Supporting Material

Thrombin activity propagates in space during blood coagulation as an excitation wave

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Data analysis

Algorithm overview.

The videomicroscopy system designed in this study monitored AMC (7-amino-4 methylcoumarin) cleavage from a fluorogenic substrate by thrombin during blood coagulation in the experimental chamber. Each experiment produced a series of images of i) light scattering intensity and ii) AMC fluorescence. These data were processed to calculate the spatial and temporal distributions of fibrin and thrombin in space and time, similarly to the strategy used in homogeneous thrombin generation assays (1). Images from red and UV light were processed similarly. To obtain light scattering or AMC fluorescence intensity profiles, the light intensity along the line perpendicular to the activator surface was measured for each frame. Values were averaged for 150–300 lines.

The clot growth rate was calculated from the movement of the point of half-maximal intensity of the light scattering profiles (2). The initial rate was calculated as the mean tangent of the size vs. time curve over the first 10 min of clot growth. The stationary rate was calculated similarly after 40 min of clot growth when the clot edge moved far enough away from the activator, such that its effect on clot growth became negligible.

To reconstruct the spatiotemporal distribution of proteolytic enzyme activity, the reaction-diffusion equation was used:

$$
\frac{\partial [AMC]}{\partial t} = D_{AMC} \cdot \frac{\partial^2 [AMC]}{\partial x^2} + \frac{k_{cat} \cdot [S] \cdot [Ha]}{K_M + [S]},
$$
\n(S1)

where $[AMC]$ is the AMC concentration; D_{AMC} is the AMC diffusion coefficient; k_{cat} and K_M are the catalytic and Michaelis constants of substrate cleavage, respectively; $|I1a|$ is the thrombin concentration; and $|S|$ is the substrate concentration. This equation can be used to derive the thrombin concentration:

$$
[IIA] = \frac{K_M + [S]}{k_{cat} \cdot [S]} \cdot \left(\frac{\partial [AMC]}{\partial t} - D_{AMC} \cdot \frac{\partial^2 [AMC]}{\partial x^2} \right),
$$
 (S2)

Numerical differentiation of an experimentally obtained signal is an "incorrectly posed problem", in which large noises in the final outcome can easily occur from slight noises in the initial signal. In addition, the procedure of thrombin reconstruction can be subject to numerous disturbances, such as nonuniformity of illumination, AMC bleaching, fluorescence distortion by fibrin clot, or binding of AMC in plasma, leading to altered diffusion velocity. Therefore, additional steps were included in the algorithm, as detailed below. Parameters for the algorithm are given in Table S1.

Calculating AMC concentration from fluorescence.

The AMC intensity profiles were transformed into concentration profiles by calibration. The calibration intensity profile *Ical* was calculated on the basis of an image of uniform distribution of known AMC concentration in the same plasma. The AMC concentration at each point $[AMC_i]$ was calculated from the fluorescence intensity I_i by using the background fluorescence profile I_{bgr} and AMC concentration in the calibration sample $[AMC_{cal}]$:

$$
[AMCi] = \frac{(Ii - Ibgr) * [AMCcal]}{(Ical - Ibgr)}
$$
\n(S3)

Correction of fibrin-clot-induced distortion.

The fluorescence intensity was increased in the presence of fibrin clot. This increase was proportional to both fibrin and AMC concentrations. (Fig. S1) To correct for fluorescence distortion, the following equation was used:

$$
[AMC_{app}] = [AMC] + (k_1 + k_2 \cdot [AMC]) \cdot \frac{F}{k_3}, \qquad (S4)
$$

where $\left[AMC_{app}\right]$ is the apparent AMC concentration obtained from the previous procedure, $[AMC]$ is the true AMC concentration, F is the fibrin light scattering intensity, and k_1-k_3 are the experimental setup-dependent parameters.

Effect of AMC binding in plasma on the diffusion velocity.

Experiments to assess AMC binding in plasma revealed that there are binding sites for AMC provided (most likely) by serum albumin, and that a significant quantity of AMC is always in a bound state. (Fig. S2) Therefore, the effective diffusion of AMC was reduced. This finding led to a correction of the initial Eq. 1:

$$
\frac{\partial [AMC]}{\partial t} = D_{AMC} \cdot \frac{\partial^2 [AMC^{free}]}{\partial x^2} + D_{BSA} \cdot \frac{\partial^2 [AMC^{BSA}]}{\partial x^2} + \frac{k_{cat} \cdot [S] \cdot [IIa]}{K_M + [S]},
$$
(S5)

$$
[AMC^{bound}] = [AMC] - [AMC^{free}] = 0.67 \cdot [AMC], \qquad (S6)
$$

Effect of fibrin clot on thrombin: a control.

