A Health Production Model with Endogenous Retirement

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1. Online Appendix: detailed derivations for scenarios A through F

This appendix provides detailed derivations of the solutions for each of the scenarios A through F (see Figure 1).

1.1. Scenario A

1.1.1. Scenario A: $0 \leq t \leq t_1$

Figure 1 shows how in scenario A initial health $H(0)$ is above the initial health threshold $H_*(0)$ and individuals do not invest in health $m(t) = 0$. As a result health deteriorates with rate $d(t)$ untill age t_1 when health reaches the health threshold $H_*(t_1)$. We have the following condition $[H(t_1) = H_*(t_1)]$:

$$H(0)e^{-\int_0^{t_1} d(s)ds} = (1-\zeta)\Lambda_A \left[\pi_H(t_1) - \varphi(t_1)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t_1}, \tag{1}$$

and the following solutions for consumption $C(t)$, health $H(t)$ and health investment $m(t)$:

$$\begin{aligned}
H(t) &= H(0)e^{-\int_0^{t} d(s)ds} && (2)\\
&= (1-\zeta)\Lambda_A \left[\pi_H(t_1) - \varphi(t_1)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t_1} e^{\int_t^{t_1} d(s)ds} && (3)\\
C(t) &= \zeta\Lambda_A^{1/\chi}(1-\zeta)^{\frac{1-\chi}{\chi}} H(0)^{-\left(\frac{1-\chi}{\chi}\right)} e^{\left(\frac{1-\chi}{\chi}\right)\int_0^{t} d(s)ds} e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} && (4)\\
&= \zeta\Lambda_A \left[\pi_H(t_1) - \varphi(t_1)\right]^{1-\chi} e^{\left(\frac{1-\chi}{\chi}\right)\left(\frac{\beta-\delta}{\rho}\right)t_1} e^{-\left(\frac{1-\chi}{\chi}\right)\int_t^{t_1} d(s)ds} e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} && (5)\\
m(t) &= 0. && (6)
\end{aligned}$$

1.1.2. Scenario A: $t_1 < t \leq R$

Between the age t_1 and retirement R individuals invest in health $m(t) > 0$ and follow the health threshold: $H_*(t)$, $C_*(t)$, and $m_*(t)$.

$$\begin{aligned}
H_*(t) &= (1-\zeta)\Lambda_A \left[\pi_H(t) - \varphi(t)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t}, && (7)\\
C_*(t) &= \zeta\Lambda_A \left[\pi_H(t) - \varphi(t)\right]^{1-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t}, && (8)\\
m_*(t) &= \frac{1}{\mu(t)} e^{-\int_0^t d(s)ds} \frac{d}{dt}\left((1-\zeta)\Lambda_A \left[\pi_H(t) - \varphi(t)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t} e^{\int_0^t d(s)ds}\right). && (9)
\end{aligned}$$

1.1.3. Scenario A: $R < t \leq t_2$

At retirement the health threshold drops to $H_*(R_+)$ and once more individuals do not invest in health $(m(t) = 0)$ till age t_2 when health reaches the health threshold $H_*(t_2)$. We have the following condition $[H(t_2) = H_*(R_-)e^{-\int_R^{t_2} d(s)ds} = H_*(t_2)]$:

$$\begin{aligned}
&(1-\zeta)\Lambda_A \left[\pi_H(R) - \varphi(R)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)R} e^{-\int_R^{t_2} d(s)ds}\\
=\ & k^{1/\rho}(1-\zeta)\Lambda_A \left[\pi_H(t_2)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t_2}, \qquad (10)
\end{aligned}$$

and the following solutions for consumption $C(t)$, health $H(t)$ and health investment $m(t)$:

