ for } m = 1, 2; \\
\boldsymbol{\beta}_k &= (\beta_{k0}, \beta_{k1}, \ldots, \beta_{k7})^T \text{ for } k = 1, \ldots, 4, \\
\boldsymbol{\theta} &= \{\boldsymbol{\Phi}_1, \ldots, \boldsymbol{\Phi}_4, \boldsymbol{\Sigma}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_4, \boldsymbol{h}\}.
\end{aligned}
$$

Then, the joint probability density function of $(\boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{\theta})$ given $(\boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{t})$ is proportional to

$$
\begin{aligned}
&|\boldsymbol{\Sigma}|^{-\frac{N+\rho_0-3}{2}} (\prod_{k=1}^{4} |\boldsymbol{\Phi}_k|^{-\frac{n+\rho_{\phi k}^0-3}{2}}) \exp\Bigg[-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{3} (\boldsymbol{Y}_i(\boldsymbol{t}_{ij}) - \boldsymbol{\eta}_i(\boldsymbol{t}_{ij}, \boldsymbol{b}_i))^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{Y}_i(\boldsymbol{t}_{ij}) - \boldsymbol{\eta}_i(\boldsymbol{t}_{ij}, \boldsymbol{b}_i)) \\
&+ \sum_{m=1}^{2}\sum_{l=1}^{L}\sum_{i=1}^{n} \left\{\delta_{im}\left(\alpha_{m1}\eta_{i1}(t_{mil}^*, \boldsymbol{b}_{i1}) + \ldots + \alpha_{m4}\eta_{i4}(t_{mil}^*, \boldsymbol{b}_{i4}) + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\right) - h_{ml}B_{mil}\right\} \\
&+ \sum_{m=1}^{2}\sum_{l=1}^{L} d_{ml}\log(h_{ml}) - \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{4} \boldsymbol{b}_{ik}^T\boldsymbol{\Phi}_k^{-1}\boldsymbol{b}_{ik} - \frac{1}{2}\sum_{m=1}^{2} (\boldsymbol{\alpha}_m - \boldsymbol{\alpha}_m^0)^T (\boldsymbol{H}_\alpha^0)^{-1} (\boldsymbol{\alpha}_m - \boldsymbol{\alpha}_m^0) \qquad (14) \\
&- \frac{1}{2}\mathrm{tr}(\boldsymbol{\Sigma}^{-1}\boldsymbol{R}_0^{-1}) - \frac{1}{2}\sum_{m=1}^{2}(\boldsymbol{\gamma}_m - \boldsymbol{\gamma}_m^0)^T (\boldsymbol{H}_\gamma^0)^{-1} (\boldsymbol{\gamma}_m - \boldsymbol{\gamma}_m^0) - \sum_{m=1}^{2}\sum_{l=1}^{L} \tau_{1l}h_{ml} \\
&- \frac{1}{2}\sum_{k=1}^{4}(\boldsymbol{\beta}_k - \boldsymbol{\beta}_k^0)^T (\boldsymbol{H}_\beta^0)^{-1} (\boldsymbol{\beta}_k - \boldsymbol{\beta}_k^0) - \frac{1}{2}\sum_{k=1}^{4} \mathrm{tr}(\boldsymbol{\Phi}_k^{-1}(\boldsymbol{R}_{\phi k}^0)^{-1})\Bigg] \left(\prod_{m=1}^{2}\prod_{l=1}^{L} h_{ml}^{\tau_{0l}-1}\right),
\end{aligned}
$$

where $N = 1664$, $n = 832$, d_{ml} is the number of failures in the lth time interval $I_{ml} = (c_{m,l-1}, c_{m,l}]$, and t^*_{mil} denotes the nearest past time point where responses are taken. Moreover, B_{mil} is defined as follows:

(i) if $T_{im} < c_{m,l-1}$, $B_{mil} = 0$;

(ii) if $T_{im} > c_{m,l}$, letting $(h_{mi1}, s_{mi1}) = \max\{(h,s) : A^*_{ihs} \le c_{m,l-1}\}$ and $(h_{mi2}, s_{mi2}) = \max\{(h,s) : A^*_{ihs} \le c_{m,l}\}$, where A^*_{ihs} is the rescaled t_{ihs} so that A^*_{ihs} has the same unit as T_{im}, then if $h_{mi1} = h_{mi2}$ and $s_{mi1} = s_{mi2}$,

$$B_{mil} = (c_{m,l} - c_{m,l-1}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi1},s_{mi1}}, \boldsymbol{b}_{i1}) + \ldots + \alpha_{m4}\eta_{i4}(t_{i,h_{mi1},s_{mi1}}, \boldsymbol{b}_{i4}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\};$$

if $h_{mi1} = h_{mi2}$ and $s_{mi1} < s_{mi2}$,

$$\begin{aligned} B_{mil} = \ & (A^*_{i,h_{mi1},s_{mi1}+1} - c_{m,l-1}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi1},s_{mi1}+1}, \boldsymbol{b}_{i1}) + \alpha_{m2}\eta_{i2}(t_{i,h_{mi1},s_{mi1}+1}, \boldsymbol{b}_{i2}) \\ & + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\} + \sum_{l=s_{mi1}+1}^{s_{mi2}} (A^*_{i,h_{mi1},l+1} - A^*_{i,h_{mi1},l}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi1},l}, \boldsymbol{b}_{i1}) + \ldots \\ & + \alpha_{m4}\eta_{i4}(t_{i,h_{mi1},l}, \boldsymbol{b}_{i4}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\} + (c_{m,l} - A^*_{i,h_{mi1},s_{mi2}}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi1},s_{mi2}}, \boldsymbol{b}_{i1}) \\ & + \ldots + \alpha_{m4}\eta_{i4}(t_{i,h_{mi1},s_{mi2}}, \boldsymbol{b}_{i4}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\}; \end{aligned}$$

if $s_{mi1} = s_{mi2}$ and $h_{mi1} < h_{mi2}$,

$$\begin{aligned} B_{mil} = \ & (A^*_{i,h_{mi1}+1,s_{mi1}} - c_{m,l-1}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi1}+1,s_{mi1}}, \boldsymbol{b}_{i1}) + \ldots + \alpha_{m4}\eta_{i4}(t_{i,h_{mi1}+1,s_{mi1}}, \boldsymbol{b}_{i4}) \\ & + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\} + \sum_{l=h_{mi1}+1}^{h_{mi2}} (A^*_{i,l+1,s_{mi1}} - A^*_{i,l,s_{mi1}}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,l,s_{mi1}}, \boldsymbol{b}_{i1}) + \ldots \\ & + \alpha_{m4}\eta_{i4}(t_{i,l,s_{mi1}}, \boldsymbol{b}_{i4}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\} + (c_{m,l} - A^*_{i,h_{mi2},s_{mi1}}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi2},s_{mi1}}, \boldsymbol{b}_{i1}) \\ & + \ldots + \alpha_{m4}\eta_{i4}(t_{i,h_{mi2},s_{mi1}}, \boldsymbol{b}_{i4}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\}; \end{aligned}$$

if $h_{mi1} < h_{mi2}$ and $s_{mi1} < s_{mi2}$,

$$\begin{aligned} B_{mil} = \ & (A^*_{i,h_{mi1}+1,s_{mi1}+1} - c_{m,l-1}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi1}+1,s_{mi1}+1}, \boldsymbol{b}_{i1}) + \ldots + \alpha_{m2}\eta_{i2}(t_{i,h_{mi1}+1,s_{mi1}+1}, \boldsymbol{b}_{i2}) \\ & + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\} + \sum_{\kappa=h_{mi1}+1}^{h_{mi2}} \sum_{l=s_{mi1}+1}^{s_{mi2}} (A^*_{i,\kappa+1,l+1} - A^*_{i\kappa l}) \exp\{\alpha_{m1}\eta_{i1}(t_{i\kappa l}, \boldsymbol{b}_{i1}) + \ldots \\ & + \alpha_{m4}\eta_{i4}(t_{i\kappa l}, \boldsymbol{b}_{i4}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\} + (c_{m,l} - A^*_{i,h_{mi2},s_{mi2}}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi2},s_{mi2}}, \boldsymbol{b}_{i1}) \\ & + \ldots + \alpha_{m4}\eta_{i4}(t_{i,h_{mi2},s_{mi2}}, \boldsymbol{b}_{i4}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\}; \end{aligned}$$