To investigate if clot formation regulates thrombin generation via reversible inhibition of thrombin by fibrin, or via protection of thrombin from inhibition by antithrombin III, or via acceleration of factor XI activation, we depleted fibrinogen using a fibrin-cleaving protease from snake venom. In defibrinated plasma, thrombin spatial propagation was qualitatively similar (Fig. S3) indicating that thrombin wave propagation is not determined by formation of polymerized fibrin clot.

Selection of a numerical method for calculation of the derivatives.

To summarize, the set of equations required to obtain thrombin from the AMC distribution were of the form:

$$
\begin{cases}\n[IIA] = \frac{K_M + [S]}{k_{cat} \cdot [S]} \cdot \left(\frac{\partial [AMC]}{\partial t} - D_{AMC} \cdot \frac{\partial^2 [AMC^{free}]}{\partial x^2} - D_{BSA} \cdot \frac{\partial^2 [AMC^{BSA}]}{\partial x^2} \right) \\
[AMC^{bound}] = [AMC] - [AMC^{free}] = 0.67 \cdot [AMC] \\
\frac{\partial [S]}{\partial t} = D_S \cdot \frac{\partial^2 [S]}{\partial x^2} - \frac{k_{cat} \cdot [S] \cdot [IIA]}{K_M + [S]} \\
S|_{t=0} = S_0\n\end{cases}
$$
\n(S7)

The contribution of substrate diffusion was extremely small and was usually neglected. To reduce noise, the following algorithm was used to calculate derivatives:

$$
\frac{\partial [AMC]}{\partial t} = \frac{[AMC]_{x,t+j\Delta t} - [AMC]_{x,t}}{j \cdot \Delta t}
$$
 (S8)

$$
\frac{\partial^2 [AMC]}{\partial x^2} = \frac{[AMC]_{x+i \Delta x,t} - 2 \cdot [AMC]_{x,t} + [AMC]_{x-i \Delta x,t}}{(i \cdot \Delta x)^2}
$$
(S9)

where ∆*t* is the time between frames (usually 1 min) and ∆*x* is pixel size (usually 4.3 microns). The parameter values $i = 60$ and $j = 3$ were chosen to be optimal for maximal noise reduction and minimal signal distortion.

This algorithm was able to reconstruct thrombin activity distribution in space and time (Figure 1). However, the regions very close to the activator (200–300 μm) were not reconstructed, and the contribution of the α_2 -macroglobulin complex with thrombin to AMC production (3) was neglected.

Computer simulations

Model design.

Blood coagulation was simulated as a 1D process on a $(0, L)$ interval, with $L = 4$ mm and tissue factor (TF) located at $x = 0$. The model included 24 partial differential equations for reactants in plasma and two ordinary differential equations for the density of the TF-containing complexes on the activator. The model was numerically integrated by using the embedded Runge-Kutta-Fehlberg method of the order 2(3) as described (4).

Notation.

The following notation is used throughout this paper. Concentration designations are as follows: x_j , concentration of the j^{th} active coagulation factor; y_j , concentration of the inactive precursor of the j^{th} coagulation factor; i_j , concentration of the j^{th} inhibitor; x_{i-j} , concentration of the complex of the i^{th} and j^{th} factors; x_i^F x_j^F , concentration of the free *j*th factor (not bound to another factor or phospholipid membrane); $x_i^{B^F}$ x_j^{B} ^F, concentration of the free j^{th} factor on the membrane (not in complex with another factor on the same membrane); and x_i^s x_j^s , surface densities of TF, factor VII, factor VIIa, and their complexes. All initial concentrations are summarized in Table S2.

Parameter designations are as follows: k_j , kinetic constant or forward rate constant of the j^{th} reaction; k_j , backward rate constant of the j^{th} reaction; K_j , the Michaelis or equilibrium constant of the j^{th} reaction; n_j , number of membrane binding sites for the j^{th} factor; and h_j , rate constant of the *j*th inhibition reaction. All constants are summarized in Table S3.

Dimensions are as follows: distance $[x] = mm$, time $[t] = min$, volume concentrations $[x_j,$ *y*_{*j*}, etc.] = nM, and surface densities $[x_3, x_7, y_7, x_{7-3}, y_{7-3}] =$ nmol/mm².

Model description.