$$
\begin{aligned}
H(t) &= H_*(R_-)e^{-\int_R^t d(s)ds} && (11)\\
&= (1-\zeta)\Lambda_A\left[\pi_H(R)-\varphi(R)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)R} e^{-\int_R^t d(s)ds} && (12)\\
C(t) &= k^{1/\rho\chi}\zeta\Lambda_A^{1/\chi}(1-\zeta)^{\frac{1-\chi}{\chi}} H_*(R_-)^{-\left(\frac{1-\chi}{\chi}\right)} e^{\left(\frac{1-\chi}{\chi}\right)\int_R^t d(s)ds} e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} && (13)\\
&= k^{1/\rho\chi}\zeta\Lambda_A\left[\pi_H(R)-\varphi(R)\right]^{1-\chi} e^{\left(\frac{1-\chi}{\chi}\right)\left(\frac{\beta-\delta}{\rho}\right)R} e^{\left(\frac{1-\chi}{\chi}\right)\int_R^t d(s)ds} e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} && (14)\\
m(t) &= 0. && (15)
\end{aligned}
$$

1.1.4. Scenario A: $t_2 < t \leq T$

Between the age t_2 and the end of life T individuals invest once again in health $(m(t) > 0)$ and follow the health threshold: $H_*(t)$, $C_*(t)$, and $m_*(t)$.

$$
\begin{aligned}
H_*(t) &= k^{1/\rho}(1-\zeta)\Lambda_A\left[\pi_H(t)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t}, && (16)\\
C_*(t) &= k^{1/\rho}\zeta\Lambda_A\left[\pi_H(t)\right]^{1-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t}, && (17)\\
m_*(t) &= k^{1/\rho}\frac{1}{\mu(t)}e^{-\int_0^t d(s)ds}\frac{d}{dt}\left((1-\zeta)\Lambda_A\left[\pi_H(t)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t} e^{\int_0^t d(s)ds}\right). && (18)
\end{aligned}
$$

1.1.5. Scenario A: determination of Λ_A

Using the life-time budget constraint (equation 38 for $t = T$) and substituting the solutions for health $H(t)$, consumption $C(t)$ and health investment $m(t)$ we can determine the constant Λ_A. Define:

$$\Lambda_A \equiv \frac{\Lambda_{An}}{\Lambda_{Ad}}, \quad (19)$$

where Λ_{An} is the numerator and Λ_{Ad} is the denominator of Λ_A. We find:

$$
\begin{aligned}
\Lambda_{An} &= A(0) - A(T)e^{-\delta T} + \int_0^R w_0(x)e^{-\delta x}dx + \int_R^T b(x)e^{-\delta x}dx \\
&+ H(0)\int_0^{t_1} \varphi(x)e^{-\int_0^x d(s)ds}e^{-\delta x}dx
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
\Lambda_{Ad} &= \int_{t_1}^R [\pi_H(x) - \varphi(x)]^{1-\chi} e^{-\kappa x}dx + k^{1/\rho}\int_{t_2}^T [\pi_H(x)]^{1-\chi} e^{-\kappa x}dx \\
&+ \zeta [\pi_H(t_1) - \varphi(t_1)]^{1-\chi} e^{\left(\frac{\beta-\delta}{\rho}\right)\left(\frac{1-\chi}{\chi}\right)t_1} \int_0^{t_1} e^{-\left(\frac{1-\chi}{\chi}\right)\int_x^{t_1} d(s)ds} e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)x} e^{-\delta x}dx \\
&+ \zeta k^{1/\rho\chi} [\pi_H(R) - \varphi(R)]^{1-\chi} e^{\left(\frac{\beta-\delta}{\rho}\right)\left(\frac{1-\chi}{\chi}\right)R} \int_R^{t_2} e^{\left(\frac{1-\chi}{\chi}\right)\int_R^x d(s)ds} e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)x} e^{-\delta x}dx \\
&+ (1-\zeta)\frac{p(R)}{\mu(R)} [\pi_H(R) - \varphi(R)]^{-\chi} e^{-\kappa R} - (1-\zeta)\frac{p(t_1)}{\mu(t_1)} [\pi_H(t_1) - \varphi(t_1)]^{-\chi} e^{-\kappa t_1} \\
&+ (1-\zeta)k^{1/\rho}\frac{p(T)}{\mu(T} [\pi_H(T)]^{-\chi} e^{-\kappa T} - (1-\zeta)k^{1/\rho}\frac{p(t_2)}{\mu(t_2)} [\pi_H(t_2)]^{-\chi} e^{-\kappa t_2},
\end{aligned} \tag{21}
$$