(iii) if $c_{m,l-1} < T_{im} \le c_{m,l}$, using $h_{mi1}, h_{mi2}, s_{mi1}$ and s_{mi2} given in (ii), then if $(h_{mi1}, s_{mi1}) = (h_{mi2}, s_{mi2})$ or $T_{im} \le A^*_{i,h_{mi1}+1,s_{mi1}}$ when $h_{mi1} < h_{mi2}$ and $s_{mi1} = s_{mi2}$ or $T_{im} \le A^*_{i,h_{mi1},s_{mi1}+1}$ when $h_{mi1} = h_{mi2}$ and $s_{mi1} < s_{mi2}$ or $T_{im} \le A^*_{i,h_{mi1}+1,s_{mi1}+1}$ when $h_{mi1} < h_{mi2}$ and $s_{mi1} < s_{mi2}$,

$$B_{mil} = (T_{im} - c_{m,l-1}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi1},s_{mi1}}, \boldsymbol{b}_{i1}) + \ldots + \alpha_{m4}\eta_{i4}(t_{i,h_{mi1},s_{mi1}}, \boldsymbol{b}_{i4}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\},$$

and otherwise, we define

$$\begin{aligned} B_{mil} = \quad & (A^*_{i,h_{mi1}+1,s_{mi1}+1} - c_{m,l-1}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,h_{mi1}+1,s_{mi1}+1}, \boldsymbol{b}_{i1}) + \ldots + \alpha_{m4}\eta_{i4}(t_{i,h_{mi1}+1,s_{mi1}+1}, \boldsymbol{b}_{i4}) \\ & + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\} + \sum_{\kappa=h_{mi1}+1}^{\iota_{i1}} \sum_{l=s_{mi1}+1}^{\iota_{i2}} (A^*_{i,\kappa+1,l+1} - A^*_{i\kappa l}) \exp\{\alpha_{m1}\eta_{i1}(t_{i\kappa l}, \boldsymbol{b}_{i1}) + \ldots \\ & + \alpha_{m4}\eta_{i4}(t_{i\kappa l}, \boldsymbol{b}_{i4}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\} + (T_{im} - A^*_{i,\iota_{i1},\iota_{i2}}) \exp\{\alpha_{m1}\eta_{i1}(t_{i,\iota_{i1},\iota_{i2}}, \boldsymbol{b}_{i1}) + \ldots \\ & + \alpha_{m4}\eta_{i4}(t_{i,\iota_{i1},\iota_{i2}}, \boldsymbol{b}_{i4}) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m\}, \end{aligned}$$

where $h_{mi1} + 1 \le \iota_{i1} \le h_{mi2}$ and $s_{mi1} \le \iota_{i2} \le s_{mi2}$ are chosen so that $A^*_{i,\iota_{i1},\iota_{i2}} < T_{im} \le A^*_{i,\iota_{i1}+1,\iota_{i2}+1}$.

When (h_{mi1}, s_{mi1}) does not exist, we define $h_{mi1} = s_{mi1} = 1$, and the calculation of B_{mil} needs a minor adjustment.

To carry out the Bayesian analysis, we specified the following prior distributions:

$$\begin{aligned} & \boldsymbol{\alpha}_m = (\alpha_{m1}, \ldots, \alpha_{m4})^T \sim N(\boldsymbol{\alpha}_m^0, \boldsymbol{H}_\alpha^0),\ \boldsymbol{\gamma}_m \sim N(\boldsymbol{\gamma}_m^0, \boldsymbol{H}_\gamma^0),\ \boldsymbol{\Sigma}^{-1} \sim \text{Wishart}_4(\boldsymbol{R}^0, \rho^0), \\ & \boldsymbol{\beta}_k = (\beta_{k0}, \ldots, \beta_{k7})^T \sim N(\boldsymbol{\beta}_k^0, \boldsymbol{H}_\beta^0),\ h_{ml} \sim \Gamma(\tau_{0l}, \tau_{1l}),\ \boldsymbol{\Phi}_k^{-1} \sim \text{Wishart}_2(\boldsymbol{R}_{\phi k}^0, \rho_{\phi k}^0) \end{aligned} \tag{15}$$

for $m = 1$ and 2, and $k = 1, \ldots, 4$, where $\boldsymbol{h} = \{h_{ml} : m = 1, 2, l = 1, \ldots, L\}$, $\boldsymbol{\alpha}_m^0$, $\boldsymbol{H}_\alpha^0$, $\boldsymbol{\gamma}_m^0$, $\boldsymbol{H}_\gamma^0$, $\boldsymbol{R}^0$, ρ^0, $\boldsymbol{\beta}_k^0$, $\boldsymbol{H}_\beta^0$, τ_{0l}, τ_{1l}, $\boldsymbol{R}_{\phi k}^0$, and $\rho_{\phi k}^0$ are pre-specified hyper-parameters. Moreover, $\boldsymbol{\alpha}_m^0$, $\boldsymbol{\gamma}_m^0$, $\boldsymbol{\beta}_k^0$, $\boldsymbol{R}^0$, and $\boldsymbol{R}_{\phi k}^0$ were set as their Bayesian posterior estimates that were obtained from the MCMC algorithm under the noninformative prior distributions of $\boldsymbol{\alpha}_m, \boldsymbol{\gamma}_m$, $\boldsymbol{\Sigma}$, $\boldsymbol{\beta}_k$, and $\boldsymbol{\Phi}_k^{-1}$.

The Gibbs sampler is used to sample a sequence of random observations from the above joint posterior distribution given in Equation (14). Specifically, we iteratively simulate from the following full conditional distributions:

$$
\begin{aligned}
& p(\mathbf{\Phi}_k^{-1}|\boldsymbol{b}), \\
& p(\mathbf{\Sigma}^{-1}|\boldsymbol{Y}, \boldsymbol{t}, \boldsymbol{b}), \\
& p(\boldsymbol{\alpha}_m, \boldsymbol{\gamma}_m|\mathbf{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_4, \boldsymbol{h}), \\
& p(\boldsymbol{\beta}_k|\mathbf{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{h}), \\
& p(\boldsymbol{b}|\mathbf{\Delta}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_4, \boldsymbol{h}, \mathbf{\Phi}_1, \ldots, \mathbf{\Phi}_4, \mathbf{\Sigma}), \\
& p(\boldsymbol{h}|\mathbf{\Delta}, \boldsymbol{b}, \boldsymbol{T}, \boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_4).
\end{aligned}
$$

The above mentioned full conditional distributions are briefly discussed as follows.