The quantitative mechanism-driven mathematical model was based on a model published recently by the authors (5). The model was modified to take into account the use of another activator, the transition to a reaction-diffusion system (6), and introduction of a fluorogenic substrate and phospholipids. The most important modifications were as follows: 1) all variables were separated into those described by partial and ordinary equations as described above; 2) interactions of thrombin and thrombomodulin were added; 3) interactions of thrombin and factor Xa with Z-Gly-Gly-Arg-AMC were added; 4) all diffusion coefficients (6) were corrected for plasma viscosity and temperature by multiplying by 0.88; 5) only free factors (not bound to membranes) were allowed to diffuse; and 6) to account for the effects of the phospholipid membrane, the multiplicator of $p/(1+p/K_p)$ was substituted for p in all equations.

Variable pools.

All equations are for total concentrations of coagulations factors. For example, factor Xa is presumably distributed between free factor, prothrombinase complex, substrate-bound, etc:

$$
x_{10} = x_{10}^F + x_{10-5}^B + x_{10}^S + x_{10-5}^S
$$
 (S10)

To calculate the thrombin activity observed in the experiment, the following equation was used:

$$
x_{2,\exp} = x_2^S + x_2^F + \frac{x_{2-M}}{2},
$$
\n(S11)

which accounts for the fact that α_2 -macroglobulin-bound thrombin has ~50% of free thrombin activity towards Z-Gly-Gly-Arg-AMC.

Model equations.

TF^S:
$$
\frac{dx_3^{S}}{dt} = -\left(k_1 \cdot x_7 \cdot x_3^{S} - k_{-1} \cdot x_{7-3}^{S^F}\right) - \left(k_1 \cdot y_7 \cdot x_3^{S} - k_{-1} \cdot y_{7-3}^{S}\right)
$$
(S12)

VII-TF^S:
$$
\frac{dy_{7-3}}{dt} = (k_1 \cdot y_7 \cdot x_3^S - k_{-1} \cdot y_{7-3}^S) - (k_2 \cdot x_2^F + k_3 \cdot x_{10}^F) \cdot y_{7-3}^S
$$
 (S13)

VIIa-TF^S:
$$
\frac{dx_{7-3}}{dt} = \left(k_1 \cdot x_7 \cdot x_3^S - k_{-1} \cdot x_{7-3}^S\right) + \left(k_2 \cdot x_2^F + k_3 \cdot x_{10}^F\right) \cdot y_{7-3}^S - h_1 \cdot x_{7-3}^S{}^F \cdot i_3 -
$$

$$
h_2 \cdot x_{10-7-3}^s \cdot i_2 \tag{S14}
$$

VII:
$$
-D_{\text{VII}} \cdot \frac{\partial y_7}{\partial x} = -\left(k_1 \cdot y_7 \cdot x_3^S - k_{-1} \cdot y_{7-3}^S\right)
$$
(S15)

VIIa:
$$
-D_{VIIa} \cdot \frac{\partial x_{7}}{\partial x} = -\left(k_{1} \cdot x_{7} \cdot x_{3}^{S} - k_{-1} \cdot x_{7-3}^{S^{F}}\right)
$$
(S16)

IX:
$$
-D_{IX} \cdot \frac{\partial y_9}{\partial x} = -F_{IX}, \text{ where } F_{IX} = \frac{k_4}{K_4} \cdot y_9 \cdot x_{7-3}^{S^F}
$$
 (S17)

$$
\text{IXa: } -D_{\text{IXa}} \cdot \frac{\partial x_{9}}{\partial x} = \frac{F_{\text{IX}}}{1 - \frac{n_{20}}{K_{20} + x_{9}} \cdot \frac{p}{1 + \frac{p}{K_{p}}}} \cdot \left(1 - \frac{x_{9}}{K_{20} + x_{9}}\right)
$$
(S18)

$$
X: -D_x \cdot \frac{\partial y_{10}}{\partial x} = -F_{X} \text{ where } F_X = \frac{k_6}{K_6} \cdot y_{10} \cdot x_{7-3}^{\quad S^F} \tag{S19}
$$

$$
\text{Xa:} \quad -D_{x_a} \cdot \frac{\partial x_{10}}{\partial x} = \frac{F_x \cdot \left(1 + \frac{S}{K_{s,10}}\right)}{1 - \frac{x_5^B}{K_{23} \cdot \left(1 + \frac{i_{12}}{K_{24}}\right) + x_{10} + x_5^B} \cdot \frac{1}{1 + \frac{p}{K_p}} \cdot \left(1 - \frac{x_{10}}{K_{23} \cdot \left(1 + \frac{i_{12}}{K_{24}}\right) + x_{10} + x_5^B}\right)}
$$
\n(S20)

$$
\text{TFPI:} \quad -D_{TFPI} \cdot \frac{\partial i_2}{\partial x} = -h_2 \cdot x_{10-7-3}^{\text{S}} \cdot i_2 \tag{S21}
$$

$$
\text{Xa-TFPI complex:} \quad -D_{\text{Xa-TFPI}} \cdot \frac{\partial i_3}{\partial x} = -h_1 \cdot x_{7-3}^{\quad S^F} \cdot i_3 \tag{S22}
$$