where we have used the following definition:

$$
\kappa \equiv \frac{\delta\rho + \beta - \delta}{\rho}. \tag{22}
$$

1.2. Scenario B

1.2.1. Scenario B: $0 \le t \le t_1$

Figure 1 shows how similar to scenario A initial health $H(0)$ is above the health threshold $H_*(0)$ and individuals do not invest in health $m(t) = 0$. As a result health deteriorates with rate $d(t)$ untill age t_1 when health reaches the health threshold $H_*(t_1)$. The same condition $[H(t_1) = H_*(t_1)]$ holds as in scenario A (equation 1; replace Λ_A with Λ_B). Also the solutions for consumption $C(t)$, health $H(t)$ and health investment $m(t)$ are the same as in scenario A (2, 3, 4, 5, and 6; replace Λ_A with Λ_B).

1.2.2. Scenario B: $t_1 < t \le R$

As in scenario A, between the age t_1 and retirement R individuals invest in health $m(t) > 0$ and follow the health threshold: $H_*(t)$, $C_*(t)$, and $m_*(t)$ (see equations 7, 8, and 9; replace Λ_A with Λ_B).

1.2.3. Scenario A: $R < t \le T$

As in scenario A, at retirement the health threshold drops to $H_*(R_+)$ and once more individuals do not invest in health ($m(t) = 0$). In scenario B (unlike in scenario A) health, after the retirement age R, does not deteriorate to the health threshold level $H_*(t)$ before the end of life T. The solutions for consumption $C(t)$, health $H(t)$ and health investment $m(t)$ are given by equations 11, 12, 13, 14, and 15 (replace Λ_A with Λ_B) and are valid for $R < t \le T$.

1.2.4. Scenario B: determination of Λ_B

Defining

$$\Lambda_B \equiv \frac{\Lambda_{Bn}}{\Lambda_{Bd}}, \tag{23}$$

where Λ_{Bn} is the numerator and Λ_{Bd} is the denominator of Λ_B, we find:

$$\begin{aligned}
\Lambda_{Bn} &= A(0) - A(T)e^{-\delta T} + \int_0^R w_0(x)e^{-\delta x}dx + \int_R^T b(x)e^{-\delta x}dx \\
&+ H(0)\int_0^{t_1} \varphi(x)e^{-\int_0^x d(s)ds}e^{-\delta x}dx \qquad (24)\\
\Lambda_{Bd} &= \int_{t_1}^R [\pi_H(x) - \varphi(x)]^{1-\chi}\, e^{-\kappa x}dx \\
&+ \zeta\,[\pi_H(t_1) - \varphi(t_1)]^{1-\chi}\, e^{\left(\frac{\beta-\delta}{\rho}\right)\left(\frac{1-\chi}{\chi}\right)t_1}\int_0^{t_1} e^{-\left(\frac{1-\chi}{\chi}\right)\int_x^{t_1} d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)x}e^{-\delta x}dx \\
&+ \zeta k^{1/\rho\chi}\,[\pi_H(R) - \varphi(R)]^{1-\chi}\, e^{\left(\frac{\beta-\delta}{\rho}\right)\left(\frac{1-\chi}{\chi}\right)R}\int_R^T e^{\left(\frac{1-\chi}{\chi}\right)\int_R^x d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)x}e^{-\delta x}dx \\
&+ (1-\zeta)\frac{p(R)}{\mu(R)}\,[\pi_H(R) - \varphi(R)]^{-\chi}\, e^{-\kappa R} \\
&- (1-\zeta)\frac{p(t_1)}{\mu(t_1)}\,[\pi_H(t_1) - \varphi(t_1)]^{-\chi}\, e^{-\kappa t_1}. \qquad (25)
\end{aligned}$$