First, it is easily shown from (14) and (15) that

$$
\begin{aligned}
& p(\mathbf{\Phi}_k^{-1}|\boldsymbol{b}) \sim \text{Wishart}_4(\rho_{\phi k}^0 + n, (\boldsymbol{R}_{\phi k}^0)^{-1} + \sum_{i=1}^n \boldsymbol{b}_{ik}\boldsymbol{b}_{ik}^T) \text{ for } k = 1, \ldots, 4, \\
& p(\mathbf{\Sigma}^{-1}|\boldsymbol{Y}, \boldsymbol{t}, \boldsymbol{b}) \sim \text{Wishart}_2(\rho^0 + 2n, \{(\boldsymbol{R}^0)^{-1} + \sum_{i=1}^n \sum_{j=1}^3 [\boldsymbol{Y}_i(\boldsymbol{t}_{ij}) - \boldsymbol{\eta}_i(\boldsymbol{t}_{ij}, \boldsymbol{b}_i)]^{\otimes 2}\}^{-1}), \\
& p(h_{ml}|\boldsymbol{b}, \boldsymbol{t}, \boldsymbol{T}, \boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_4) \sim \Gamma(\tau_{0l} + d_{ml}, \tau_{1l} + \sum_{i=1}^n B_{mil}).
\end{aligned}
$$

It is easily shown from (14) and (15) that $p(\boldsymbol{\alpha}_m, \boldsymbol{\gamma}_m|\mathbf{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_4, \boldsymbol{h})$ is proportional to

$$
\begin{aligned}
\exp\Bigg[& \sum_{l=1}^L \sum_{i=1}^n \left\{\delta_{im}\left(\alpha_{m1}\eta_{i1}(t_{mil}^*, \boldsymbol{b}_{i1}) + \ldots + \alpha_{m4}\eta_{i4}(t_{mil}^*, \boldsymbol{b}_{i4}) + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\right) - h_{ml}B_{mil}\right\} \\
& -\frac{1}{2}(\boldsymbol{\alpha}_m - \boldsymbol{\alpha}_m^0)^T(\boldsymbol{H}_\alpha^0)^{-1}(\boldsymbol{\alpha}_m - \boldsymbol{\alpha}_m^0) - \frac{1}{2}(\boldsymbol{\gamma}_m - \boldsymbol{\gamma}_m^0)^T(\boldsymbol{H}_\gamma^0)^{-1}(\boldsymbol{\gamma}_m - \boldsymbol{\gamma}_m^0)\Bigg]. \qquad (16)
\end{aligned}
$$

Also, $p(\boldsymbol{\beta}_k|\mathbf{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{h})$ is proportional to

$$
\begin{aligned}
\exp\Bigg[& -\frac{1}{2}\sum_{i=1}^n \sum_{j=1}^3 (\boldsymbol{Y}_i(\boldsymbol{t}_{ij}) - \boldsymbol{\eta}_i(\boldsymbol{t}_{ij}, \boldsymbol{b}_i))^T\mathbf{\Sigma}^{-1}(\boldsymbol{Y}_i(\boldsymbol{t}_{ij}) - \boldsymbol{\eta}_i(\boldsymbol{t}_{ij}, \boldsymbol{b}_i)) + \sum_{m=1}^2 \sum_{l=1}^L \sum_{i=1}^n (\delta_{im}\alpha_{mk} \\
& \times\eta_{ik}(t_{mil}^*, \boldsymbol{b}_{ik}) - h_{ml}B_{mil}) - \frac{1}{2}(\boldsymbol{\beta}_k - \boldsymbol{\beta}_k^0)^T(\boldsymbol{H}_\beta^0)^{-1}(\boldsymbol{\beta}_k - \boldsymbol{\beta}_k^0)\Bigg]. \qquad (17)
\end{aligned}
$$

Again, $p(\boldsymbol{b}_{ik}|\boldsymbol{\Delta}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_4, \boldsymbol{h}, \boldsymbol{\Phi}_k, \boldsymbol{\Sigma})$ is proportional to

$$\exp\left[-\frac{1}{2}\sum_{j=1}^{2}(\boldsymbol{Y}_i(\boldsymbol{t}_{ij}) - \boldsymbol{\eta}_i(\boldsymbol{t}_{ij}, \boldsymbol{b}_i))^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{Y}_i(\boldsymbol{t}_{ij}) - \boldsymbol{\eta}_i(\boldsymbol{t}_{ij}, \boldsymbol{b}_i)) \right. \tag{18}$$
$$\left. + \sum_{m=1}^{2}\sum_{l=1}^{L}\{\delta_{im}\alpha_{mk}\eta_{ik}(t^*_{mil}, \boldsymbol{b}_{ik}) - h_{ml}B_{mil}\} - \frac{1}{2}\boldsymbol{b}_{ik}^T\boldsymbol{\Phi}_k^{-1}\boldsymbol{b}_{ik}\right].$$

To simulate observations from the full conditional distribution relating to (16), we define

$$\boldsymbol{\Omega}_u^{-1} = \mathrm{diag}((\boldsymbol{H}_\alpha^0)^{-1}, (\boldsymbol{H}_\gamma^0)^{-1}) + \sum_{i=1}^{n}\sum_{l=1}^{L}h_{ml}\mathcal{B}_{mil},$$

where $\mathcal{B}_{mil} = \partial^2 B_{mil}/\partial\boldsymbol{u}_m\partial\boldsymbol{u}_m^T|_{\boldsymbol{u}_m=\boldsymbol{0}}$ with $\boldsymbol{u}_m = (\boldsymbol{\alpha}_m^T, \boldsymbol{\gamma}_m^T)^T$. The MH algorithm is implemented as follows. At the $(d+1)$st iteration with a $\boldsymbol{u}_m^{(d)}$, a new candidate $\boldsymbol{u}_m$ is generated from a $N(\boldsymbol{u}_m^{(d)}, \sigma_u^2\boldsymbol{\Omega}_u)$ distribution and is accepted with probability

$$\min\left\{1, \frac{p(\boldsymbol{\alpha}_m, \boldsymbol{\gamma}_m|\boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{h})}{p(\boldsymbol{\alpha}_m^{(d)}, \boldsymbol{\gamma}_m^{(d)}|\boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{h})}\right\}.$$

σ_u^2 is selected such that the average acceptance rate is about 25% or more.

Similarly, the MH algorithm for sampling $\boldsymbol{\beta}_k$ from $p(\boldsymbol{\beta}_k|\boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{r}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{h})$ given in (17) is implemented as follows. Let σ^{kk} be the (k,k)th component of $\boldsymbol{\Sigma}^{-1}$. At the $(d+1)$st iteration with a $\boldsymbol{\beta}_k^{(d)}$, a new candidate $\boldsymbol{\beta}_k$ is simulated from the proposal distribution $N(\boldsymbol{\beta}_k^{(d)}, \sigma_\beta^2\boldsymbol{\Omega}_\beta)$, where $\boldsymbol{\Omega}_\beta^{-1} = 2\sum_{i=1}^n \sigma^{kk}\boldsymbol{q}_i\boldsymbol{q}_i^T + \sum_{m=1}^2\sum_{l=1}^L\sum_{i=1}^n h_{ml}\mathcal{C}_{mil} + (\boldsymbol{H}_\beta^0)^{-1}$ with $\boldsymbol{q}_i^T = (1, \boldsymbol{x}_i^T)$ and $\mathcal{C}_{mil} = \partial^2 B_{mil}/\partial\boldsymbol{\beta}_k\partial\boldsymbol{\beta}_k^T|_{\boldsymbol{\beta}_k=\boldsymbol{0}}$. The acceptance probability is

$$\min\left\{1, \frac{p(\boldsymbol{\beta}_k|\boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{h})}{p(\boldsymbol{\beta}_k^{(d)}|\boldsymbol{\Delta}, \boldsymbol{b}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{h})}\right\}.$$