VII:
$$
\frac{\partial y_7}{\partial t} = D_{VII} \cdot \frac{\partial^2 y_7}{\partial x^2} - k_2 \cdot y_7 \cdot x_2^F
$$
 (S23)

VIIa:
$$
\frac{\partial x_7}{\partial t} = D_{VIIa} \cdot \frac{\partial^2 x_7}{\partial x^2} + k_2 \cdot y_7 \cdot x_2^F
$$
 (S24)

IX:
$$
\frac{\partial y_9}{\partial t} = D_K \cdot \frac{\partial^2 y_9}{\partial x^2} - R_{IX}
$$
, where $R_{IX} = \frac{k_5 \cdot y_9 \cdot x_{11}}{K_5 + y_9}$ (S25)

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$$
\begin{split} \text{IXa: } \frac{\partial x_{9}}{\partial t} &= D_{Xa} \cdot \frac{\partial^{2}}{\partial x^{2}} \left(x_{9} - x_{9}^{B^{F}} \right) + \text{R}_{IX} - h_{3} \cdot i_{1} \cdot x_{9} \\ \text{X: } \frac{\partial y_{10}}{\partial t} &= D_{X} \cdot \frac{\partial^{2}}{\partial x^{2}} \left(y_{10} - y_{10}^{B} \right) - \text{R}_{X}, \text{ where } \text{R}_{X} = \frac{k_{7} \cdot x_{9}^{B^{F}} \cdot y_{10}^{B}}{p \cdot K_{7}} + \\ &+ \frac{k_{8} \cdot x_{9}^{B^{F}} \cdot x_{8}^{B^{F}} \cdot y_{10}^{B}}{p^{2} \cdot K_{9} \cdot K_{8}} \end{split} \tag{S27}
$$

$$
\text{Xa: } \frac{\partial x_{10}}{\partial t} = D_{xa} \cdot \frac{\partial^2}{\partial x^2} \left(x_{10}^F \cdot \left(1 + \frac{S}{K_{s,10}} \right) \right) + \text{R}_X - \left(k_{11} \cdot x_{10}^F \cdot i_2 - k_{-11} \cdot i_3 \right) - (h_4 \cdot i_1 + h_5 \cdot i_6 + h_6 \cdot i_7 + h_7 \cdot i_{10}) \cdot x_{10}^F - h_8 \cdot i_1 \cdot x_{10}^B \tag{S28}
$$

II:
$$
\frac{\partial y_2}{\partial t} = D_{II} \cdot \frac{\partial^2}{\partial x^2} \left(y_2 - y_2 \right)^B - R_{II},
$$

where
$$
R_{II} = k_{12} \cdot \frac{x_{10}^F \cdot p}{\left(1 + \frac{p}{K_p}\right)^2} \cdot y_2 + k_{13} \cdot \frac{x_{10-5}^B}{p \cdot \left(1 + \frac{S}{K_{s,10}}\right)} \cdot y_2^B
$$
 (S29)

$$
\text{IIa: } \frac{\partial x_2}{\partial t} = D_{\text{IIa}} \cdot \frac{\partial^2}{\partial x^2} \left(x_2^F + x_2^S \right) + D_{\text{Fg}} \cdot \frac{\partial^2}{\partial x^2} x_2^{\text{Fg}} + D_{\text{Fn}} \cdot \frac{\partial^2}{\partial x^2} x_2^{\text{Fn}} + R_{\text{II}} - (h_9 \cdot i_1 + h_{11} \cdot i_7 + h_{12} \cdot i_{10} + h_{13} \cdot i_9) \cdot x_2^F - R_{\text{IIa-M}} - (k_{28} \cdot i_{13} \cdot x_2^F - k_{28} \cdot x_{2\text{-Tm}}) \tag{S30}
$$

$$
\text{IIa-}\alpha_2 \mathbf{M}: \frac{\partial x_{2-M}}{\partial t} = D_{\alpha_2 M} \cdot \frac{\partial^2 x_{2-M}}{\partial x^2} + \mathbf{R}_{\text{IIa-M}}, \text{ where } \mathbf{R}_{\text{IIa-M}} = h_{10} \cdot i_6 \cdot \left(x_2^F + x_2^S\right) \tag{S31}
$$

$$
\begin{split} \text{IIa-Tm:} \quad & \frac{\partial x_{2-Tm}}{\partial t} = D_{\text{Ila-Tm}} \cdot \frac{\partial^2 x_{2-Tm}}{\partial x^2} + \left(k_{28} \cdot i_{13} \cdot x_2^F - k_{28} \cdot x_{2-Tm}\right) - \\ & - \left(h_{25} \cdot i_1 + h_{26} \cdot i_{10}\right) \cdot x_{2-Tm} \end{split} \tag{S32}
$$