1.3. Scenario C

1.3.1. Scenario C: $0 \leq t \leq R$

Figure 1 shows how similar to scenarios A and B initial health $H(0)$ is above the initial health threshold $H_*(0)$ and individuals do not invest in health $m(t) = 0$. But unlike scenarios A and B, health reaches the health threshold $H_*(t_2)$ only at age t_2, after the retirement age R. Individuals thus only invest in health during retirement and not during working life. A similar condition $[H(t_2) = H_*(t_2)]$ holds as in scenarios A and B (equation 1). We have:

$$H(0)e^{-\int_0^{t_2} d(s)ds} = k^{1/\rho}(1-\zeta)\Lambda_C\,[\pi_H(t_2)]^{-\chi}\, e^{-\left(\frac{\beta-\delta}{\rho}\right)t_2}, \tag{26}$$

and the following solutions for consumption $C(t)$, health $H(t)$ and health investment $m(t)$:

$$\begin{aligned}
H(t) &= H(0)e^{-\int_0^t d(s)ds} \qquad (27)\\
&= k^{1/\rho}(1-\zeta)\Lambda_C\,[\pi_H(t_2)]^{-\chi}\, e^{-\left(\frac{\beta-\delta}{\rho}\right)t_2}e^{\int_t^{t_2} d(s)ds} \qquad (28)\\
C(t) &= \zeta\Lambda_C^{1/\chi}(1-\zeta)^{\frac{1-\chi}{\chi}}H(0)^{-\left(\frac{1-\chi}{\chi}\right)}e^{\left(\frac{1-\chi}{\chi}\right)\int_0^t d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} \qquad (29)\\
&= k^{-\left(\frac{1-\chi}{\rho\chi}\right)}\zeta\Lambda_C\,[\pi_H(t_2)]^{1-\chi}\, e^{\left(\frac{1-\chi}{\chi}\right)\left(\frac{\beta-\delta}{\rho}\right)t_2}e^{-\left(\frac{1-\chi}{\chi}\right)\int_t^{t_2} d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} \qquad (30)\\
m(t) &= 0. \qquad (31)
\end{aligned}$$

1.3.2. Scenario C: $R < t \leq t_2$

The solutions for consumption $C(t)$, health $H(t)$ and health investment $m(t)$ are:

$$
\begin{aligned}
H(t) &= H(0)e^{-\int_0^t d(s)ds} && (32)\\
&= k^{1/\rho}(1-\zeta)\Lambda_C\left[\pi_H(t_2)\right]^{-\chi} e^{-\left(\frac{\beta-\delta}{\rho}\right)t_2} e^{\int_t^{t_2} d(s)ds} && (33)\\
C(t) &= k^{1/\rho\chi}\zeta\Lambda_C^{1/\chi}(1-\zeta)^{\frac{1-\chi}{\chi}} H(0)^{-\left(\frac{1-\chi}{\chi}\right)} e^{\left(\frac{1-\chi}{\chi}\right)\int_0^t d(s)ds} e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} && (34)\\
&= k^{1/\rho}\zeta\Lambda_C\left[\pi_H(t_2)\right]^{1-\chi} e^{\left(\frac{1-\chi}{\chi}\right)\left(\frac{\beta-\delta}{\rho}\right)t_2} e^{-\left(\frac{1-\chi}{\chi}\right)\int_t^{t_2} d(s)ds} e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} && (35)\\
m(t) &= 0. && (36)
\end{aligned}
$$

1.3.3. Scenario C: $t_2 < t \leq T$

Between the age t_2 and the end of life T individuals invest once again in health ($m(t) > 0$) and follow the health threshold: $H_*(t)$, $C_*(t)$, and $m_*(t)$. The equations are the same as in scenario A (equations 16, 17, and 18; replace Λ_A with Λ_C).