Sampling $\boldsymbol{b}_{ik}$ from $p(\boldsymbol{b}_{ik}|\boldsymbol{\Delta}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_4, \boldsymbol{h}, \boldsymbol{\Phi}_k, \boldsymbol{\Sigma})$ as given in Equation (18) can be implemented as follows. At the $(d+1)$st iteration with a current value $\boldsymbol{b}_{ik}^{(d)}$, a new candidate $\boldsymbol{b}_{ik}$ is simulated from the proposal distribution $N(\boldsymbol{b}_{ik}^{(d)}, \sigma_b^2\boldsymbol{\Omega}_{bk})$ and is accepted with probability

$$\min\left\{1, \frac{p(\boldsymbol{b}_{ik}|\boldsymbol{\Delta}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_4, \boldsymbol{h}, \boldsymbol{\Phi}_k, \boldsymbol{\Sigma})}{p(\boldsymbol{b}_{ik}^{(d)}|\boldsymbol{\Delta}, \boldsymbol{Y}, \boldsymbol{T}, \boldsymbol{X}, \boldsymbol{Z}, \boldsymbol{t}, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_4, \boldsymbol{h}, \boldsymbol{\Phi}_k, \boldsymbol{\Sigma})}\right\},$$

where $\boldsymbol{\Omega}_b = \boldsymbol{\Phi}_k^{-1} + \sum_{j=1}^2 \sigma^{kk} \boldsymbol{A}_{ijk} \boldsymbol{A}_{ijk}^T + \sum_{m=1}^2 \sum_{l=1}^L h_{ml} \partial^2 B_{mil} / \partial \boldsymbol{b}_{ik} \partial \boldsymbol{b}_{ik}^T |_{\boldsymbol{b}_{ik}=\mathbf{0}}$ with $\boldsymbol{A}_{ijk} = (1, t_{ijk})^T$.

3.2 Influence analysis

The first perturbation is a single-case perturbation obtained by perturbing each subject's longitudinal profile as follows:

$$y_{ik}(t_{ijk}, \omega_{ijk}) = \beta_{k0} + \beta_{k1} x_{i1} + \ldots + \beta_{k6} x_{i6} + \beta_{k7} t_{ijk} + b_{ik0} + b_{ik1} t_{ijk} + \varepsilon_{ijk} / \omega_{ijk}. \tag{19}$$

In this case, $\boldsymbol{\omega} = (\boldsymbol{\omega}_{11}^T, \ldots, \boldsymbol{\omega}_{1n_1}^T, \ldots, \boldsymbol{\omega}_{n1}^T, \ldots, \boldsymbol{\omega}_{nn_n}^T)^T$, in which $\boldsymbol{\omega}_{ij} = (\omega_{ij1}, \ldots, \omega_{ij4})^T$ for $i = 1, \ldots, n = 832$ and $j = 1, 2, 3$, and $\boldsymbol{\omega}^0 = \mathbf{1}$ presents no perturbation, where $\mathbf{1}$ is a vector with all ones. Let $\boldsymbol{W}_{ij} = \mathrm{diag}(\omega_{ij1}, \ldots, \omega_{ij4})$ for all i, j. The perturbed log-posterior likelihood $l(\boldsymbol{\omega})$ is given by

$$l(\boldsymbol{\omega}) = \sum_{i=1}^n \sum_{j=1}^{n_i} \{ \sum_{k=1}^4 \log(\omega_{ijk}) - \frac{1}{2} [\boldsymbol{Y}_i(\boldsymbol{t}_{ij}) - \boldsymbol{\eta}_i(\boldsymbol{t}_{ij}, \boldsymbol{b}_i)]^T \boldsymbol{W}_{ij} \boldsymbol{\Sigma}^{-1} \boldsymbol{W}_{ij} [\boldsymbol{Y}_i(\boldsymbol{t}_{ij}) - \boldsymbol{\eta}_i(\boldsymbol{t}_{ij}, \boldsymbol{b}_i)] \} + C,$$

where C is a constant that does not depend on $\boldsymbol{\omega}$. It can be shown that $\boldsymbol{G}(\boldsymbol{\omega}^0) = \mathrm{diag}(\boldsymbol{A}, \ldots, \boldsymbol{A})$, where

$$\boldsymbol{A} = \boldsymbol{I}_4 + \begin{pmatrix} E(\sigma^{11}\sigma_{11}) & E(\sigma^{12}\sigma_{12}) & E(\sigma^{13}\sigma_{13}) & E(\sigma^{14}\sigma_{14}) \\ \vdots & \vdots & \vdots & \vdots \\ E(\sigma^{41}\sigma_{41}) & E(\sigma^{42}\sigma_{42}) & E(\sigma^{43}\sigma_{43}) & E(\sigma^{44}\sigma_{44}) \end{pmatrix},$$

in which σ^{kl} and σ_{kl} are the (k, l)th component of matrix $\boldsymbol{\Sigma}^{-1}$ and $\boldsymbol{\Sigma}$, respectively, and the expectation is taken with respect to the prior distribution of Σ.

The second perturbation is also a single-case perturbation obtained by perturbing the marginal hazard function as follows:

$$\lambda_m(t | \boldsymbol{b}_i, \boldsymbol{z}_i, \omega_{mi}) = \lambda_{m0}(t) \exp\{\alpha_{m1} \eta_{i1}(t, \boldsymbol{b}_i) + \ldots + \alpha_{m4} \eta_{i4}(t, \boldsymbol{b}_i) + \boldsymbol{z}_i^T \boldsymbol{\gamma}_m + \omega_{mi}\},$$

where $\eta_{ik}(t, \boldsymbol{b}_i) = \beta_{k0} + \beta_{k1}x_{i1} + \ldots + \beta_{k6}x_{i6} + \beta_{k7}t + b_{ik0} + b_{ik1}t$ for $k = 1, \ldots, 4$. In this case, $\boldsymbol{\omega} = (\omega_{11}, \omega_{21}, \ldots, \omega_{n1}, \omega_{n2})^T$ and $\boldsymbol{\omega}^0 = \mathbf{0}$ represents no perturbation. Then the corresponding perturbed log-posterior is given by

$$l(\boldsymbol{\omega}) = \sum_{i=1}^{n}\sum_{l=1}^{L}\sum_{m=1}^{M}\{\delta_{mi}[\alpha_{m1}\eta_{i1}(t^*_{mil}, \boldsymbol{b}_i) + \ldots + \alpha_{m4}\eta_{i4}(t^*_{mil}, \boldsymbol{b}_i) + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m + \omega_{mi}] - h_{ml}B_{mil}(\omega_{mi})\} + C,$$

where $B_{mil}(\omega_{im})$ can be obtained by using $\boldsymbol{z}_i^T\boldsymbol{\gamma}_m + \omega_{mi}$ to replace $\boldsymbol{z}_i^T\boldsymbol{\gamma}_m$ in B_{mil}. Similarly, we can show that

$$\boldsymbol{G}(\boldsymbol{\omega}^0) = \mathrm{diag}(g_{11}, g_{21}, \ldots, g_{n1}, g_{n2}),$$

where $g_{mi} = \sum_{l=1}^{L} E(h_{ml}B_{mil})$ for $m = 1, 2$ and $i = 1, \ldots, n$, and $E(\cdot)$ represents the expectation taken with respect to the distribution of $\boldsymbol{b}_{ik}$ and the priors for $\boldsymbol{\beta}_k$ and (h_{ml}, $\boldsymbol{\gamma}_m$, $\boldsymbol{\alpha}_m$), in which $\boldsymbol{\beta}_k = (\beta_{k0}, \beta_{k1}, \ldots, \beta_{k6})^T$ and $\boldsymbol{\alpha}_m = (\alpha_{m1}, \ldots, \alpha_{m4})^T$.