Fibrin:
$$
\frac{\partial x_1}{\partial t} = D_{Fn} \cdot \frac{\partial^2 x_1}{\partial x^2} + R_{Fn}
$$
, where $R_{Fn} = \frac{k_{14}}{K_{14}} \cdot y_1 \cdot x_2^F$ (S33)

Fibrinogen:
$$
\frac{\partial y_1}{\partial t} = D_{Fg} \cdot \frac{\partial^2 y_1}{\partial x^2} - R_{Fn}
$$
 (S34)

VIII:
$$
\frac{\partial y_8}{\partial t} = D_{VIII} \cdot \frac{\partial^2 y_8}{\partial x^2} - R_{VIII} \text{, where } R_{VIII} = \frac{k_{15} \cdot y_8 \cdot x_2^F}{K_{15} + x_2^F}
$$
 (S35)

VIIIa:
$$
\frac{\partial x_8}{\partial t} = D_{VIIIa} \cdot \frac{\partial^2}{\partial x^2} \left(x_8 - x_8^{\ BF} \right) + R_{VIII} - h_{14} \cdot x_8
$$
 (S36)

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$$
V: \frac{\partial y_5}{\partial t} = D_V \cdot \frac{\partial^2 y_5}{\partial x^2} - R_V, \text{ where } R_V = \frac{k_{16} \cdot y_5 \cdot x_2^F}{K_{16} + x_2^F}
$$
 (S37)

$$
\text{Va: } \frac{\partial x_5}{\partial t} = D_{\text{Va}} \cdot \frac{\partial^2}{\partial x^2} \left(x_5 - x_5^{\ B} \right) + \text{R}_{\text{V}} - h_{15} \cdot i_4 \cdot x_5^{\ B^F} \tag{S38}
$$

$$
\text{XI: } \frac{\partial y_{11}}{\partial t} = D_{\text{XI}} \cdot \frac{\partial^2 y_{11}}{\partial x^2} - \text{R}_{\text{XI}} \text{, where } \text{R}_{\text{XI}} = k_{17} \cdot y_{11} \cdot x_2^F \cdot \frac{p}{1 + \frac{p}{K_p}}
$$
\n
$$
\tag{S39}
$$

XIa:
$$
\frac{\partial x_{11}}{\partial t} = D_{x1a} \cdot \frac{\partial^2 x_{11}}{\partial x^2} + R_{x1} - (h_{16} \cdot i_1 + h_{17} \cdot i_8 + h_{18} \cdot i_7 + h_{19} \cdot i_{10} + h_{20} \cdot i_{11}) \cdot x_{11}
$$
 (S40)

AT-III:
$$
\frac{\partial i_1}{\partial t} = D_{ATJI} \cdot \frac{\partial^2 i_1}{\partial x^2} - (h_3 \cdot x_9 + h_4 \cdot x_{10}^F + h_8 \cdot x_{10-5}^B + h_9 \cdot x_2^F + h_{16} \cdot x_{11}) \cdot i_1 -
$$

$$
-h_{25} \cdot i_1 \cdot x_{2-Tm}
$$
 (S41)

TFPI:
$$
\frac{\partial i_2}{\partial t} = D_{TFPI} \cdot \frac{\partial^2 i_2}{\partial x^2} - (k_{11} \cdot x_{10}^F \cdot i_2 - k_{-11} \cdot i_3) - h_2 \cdot x_{10-7-3}^V \cdot i_2
$$
 (S42)

Xa-TFPI:
$$
\frac{\partial i_3}{\partial t} = D_{xa-TFPI} \cdot \frac{\partial^2 i_3}{\partial x^2} + (k_{11} \cdot x_{10}^F \cdot i_2 - k_{-11} \cdot i_3) - h_1 \cdot x_{7-3}^{\quad V^F} \cdot i_3
$$
 (S43)

