Supporting Information Legends

Estimating Number of Principal Components

The spectral decomposition yields a parsimonious expansion of the subject level functions $A_{iq}(t) = \sum_{j=1}^{\infty} \xi_{ijq}\psi_{jq}(t)$. Necessarily, we truncate the decomposition at L terms so that $A_{iq}(t)$ has a finite decomposition expression, $A_{iq}(t) = \sum_{j=1}^{L} \xi_{ijq}\psi_{jq}(t)$.

We follow the approach proposed by [?] to estimate L based on proportion of variance explained. Let P_1 and P_2 be two thresholds, and define

$$L_q = \min\{k : \sum_{j=1}^{k} \lambda_{jq} / \sum_{j'=1}^{\infty} \lambda_{j'q} \geq P_1, \lambda_{kq} < P_2\}.$$

Here, P_1 is a threshold on the cumulative explained variance while P_2 is a threshold on the individual explained variance. In this manuscript, we choose $P_1 = 0.95$ and $P_2 = 0.02$. These choices work well in our simulations and application. However, they should be carefully tuned in other settings, perhaps using simulations.

Variance Matrix Smoothing

Instead of the true mixing matrix functions, $A_{iq}(t)$, we obtain the model-based estimates from the ICA algorithm, $\hat{A}_{iq}(t)$. Assume a measurement error model so that $\hat{A}_{iq}(t) = A_{iq}(t) + \epsilon_{iq}(t)$, where $\epsilon_i(t)$ is a white noise process with variance σ_q^2. Thus a smoothing step is desirable.

Under the assumed model $\hat{A}_{iq}(t) = A_{iq}(t) + \epsilon_{iq}(t)$, the covariance operator for the observed data is $K_q^W(s,t) = K_q(s,t) + \sigma_q^2 \delta_{ts}$, where $K_q^W(s,t) = \mathrm{Cov}\{\hat{A}_{iq}(s), \hat{A}_{iq}(t)\}$ and $\delta_{ts} = 1$ if $t = s$ and 0 otherwise [?]. This equation reveals that the diagonal elements of $K_q^W(s,t)$ includes the nugget measurement error. A simple and natural solution is to drop the diagonal elements and smooth the covariance matrix. We use the standard (moment based) estimate $\hat{K}_q^W(s,t)$ from the observed data, $\sum_{i=1}^{I} \hat{A}_{iq}(t)\hat{A}_{iq}(s)/I$ and then estimate $\hat{K}^A(s,t)$ by smoothing the estimate for $s \neq t$ [?, ?]. The eigenvalues, λ_{jq}, and eigenfunctions, ψ_{jq} can then be derived from this estimated covariance matrix.