The third perturbation is to simultaneously perturb the shared random effects $\mathbf{b}_i$ in both the longitudinal profile and the marginal hazard functions:

$$\begin{aligned} y_{ik}(t_{ijk}, \omega_{ik}) &= \beta_{k0} + \beta_{k1}x_{i1} + \ldots + \beta_{k6}x_{i6} + \omega_{ik}(b_{ik0} + b_{ik1}t_{ijk}) + \varepsilon_{ijk} \\ &\triangleq \eta_{ik}(t_{ijk}, \boldsymbol{b}_{ik}, \omega_{ik}) + \varepsilon_{ijk}, \end{aligned}$$

$$\lambda_m(t|\boldsymbol{b}_i, \boldsymbol{z}_i, \boldsymbol{\omega}_i) = \lambda_{m0}(t)\exp\{\alpha_{m1}\eta_{i1}(t, \boldsymbol{b}_{i1}, \omega_{i1}) + \ldots + \alpha_{m4}\eta_{i4}(t, \boldsymbol{b}_{i4}, \omega_{i4}) + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m\},$$

where $\boldsymbol{\omega}_i = (\omega_{i1}, \ldots, \omega_{i4})$. In this case, $\boldsymbol{\omega} = (\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_n)$, and $\boldsymbol{\omega}^0 = \mathbf{1}$ presents no perturbation. Then the perturbed log-posterior is

$$\begin{aligned} l(\boldsymbol{\omega}) = \; & C - \tfrac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n_i}(\boldsymbol{Y}_i(\boldsymbol{t}_{ij}) - \boldsymbol{\eta}_i(\boldsymbol{t}_{ij}, \boldsymbol{b}_i, \boldsymbol{\omega}_i))^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{Y}_i(\boldsymbol{t}_{ij}) - \boldsymbol{\eta}_i(\boldsymbol{t}_{ij}, \boldsymbol{b}_i, \boldsymbol{\omega}_i)) \\ & + \sum_{i=1}^{n}\sum_{l=1}^{L}\sum_{m=1}^{M}\{\delta_{mi}(\alpha_{m1}\eta_{i1}(t^*_{mil}, \boldsymbol{b}_{i1}, \boldsymbol{\omega}_{i1}) + \ldots + \alpha_{m4}\eta_{i4}(t^*_{mil}, \boldsymbol{b}_{i4}, \boldsymbol{\omega}_{i4}) + \boldsymbol{z}_i^T\boldsymbol{\gamma}_m) - h_{ml}B_{mil}(\boldsymbol{\omega}_i)\}, \end{aligned}$$

where $\boldsymbol{\eta}_i(\boldsymbol{t}_{ij}, \boldsymbol{b}_i, \boldsymbol{\omega}_i) = (\eta_{i1}(t_{ij1}, \boldsymbol{b}_{i1}, \omega_{i1}), \ldots, \eta_{i4}(t_{ij4}, \boldsymbol{b}_{i4}, \omega_{i4}))^T$, and $B_{mil}(\boldsymbol{\omega}_i)$ can be obtained by using $\omega_{ik}(b_{ik0} + b_{ik1}t)$ to replace $b_{ik0} + b_{ik1}t$ in B_{mil}. Thus, we can shown that $\boldsymbol{G}(\boldsymbol{\omega}^0) = \mathrm{diag}(g_{11}, \ldots, g_{14}, \ldots, g_{n1}, \ldots, g_{n4})$, where

$$g_{ik} = \rho_0 R_{kk,0}\sum_{j=1}^{n_i}\boldsymbol{v}_{ijk}^T(\boldsymbol{R}^0_{\phi k})^{-1}\boldsymbol{v}_{ijk}/(\rho^0_{\phi k} - 3) + \sum_{l=1}^{L}\sum_{m=1}^{M}E\{h_{ml}B_{mil}\alpha^2_{mk}(b_{ik0} + b_{ik1}t^*_{mil})^2\},$$

where $R_{kk,0}$ is the (k,k)th element of the matrix $\boldsymbol{R}_0$, $\boldsymbol{v}_{ijk} = (1, t_{ijk})^T$, and $E(\cdot)$ represents the expectation taken with respect to the distribution of $\boldsymbol{b}_{ik}$ and the priors for $\boldsymbol{\beta}_k$ and (h_{ml}, $\boldsymbol{\gamma}_m$, $\boldsymbol{\alpha}_m$) for $i = 1, \ldots, n$ and $k = 1, \ldots, 4$.

The fourth perturbation is to perturb the prior distributions of all the parameters as follows:

$$\begin{aligned}
&\boldsymbol{\beta}_k \sim N_7(\boldsymbol{\beta}_k^0 + \omega_{\beta 0}\mathbf{1}_7, \boldsymbol{H}_{\beta k}^0/\omega_{\beta 1}),\ \boldsymbol{\alpha}_m \sim N_4(\boldsymbol{\alpha}_m^0 + \omega_{\alpha 0}\mathbf{1}_4, \boldsymbol{H}_{\alpha m}^0/\omega_{\alpha 1}),\\
&\boldsymbol{\Sigma}^{-1} \sim \text{Wishart}_4(\rho_0, \omega_\Sigma^{-1}\boldsymbol{R}^0),\ \boldsymbol{\Phi}_k^{-1} \sim \text{Wishart}_2(\rho_k^0, \omega_\phi^{-1}\boldsymbol{R}_{\phi k}^0),\\
&h_{ml} \sim \Gamma(\tau_{\lambda 0}, \omega_h \tau_{\lambda 1}),\ \boldsymbol{\gamma}_m \sim N_6(\boldsymbol{\gamma}_m^0 + \omega_{\gamma 0}\mathbf{1}_6, \boldsymbol{H}_{\gamma m}^0/\omega_{\gamma 1}).
\end{aligned} \tag{20}$$

In this case, $\boldsymbol{\omega} = \{\omega_{\beta 0}, \omega_{\beta 1}, \omega_{\alpha 0}, \omega_{\alpha 1}, \omega_{\gamma 0}, \omega_{\gamma 1}, \omega_h, \omega_\Sigma, \omega_\phi\}$, and $\boldsymbol{\omega}^0 = (0,1,0,1,0,1,1,1,1)$ represents no perturbation. The perturbed log-posterior is given by

$$\begin{aligned}
l(\boldsymbol{\omega}) = \ & 14\log(\omega_{\beta 1}) + 2M\log(\omega_{\alpha 1}) + 3M\log(\omega_{\gamma 1})\\
& -0.5\omega_{\beta 1}\sum_{k=1}^{4}(\boldsymbol{\beta}_k - \boldsymbol{\beta}_k^0 - \omega_{\beta 0}\mathbf{1}_7)^T(\boldsymbol{H}_{\beta k}^0)^{-1}(\boldsymbol{\beta}_k - \boldsymbol{\beta}_k^0 - \omega_{\beta 0}\mathbf{1}_7)\\
& -0.5\omega_{\alpha 1}\sum_{k=1}^{M}(\boldsymbol{\alpha}_m - \boldsymbol{\alpha}_m^0 - \omega_{\alpha 0}\mathbf{1}_4)^T(\boldsymbol{H}_{\alpha m}^0)^{-1}(\boldsymbol{\alpha}_m - \boldsymbol{\alpha}_m^0 - \omega_{\alpha 0}\mathbf{1}_4)\\
& -0.5\omega_{\gamma 1}\sum_{m=1}^{M}(\boldsymbol{\gamma}_m - \boldsymbol{\gamma}_m^0 - \omega_{\gamma 0}\mathbf{1}_6)^T(\boldsymbol{H}_{\gamma m}^0)^{-1}(\boldsymbol{\gamma}_m - \boldsymbol{\gamma}_m^0 - \omega_{\gamma 0}\mathbf{1}_6)\\
& -\omega_h\tau_{\lambda 1}\sum_{m=1}^{M}\sum_{l=1}^{L}h_{ml} + ML\tau_{\lambda 0}\log(\omega_h) + 2\rho_0\log(\omega_\Sigma)\\
& -0.5\omega_\Sigma \text{tr}(\boldsymbol{R}^{0^{-1}}\boldsymbol{\Sigma}^{-1}) + (\sum_{k=1}^{4}\rho_k^0)\log(\omega_\phi) - 0.5\omega_\phi\sum_{k=1}^{4}\text{tr}(\boldsymbol{R}_{\phi k}^{0\ -1}\boldsymbol{\Phi}_k^{-1}) + C,
\end{aligned}$$