PC:
$$
\frac{\partial i_5}{\partial t} = D_{PC} \cdot \frac{\partial^2 i_5}{\partial x^2} - \text{R}_{PC}, \text{ where } \text{R}_{PC} = k_{18} \cdot i_5 \cdot x_2^F + \frac{k_{29} \cdot i_5 \cdot x_{2-Tm}}{K_{29} + i_5}
$$
 (S44)

APC:
$$
\frac{\partial i_4}{\partial t} = D_{PCa} \cdot \frac{\partial^2 i_4}{\partial x^2} + \text{R}_{PC} - (h_{21} \cdot i_6 + h_{22} \cdot i_8 + h_{23} \cdot i_7 + h_{24} \cdot i_{10}) \cdot i_4
$$
 (S45)

Tm:
$$
\frac{\partial i_{13}}{\partial t} = D_{Tm} \cdot \frac{\partial^2 i_{13}}{\partial x^2} - (k_{28} \cdot i_{13} \cdot x_2^F - k_{28} \cdot x_{2-Tm}) + (h_{25} \cdot i_1 + h_{26} \cdot i_{10}) \cdot x_{2-Tm}
$$
(S46)

Free VIIa–TF: 6 10 4 9 $7 - 3$ $7 - 3$ 1 *K y K y* $x_{7-3}^{S^F} = \frac{x}{x}$ S **F** X_{7-3} ^S $+\frac{y_9}{1}$ + $= -\frac{\lambda_{7-}}{2}$ \overline{a} (S47)

$$
Xa-VIIa-TF: x_{10-7-3}^{s} = \frac{k_6}{K_6 \cdot k_{-19}} \cdot y_{10} \cdot x_{7-3}^{sF}
$$
 (S48)

Free lipid-bound IXa:
$$
x_9^{B^F} = \frac{x_9 \cdot n_{20}}{K_{20} + x_9} \cdot \frac{p}{1 + \frac{p}{K_p}}
$$
 (S49)

Free lipid-bound VIIIa: $x_8^{B^F}$ = $(K_{21}+x_8)\cdot\left(1+\frac{y_{10}}{p\cdot K_{10}}\right)\cdot\left(1+\frac{y_{12}}{K_{22}}\right)^{-1}$ *B K p p K i* $p \cdot K$ $K_{21} + x_8$). $\left(1 + \frac{y}{x_2}\right)$ $x_{8} \cdot n$ $\ddot{}$. $\overline{}$ J \backslash $\overline{}$ \setminus ſ $\left|\cdot\right|1+$ J \backslash $\overline{}$ \setminus ſ . $+ x_8$) $\cdot | 1 +$. $1 + \frac{y_{10}}{1} \cdot \left(1 + \frac{l_{12}}{1} \right)$ 1 $\frac{1}{22}$ $\frac{1}{12}$ $\frac{1}{10}$ $(x_2 + x_8) \cdot \left(1 + \frac{y_{10}}{n \cdot k}\right)$ $8 \cdot n_{21}$ (S50)

Lipid-bound Xa–Va:
$$
x_{10-5}^B = \frac{x_{10} \cdot x_5^B}{K_{23} \cdot \left(1 + \frac{i_{12}}{K_{24}} + \frac{x_{10}}{K_{23}}\right) + x_5^B} \cdot \frac{1}{1 + \frac{p}{K_p}} \cdot \frac{1}{1 + \frac{S}{K_{S,10}}}
$$
 (S51)

Free Xa:

$$
x_{10}^{F} = \frac{x_{10}}{1 + \frac{S}{K_{s,10}}} - x_{10-5}^{B}
$$
 (S52)

Xa– Z-Gly-Gly-Arg-AMC complex: ,10 $_{10} - \lambda_{10}$ *S* $S \sim F$ *K* $x_{10}^S = x_{10}^F \cdot \frac{S}{I}$

(S53)

Lipid-bound Va-Xa– Z-Gly-Gly-Arg-AMC:
$$
x_{10-5}^{S} = x_{10-5}^{B} \cdot \frac{S}{K_{S,10}}
$$
 (S54)

Lipid-bound X:
$$
y_{10}^B = \frac{y_{10} \cdot n_{25}}{K_{25} \cdot \left(1 + \frac{y_{10}}{K_{25}} + \frac{y_2}{K_{26}}\right)} \cdot \frac{p}{1 + \frac{p}{K_p}}
$$
(S55)

Free thrombin:
$$
x_2^F = \frac{x_2}{1 + \frac{x_1 + y_1}{K_{14}} + \frac{S}{K_{s,2}}}
$$
 (S56)

$$
\text{IIa} - \text{Z-Gly-Gly-Arg-AMC: } x_2^S = x_2^F \cdot \frac{S}{K_{S,2}} \tag{S57}
$$