1.3.4. Scenario C: determination of Λ_C

Defining

$$\Lambda_C \equiv \frac{\Lambda_{Cn}}{\Lambda_{Cd}}, \quad (37)$$

where Λ_{Cn} is the numerator and Λ_{Cd} is the denominator of Λ_C, we find:

$$
\begin{aligned}
\Lambda_{Cn} &= A(0) - A(T)e^{-\delta T} + \int_0^R w_0(x)e^{-\delta x}dx + \int_R^T b(x)e^{-\delta x}dx\\
&+ H(0)\int_0^R \varphi(x)e^{-\int_0^x d(s)ds}e^{-\delta x}dx && (38)\\
\Lambda_{Cd} &= k^{1/\rho}\int_{t_2}^T \left[\pi_H(x)\right]^{1-\chi} e^{-\kappa x}dx\\
&+ k^{-\left(\frac{1-\chi}{\rho\chi}\right)}\zeta\left[\pi_H(t_2)\right]^{1-\chi} e^{\left(\frac{\beta-\delta}{\rho}\right)\left(\frac{1-\chi}{\chi}\right)t_2}\int_0^R e^{-\left(\frac{1-\chi}{\chi}\right)\int_x^{t_2} d(s)ds} e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)x} e^{-\delta x}dx\\
&+ k^{1/\rho}\zeta\left[\pi_H(t_2)\right]^{1-\chi} e^{\left(\frac{\beta-\delta}{\rho}\right)\left(\frac{1-\chi}{\chi}\right)t_2}\int_R^{t_2} e^{-\left(\frac{1-\chi}{\chi}\right)\int_x^{t_2} d(s)ds} e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)x} e^{-\delta x}dx\\
&+ (1-\zeta)k^{1/\rho}\frac{p(T)}{\mu(T}\left[\pi_H(T)\right]^{-\chi} e^{-\kappa T} - (1-\zeta)k^{1/\rho}\frac{p(t_2)}{\mu(t_2)}\left[\pi_H(t_2)\right]^{-\chi} e^{-\kappa t_2}. && (39)
\end{aligned}
$$

1.4. Scenario D

1.4.1. Scenario D: $0 \leq t \leq R$

Figure 1 shows how similar to scenarios A, B and C initial health $H(0)$ is above the initial health threshold $H_*(0)$ and individuals do not invest in health $m(t) = 0$. But unlike scenarios A, B and C health never reaches the health threshold $H_*(t)$ at any point during the individual's life time. Individuals are sufficiently endowed with initial health capital that they never need to invest in health during working life nor during retirement.

The solutions for consumption $C(t)$, health $H(t)$ and health investment $m(t)$ are:

$$
\begin{aligned}
H(t) &= H(0)e^{-\int_0^t d(s)ds} && (40)\\
C(t) &= \zeta\Lambda_D^{1/\chi}(1-\zeta)^{\frac{1-\chi}{\chi}}H(0)^{-\left(\frac{1-\chi}{\chi}\right)}e^{\left(\frac{1-\chi}{\chi}\right)\int_0^t d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} && (41)\\
m(t) &= 0. && (42)
\end{aligned}
$$

1.4.2. Scenario D: $R < t \leq T$

The solutions for consumption $C(t)$, health $H(t)$ and health investment $m(t)$ are:

$$
\begin{aligned}
H(t) &= H(0)e^{-\int_0^t d(s)ds} && (43)\\
C(t) &= k^{1/\rho\chi}\zeta\Lambda_D^{1/\chi}(1-\zeta)^{\frac{1-\chi}{\chi}}H(0)^{-\left(\frac{1-\chi}{\chi}\right)}e^{\left(\frac{1-\chi}{\chi}\right)\int_0^t d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)t} && (44)\\
m(t) &= 0. && (45)
\end{aligned}
$$