where C is a constant that does not dependent on $\boldsymbol{\omega}$. We can show that

$$G(\boldsymbol{\omega}^0) = \text{diag}(\boldsymbol{G}_\beta^0, \boldsymbol{G}_\alpha^0, \boldsymbol{G}_\gamma^0, ML\tau_{\lambda 0}, 2\rho_0, \sum_{k=1}^{4}\rho_k^0),$$

where $\boldsymbol{G}_\beta^0 = \text{diag}(\mathbf{1}_7^T(\sum_{k=1}^4(\boldsymbol{H}_{\beta k}^0)^{-1})\mathbf{1}_7, 14)$, $\boldsymbol{G}_\alpha^0 = \text{diag}(\mathbf{1}_4^T(\sum_{m=1}^M(\boldsymbol{H}_{\alpha m}^0)^{-1})\mathbf{1}_4, 2M))$, and $\boldsymbol{G}_\gamma^0 = \text{diag}(\mathbf{1}_6^T(\sum_{k=1}^4(\boldsymbol{H}_{\gamma m}^0)^{-1})\mathbf{1}_6, 3M)$.

The last perturbation is a simultaneous perturbation of the priors, the sampling distributions, and the individual observations. Specifically, we first perturb the sampling distributions and the individual observations as follows:

$$\begin{aligned} y_{ik}(t_{ijk},\omega_i,\omega_b) &= \beta_{k0}+\beta_{k1}x_{i1}+\ldots+\beta_{k6}x_{i6}+\omega_b(b_{ik0}+b_{ik1}t_{ijk})+\varepsilon_{ijk}/\omega_i \\ &\triangleq \eta_{ik}(t_{ijk},\boldsymbol{b}_{ik},\omega_b)+\varepsilon_{ijk}/\omega_i, \end{aligned},$$

$$\lambda_m(t|\boldsymbol{b}_i,\boldsymbol{z}_i,\omega_\lambda,\omega_b)=\lambda_{m0}(t)\exp\{\alpha_{m1}\eta_{i1}(t,\boldsymbol{b}_{i1},\omega_b)+\ldots+\alpha_{m4}\eta_{i4}(t,\boldsymbol{b}_{i4},\omega_b)+\boldsymbol{z}_i^T\boldsymbol{\gamma}_m+\omega_\lambda\}.$$

Secondly, we use the same perturbation given in (20) to perturb the priors of $\boldsymbol{\beta}_k,\boldsymbol{\alpha}_m,\boldsymbol{\gamma}_m,h_{ml},\boldsymbol{\Sigma}^{-1}$ and $\boldsymbol{\Phi}_k^{-1}$. Thirdly, we also introduce a perturbation to a subset of h'_{ml}s. The perturbed log-posterior is given by

$$\begin{aligned} l(\boldsymbol{\omega})= &\sum_{i=1}^{n}\sum_{j=1}^{n_i}\{4\log(\omega_i)-\tfrac{\omega_i^2}{2}(\boldsymbol{Y}_i(\boldsymbol{t}_{ij})-\boldsymbol{\eta}_i(\boldsymbol{t}_{ij},\boldsymbol{b}_i,\omega_b))^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{Y}_i(\boldsymbol{t}_{ij})-\boldsymbol{\eta}_i(\boldsymbol{t}_{ij},\boldsymbol{b}_i,\omega_b))\} \\ &+\sum_{m=1}^{M}\sum_{l=1}^{L}\sum_{i=1}^{n}\{\delta_{mi}(\alpha_{m1}\eta_{i1}(t^*_{mil},\boldsymbol{b}_i,\omega_b)+\ldots+\alpha_{m4}\eta_{i4}(t^*_{mil},\boldsymbol{b}_i,\omega_b)+\boldsymbol{z}_i^T\boldsymbol{\gamma}_m+\omega_\lambda) \\ &-\sum_{g=1}^{G}\omega_{hg}h_{ml}B_{mil}(\omega_\lambda,\omega_b)I(l\in T_g)\}+\sum_{m=1}^{M}\sum_{l=1}^{L}\sum_{g=1}^{G}d_{ml}\log(\omega_{hg}h_{ml})I(l\in T_g) \\ &+14\log(\omega_{\beta1})+2M\log(\omega_{\alpha1})+3M\log(\omega_{\gamma1}) \\ &-0.5\omega_{\beta1}\sum_{k=1}^{4}(\boldsymbol{\beta}_k-\boldsymbol{\beta}_k^0-\omega_{\beta0}\mathbf{1}_7)^T(\boldsymbol{H}^0_{\beta k})^{-1}(\boldsymbol{\beta}_k-\boldsymbol{\beta}_k^0-\omega_{\beta0}\mathbf{1}_7) \\ &-0.5\omega_{\alpha1}\sum_{k=1}^{M}(\boldsymbol{\alpha}_m-\boldsymbol{\alpha}_m^0-\omega_{\alpha0}\mathbf{1}_4)^T(\boldsymbol{H}^0_{\alpha m})^{-1}(\boldsymbol{\alpha}_m-\boldsymbol{\alpha}_m^0-\omega_{\alpha0}\mathbf{1}_4) \\ &-0.5\omega_{\gamma1}\sum_{m=1}^{M}(\boldsymbol{\gamma}_m-\boldsymbol{\gamma}_m^0-\omega_{\gamma0}\mathbf{1}_6)^T(\boldsymbol{H}^0_{\gamma m})^{-1}(\boldsymbol{\gamma}_m-\boldsymbol{\gamma}_m^0-\omega_{\gamma0}\mathbf{1}_6) \\ &-\omega_h\tau_{\lambda1}\sum_{m=1}^{M}\sum_{l=1}^{L}h_{ml}+ML\tau_{\lambda0}\log(\omega_h)+2\rho_0\log(\omega_\Sigma) \\ &-0.5\omega_\Sigma\mathrm{tr}(\boldsymbol{R}^{0^{-1}}\boldsymbol{\Sigma}^{-1})+(\sum_{k=1}^{4}\rho_k^0)\log(\omega_\phi)-0.5\omega_\phi\sum_{k=1}^{4}\mathrm{tr}({\boldsymbol{R}^0_{\phi k}}^{-1}\boldsymbol{\Phi}_k^{-1})+C, \end{aligned}$$