Thrombin–fibrinigen complex:
$$
x_2^{F_g} = x_2^F \cdot \frac{y_1}{K_{14}}
$$
 (S58)

Thrombin–fibrin complex:
$$
x_2^{Fn} = x_2^F \cdot \frac{x_1}{K_{14}}
$$
 (S59)

Lipid-bound problem:
$$
y_2^B = \frac{y_2 \cdot n_{25}}{K_{26} \cdot \left(1 + \frac{y_{10}}{K_{25}} + \frac{y_2}{K_{26}}\right)} \cdot \frac{p}{1 + \frac{p}{K_p}}
$$
 (S60)

Lipid-bound Va: $_{27}$ + λ_5 $5 P^{11}27$ $\frac{5}{K_{27}+x}$ $x_5^B = \frac{x_5 \cdot p \cdot n}{\sigma}$ $\overline{+}$ $=\frac{x_5 \cdot p}{\sqrt{2}}$ (S61) Lipid-bound free Va: $x_5^{B^F} = x_5^B - x_{10^-5}^B$ (S62)

Supporting Figures

Fig. S1. Effect of fibrin clot on AMC fluorescence. Clot formation was initiated in recalcified plasma by a high-density TF activator in the presence of different AMC concentrations or in the absence of any fluorogenic substrate. A typical linear $(R = 0.9993)$ "apparent" increase of AMC concentration caused by fibrin clot formation is shown.

Fig. S2. AMC binding in plasma. Different concentrations of AMC were added to pooled normal plasma $(n = 3)$ incubated in a plastic cup with stirring. A PBS-filled dialysis unit with a separation threshold of 3500 kDa was inserted into the plasma. The AMC was allowed to equilibrate between the plasma and buffer. The AMC concentration in PBS was calculated on the basis of the fluorescence (λ_{ex} 355 nm, λ_{em} 460 nm) by using the calibration curve. The bound AMC concentration was determined by subtracting the free AMC concentration from the total concentration. A linear fitting revealed that $67\% \pm 1\%$ of the AMC was bound (R = 0.9995). The value was stable $(\pm 5\%)$ across different plasmas. Similar results were obtained when a solution of BSA (50 mg/mL) was used instead of plasma.

Fig. S3. Spatial thrombin propagation in defibrinated plasma. Panels display thrombin **(A, C)** or fibrin **(B, D)** distribution in time and space after stimulation of clotting in normal plasma: control plasma **(A, B)** or plasma defibrinated with ancistron **(C, D)**. Activation was caused by TF at 90 pmol/m². Experiments were performed in 0.5% agarose gel in the presence of 10 μ M phospholipids.

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Fig. S4. Regulation of spatial thrombin propagation by phospholipids. Thrombin peak height **(black squares)** and spatial velocity **(blue circles)** of fibrin clot propagation as functions of phospholipid concentration in a typical experiment (out of $n = 6$) in normal plasma. Stimulation is with TF at 90 pmol/m².

Fig. S5. Spatial thrombin propagation in factor VIII-deficient plasma. (**A**) and (**B**) Thrombin distribution in time and space after clot stimulation in factor VIII-deficient plasma supplemented with or without 1.5 IU/mL of factor VIII. (*C*) Clot size as a function of time for different factor VIII (fVIII) concentrations. A typical series of experiments is shown (out of $n = 2$).

Fig. S6. Shape of the thrombin wave in the mathematical model. Spatial distribution of thrombin activity in normal plasma with regard to the Z-GGR-AMC measured experimentally $(x_{2,exp})$, free active thrombin with regard to coagulation proteins (x_2) , and the activity of the thrombin– α_2 -macroglobulin complex after 50 min of simulation. The results are similar to the experimental findings obtained in this study. Note that a significant portion of thrombin activity near the activator is not thrombin, but the thrombin– α_2 -macroglobulin complex.

Fig. S7. Thrombin wave propagation at different factor XI concentrations in the mathematical model. Factor XI is necessary for formation of the traveling thrombin wave in the mathematical model, which is consistent with the experiments shown in Figure 3. Thus, the experimental findings of the study are consistent with current theoretical knowledge of how the coagulation system is regulated.

Fig. S8. Distinct roles of factor XI and factor VIII activation. Computer simulations of the velocity of the traveling wave of thrombin, as a function of the coagulation factor concentration, with or without 1000-fold free factor IXa efficiency increase obtained from computer simulations. Factor XI-dependent feedback is principally indispensable for the formation of the traveling wave. In other words, both factors are needed for a wave to appear. An artificial 1000 fold increase in the catalytic efficiency of free factor IXa (not bound to the intrinsic tenase complex with factor VIIIa) can stimulate wave formation even without factor VIII. Factor XI is indispensable because without it, no increase of these rate constants will produce a traveling wave.