1.4.3. Scenario D: determination of Λ_D

Defining

$$
\Lambda_D \equiv \frac{\Lambda_{Dn}}{\Lambda_{Dd}}, \qquad (46)
$$

where Λ_{Dn} is the numerator and Λ_{Dd} is the denominator of Λ_D, we find:

$$
\begin{aligned}
\Lambda_{Dn}^{1/\chi} &= A(0) - A(T)e^{-\delta T} + \int_0^R w_0(x)e^{-\delta x}dx + \int_R^T b(x)e^{-\delta x}dx\\
&+ H(0)\int_0^R \varphi(x)e^{-\int_0^x d(s)ds}e^{-\delta x}dx && (47)\\
\Lambda_{Dd}^{1/\chi} &= \zeta(1-\zeta)^{\frac{1-\chi}{\chi}}H(0)^{-\left(\frac{1-\chi}{\chi}\right)}\int_0^R e^{\left(\frac{1-\chi}{\chi}\right)\int_0^x d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)x}e^{-\delta x}dx\\
&+ \zeta(1-\zeta)^{\frac{1-\chi}{\chi}}H(0)^{-\left(\frac{1-\chi}{\chi}\right)}k^{1/\rho\chi}\int_R^T e^{\left(\frac{1-\chi}{\chi}\right)\int_0^x d(s)ds}e^{-\left(\frac{\beta-\delta}{\rho\chi}\right)x}e^{-\delta x}dx. && (48)
\end{aligned}
$$

1.5. Scenarios E and F

Figure 1 shows scenarios E and F. In these scenarios initial health $H(0)$ is below the initial health threshold $H_*(0)$. The simplified Grossman model that we employ here allows for complete repair. Case and Deaton (2005) point out that employing such technology is not realistic. Indeed wealthy individuals may have high health threshold levels and the ability to afford any kind of health investment, but they may not necessary be able to repair all types of poor health (e.g., cancer, aids, various disabilities such as blindness etc). Simply stated, not every illness has a cure. Further, while health in the formulation cannot deteriorate faster than the deterioration rate $d(t)$ there is no intrinsic constraint on the rate at which health can be repaired. As such, in scenarios E and F individuals will seek to repair their health instantaneously when they enter the workforce at age 20 ($t = 0$), effectively substituting initial assets $A(0)$ for improved initial health $H(0)$ such that initial health equals the initial health threshold $H(0) = H_*(0)$. An alternative interpretation is that individuals invest in health $m(t)$ well before they enter the workforce at age 20 ($t = 0$) to ensure their health is at the initial health threshold $H_*(0)$ at $t = 0$. Before they enter the workforce individuals don't consume yet (or at least consumption is paid for by their parents / caretakers) and have no assets $A(t)$ yet. In this case the end result is the same as if health investment were made in an infinitesimally small period of time at $t = 0$. We assume that individuals pay for the health investment themselves, i.e. they start with lower initial assets $A_*(0) = A(0) - p(0)m_*(0)$, where $m_*(0)$ is the quantity of health investment needed to arrive from initial health $H(0)$ to the initial health threshold $H_*(0)$. Approximating this initial health investment by a delta function, $m(t) = m_*(0)\delta(t - 0)$ (i.e., mathematically investment takes place at $t = 0$ during an infinitesimally small period of time) we find:

$$H_*(0) = H(0) + \mu(0)m_*(0), \tag{49}$$

and

$$\begin{aligned} A_*(0) &= A(0) - p(0)m_*(0) \\ &= A(0) - \frac{p(0)}{\mu(0)}\left[H_*(0) - H(0)\right] \\ &= A(0) - \frac{p(0)}{\mu(0)}(1-\zeta)\Lambda_{E,F}\left[\pi_H(0) - \varphi(0)\right]^{-\chi} + \frac{p(0)}{\mu(0)}H(0), \end{aligned} \tag{50}$$

where $\Lambda_{E,F}$ denotes either Λ_E for scenario E or Λ_F for scenario F.

The solution for scenarios E and F can be derived from solutions A and B, respectively, by setting $t_1 = 0$ and replacing initial assets $A(0)$ and initial health $H(0)$ with the above expressions for $A_*(0)$ and $H_*(0)$. We leave this excercise to the reader.