where $\boldsymbol{\eta}_i(\boldsymbol{t}_{ij},\boldsymbol{b}_i,\omega_i,\omega_b)=(\eta_{i1}(t_{ij1},\boldsymbol{b}_{i1},\omega_b),\ldots,\eta_{i4}(t_{ij4},\ \boldsymbol{b}_{i4},\omega_b))^T$, $T_g\in\{1,\ldots,L\}$ $(g=1,\ldots,G)$ is an index set and satisfies $T_{g_1}\bigcap T_{g_2}=\emptyset$ for every $g_1\neq g_2\in\{1,\ldots,G\}$ and $T_1\bigcup\ldots\bigcup T_G=\{1,\ldots,L\}$. In this case, $\boldsymbol{\omega}$ is given by

$$\boldsymbol{\omega}=(\omega_1,\ldots,\omega_n,\omega_b,\omega_\lambda,\omega_{h1},\ldots,\omega_{hG},\omega_{\beta0},\omega_{\beta1},\omega_{\alpha0},\omega_{\alpha1},\omega_{\gamma0},\omega_{\gamma1},\omega_h,\omega_\Sigma,\omega_\phi),$$

and $\boldsymbol{\omega}^0=\{1,\ldots,1,1,0,1,1,\ldots,1,0,\ 1,0,1,0,1,1,1,1\}$ represents no perturbation. Here, we take $L=250$ and $G=5$ and $T_1=\{1,\ldots,50\}$, $T_2=\{51,\ldots,100\}$, $T_3=\{101,\ldots,150\}$,

$T_4 = \{151, \ldots, 200\}$ and $T_5 = \{201, \ldots, 250\}$. After some calculations, we have

$$\boldsymbol{G}(\boldsymbol{\omega}^0) = \text{diag}(8n_1, \ldots, 8n_n, \boldsymbol{W}, \boldsymbol{G}_\beta^0, \boldsymbol{G}_\alpha^0, \boldsymbol{G}_\gamma^0, ML\tau_{\lambda 0}, (4\rho_0)/2, \sum_{k=1}^{4} \rho_k^0). \tag{21}$$

Let $R_{kk,0}$ be the (k,k)th element of the matrix $\boldsymbol{R}_0$ and $\boldsymbol{v}_{ijk} = (1, t_{ijk})^T$. In (21), $\boldsymbol{W}$ is given by

$$\boldsymbol{W} = \begin{pmatrix} w_1 & w_2 & v_{11} & \ldots & v_{15} \\ w_2 & w_3 & v_{21} & \ldots & v_{25} \\ v_{11} & v_{21} & \nu_1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ v_{15} & v_{25} & 0 & \ldots & \nu_5 \end{pmatrix},$$

in which

$$w_1 = \sum_{i=1}^{n}\sum_{j=1}^{n_i}\sum_{k=1}^{4} \rho_0 R_{kk,0} \boldsymbol{v}_{ijk}^T (\boldsymbol{R}_{\phi k}^0)^{-1} \boldsymbol{v}_{ijk} / (\rho_{\phi k}^0 - 3) + \sum_{i=1}^{n}\sum_{l=1}^{L}\sum_{m=1}^{M} E\{h_{ml} B_{mil} q_{mi}^2\},$$

$$w_2 = \sum_{m=1}^{M}\sum_{l=1}^{L}\sum_{i=1}^{n} E\{h_{ml} B_{mil} q_{mi}\},$$

$$w_3 = \sum_{m=1}^{M}\sum_{l=1}^{L}\sum_{i=1}^{n} E\{h_{ml} B_{mil}\},$$

$$v_{11} = \sum_{m=1}^{M}\sum_{l \in T_1}\sum_{i=1}^{n} E\{h_{ml} B_{mil} q_{mi}\},\ v_{15} = \sum_{m=1}^{M}\sum_{l \in T_5}\sum_{i=1}^{n} E\{h_{ml} B_{mil} q_{mi}\},$$

$$v_{21} = \sum_{m=1}^{M}\sum_{l \in T_1}\sum_{i=1}^{n} E\{h_{ml} B_{mil}\},\ v_{25} = \sum_{m=1}^{M}\sum_{l \in T_5}\sum_{i=1}^{n} E\{h_{ml} B_{mil}\},$$

$$\nu_1 = \sum_{m=1}^{M}\sum_{l \in T_1} d_{ml}, \nu_5 = \sum_{m=1}^{M}\sum_{l \in T_5} d_{ml} \text{ and } q_{mi} = \sum_{k=1}^{4} \alpha_{mk}(b_{ik0} + b_{ik1} t_{mil}^*),$$

and $E(\cdot)$ represents the expectation with respect to the distribution of $\boldsymbol{b}_{ik}$ and the priors of all unknown parameters for $i = 1, \ldots, n, j = 1, \ldots, n_i$, and $k = 1, \ldots, 4$.

References

Gustafson, P. (1996). Local sensitivity of posterior expectations. *Annals of Statistics*, 24:174–195.

Ibrahim, J. G., Chen, M. H., and Sinha, D. (2001). *Bayesian Survival Analysis*. New York: Springer-Verlag Inc.

Kass, R. E. and Raftery, A. E. (1995). Bayes factors. *Journal of the American Statistical Association*, 90:773–795.

Kass, R. E., Tierney, L., and Kadane, J. B. (1989). Approximate methods for assessing influence and sensitivity in Bayesian analysis. *Biometrika*, 76:663–674.

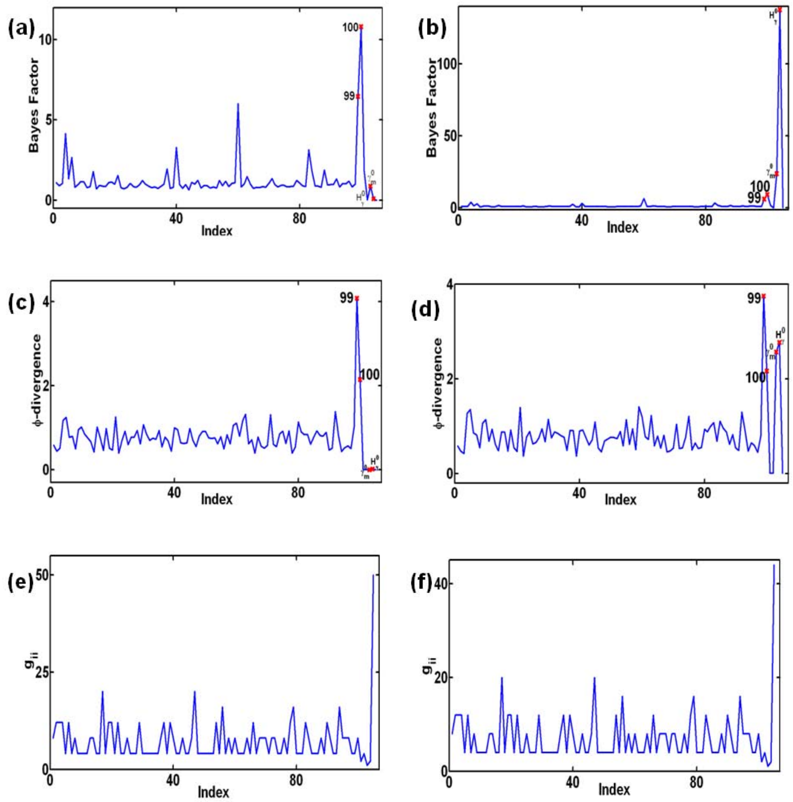

Figure 1: Index plots of local influence measures (a) $\mathbf{v}_{\max}^B$, (c) $\mathrm{SI}_{\mathrm{D}_{\phi,\mathbf{e}_j}}$ and (e) g_{ii} with $p(\boldsymbol{\gamma}_m) \stackrel{D}{=} N(\boldsymbol{\gamma}_m^0, \boldsymbol{H}_\gamma^0)$, (b) $\mathbf{v}_{\max}^B$, (d) $\mathrm{SI}_{\mathrm{D}_{\phi,\mathbf{e}_j}}$ and (f) g_{ii} with $p(\boldsymbol{\gamma}_m) \stackrel{D}{=} N(\mathbf{1}_2, 0.08\boldsymbol{I}_2)$ for the first simultaneous perturbation (10).

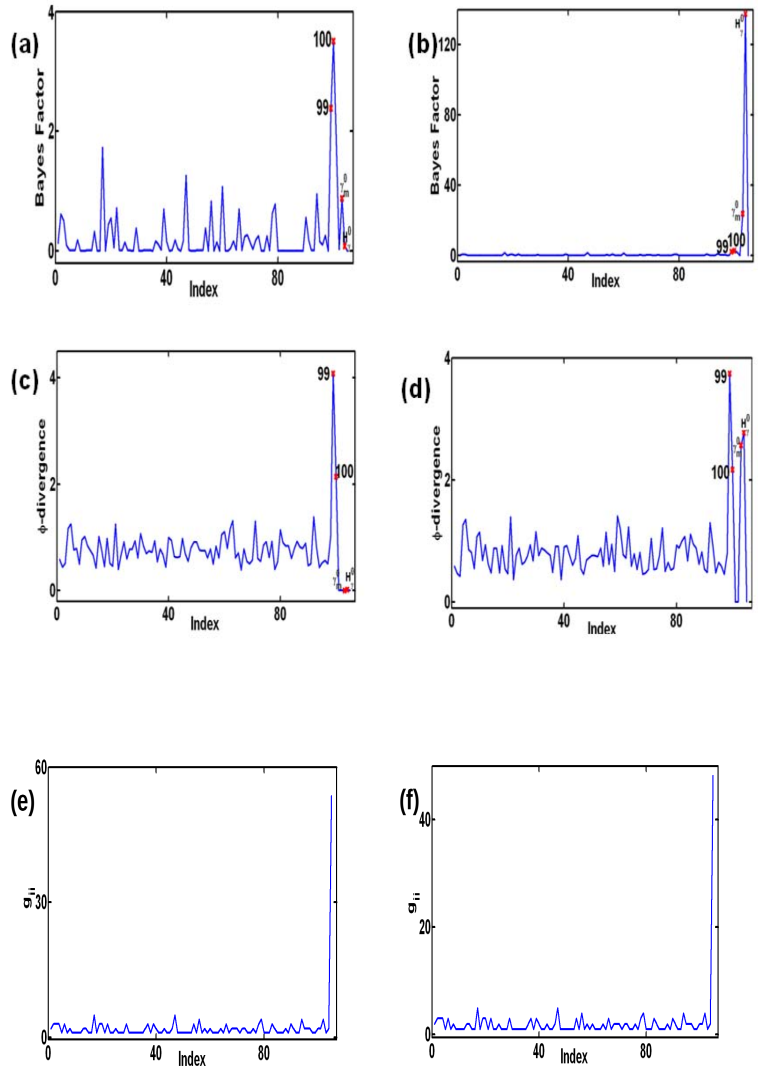

Figure 2: Index plots of local influence measures (a) $\mathbf{v}^B_{\max}$, (c) $\mathrm{SI}_{\mathrm{D}_{\phi,\mathbf{e}_j}}$ and (e) g_{ii} with $p(\boldsymbol{\gamma}_m) \stackrel{D}{=} N(\boldsymbol{\gamma}^0_m, \boldsymbol{I}_2)$, (b) $\mathbf{v}^B_{\max}$, (d) $\mathrm{SI}_{\mathrm{D}_{\phi,\mathbf{e}_j}}$ and (f) g_{ii} with $p(\boldsymbol{\mu}_\gamma) \stackrel{D}{=} N(\mathbf{1}_2, 0.08\boldsymbol{I}_2)$ for the second simultaneous perturbation.

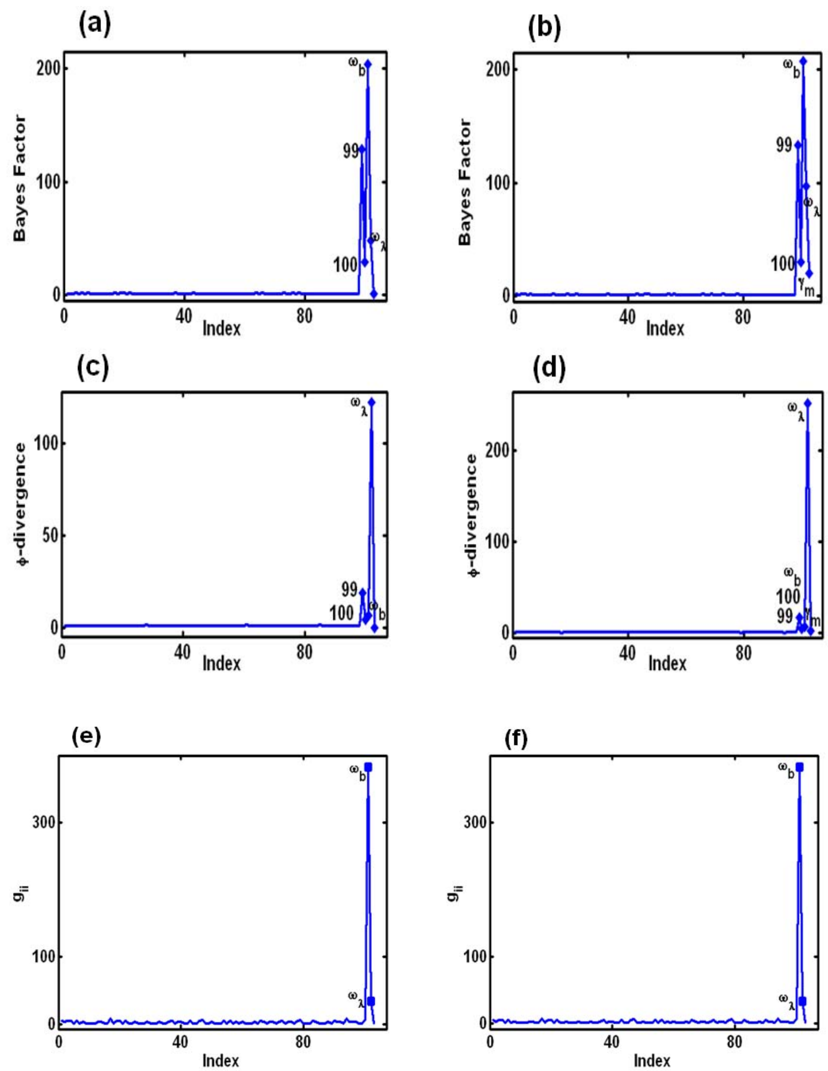

Figure 3: Index plots of local influence measures (a) $\mathbf{v}^B_{\max}$, (c) $\mathrm{SI}_{\mathrm{D}_{\phi,\mathbf{e}_j}}$ and (e) g_{ii} with $p(\boldsymbol{\gamma}_m) \stackrel{D}{=} N(\boldsymbol{\gamma}^0_m, \boldsymbol{I}_2)$, (b) $\mathbf{v}^B_{\max}$, (d) $\mathrm{SI}_{\mathrm{D}_{\phi,\mathbf{e}_j}}$ and (f) g_{ii} with $p(\boldsymbol{\mu}_\gamma) \stackrel{D}{=} N(\mathbf{1}_2, 0.08\boldsymbol{I}_2)$ for the second simultaneous perturbation.