Fig. S9. Effect of soluble thrombomodulin on thrombin propagation in the mathematical model. Computer simulation of thrombin propagation in presence of increasing concentrations of soluble thrombomudulin that is in consistent with the experiment shown in Figure 4. [TF]= 0.05 pmol/m²

Fig S10. Immobilized thrombomodulin stops thrombin wave propagation in the mathematical model. When TM is localized far from the activating surface (*A*) it still has effect on thrombin distribution. TM decreases thrombin peak height and thereby stops thrombin propagation.

Fig. S11. Effect of anti-fVIII and anti-fXI antibodies on clot growth. Clot size vs time plots for normal pooled fresh frozen plasma (FACT, George King Biomedicals) (black) and the same plasma incubated for 30 min with 100nM anti-fVIII antibodies (single-chain variable antibody fragment KM33 (7) (red) or 0.1 mg/ml anti-fXI antiboies (clone anti-FXI-2, Haematologic Technologies, Inc, (8) (blue) . The experiment was performed as described in the main text using 4µM of phospholipid vesicles and 800 µM of the fluorogenic substrate. Both antibodies inhibit clot growth in a similar manner showing that these two positive feedbacks play similar roles in the coagulation system.

Supporting Tables

Table S1. Thrombin reconstruction parameters

 a^a Concentrations are mostly from references $(9, 10)$.

^b Concentration or density is specified in figure legends for each numerical experiment.

 \textdegree Procoagulant surface in platelet-free plasma as expressed in activated platelet equivalents (6).

Parameter	Value	Reference
	Clotting initiation:	
k_1, k_1	$4.2 \text{ nM}^{-1} \text{min}^{-1}$, 1.1 min ⁻¹	$(11, 12)^{a}$
k_2	$0.0014 \text{ nM}^{-1} \text{min}^{-1}$	(13)
k_3	0.4 nM^{-1} min ⁻¹	(14)
	Clotting cascade:	
k_4, K_4	630 min ⁻¹ , 210 nM	(15, 16)
k_5, K_5	5.8 min^{-1} , 200 nM	(17)
k_6, K_6	435 min^{-1} , 238 nM	(18)
k_7, K_7	0.06 min ⁻¹ , 230 molecules/platelet	$(19, 20)^a$
k_8	$6,350 \text{ min}^{-1}$	(21)
K_8	1,216 molecules/platelet	(21)
K_9	278 molecules/platelet	(21)
K_{10}	1,655 molecules/platelet	(21)
k_{11}, k_{-11}	$0.052 \text{ nM}^{-1} \text{min}^{-1}$, 0.02 min^{-1}	(22)
k_{12}	$45 \text{ nM}^2 \text{min}^{-1}$	$(23, 24)^a$
k_{13}	1.44 min^{-1}	$(25)^{a}$
k_{14} , K_{14}	5,040 min ⁻¹ , 7,200 nM	(26)
k_{15} , K_{15}	54 min ⁻¹ , 147 nM	(27)
k_{16} , K_{16}	14 min^{-1} , 71.7 nM	(28)
k_{17}	0.03 nM ⁻² min ⁻¹	$(29)^{a}$
k_{18}	$0.0004 \text{ min}^{-1} \text{nM}^{-1}$	$(30, 31)^{a}$
k_{-19}	770 min^{-1}	(32)
n_{20} , K_{20}	260 sites/platelet, 2.57 nM	(33)
n_{21}, K_{21}	750 sites/platelet, 1.5 nM	(34)
K_{22}	150 nM	(35)
K_{23}	0.118 nM	(23)
K_{24}	200 nM	$(36)^{a}$

Table S3. Model parameters: rate constants

a Estimation based on experimental data.

Supporting Movies

Movie S1. Temporally and spatially resolved imaging of clot growth and thrombin activity in blood plasma. This movie shows the formation and propagation of thrombin activity and a fibrin clot in plasma stimulated with immobilized TF (90 pmol/m^2) . Imaging of thrombin activity in blood plasma from a healthy individual reveals a propagating wave. Upper row: The fibrin clot illuminated with red light grows from the activation surface (left). Thrombin formed during this process cleaves the substrate, and the fluorescence of released AMC is recorded (right). Lower row: Light scattering intensity (left) and distribution of thrombin concentration calculated from the AMC fluorescence intensity distribution (right).

Movie S2. Deceleration and cessation of thrombin wave in the presence of thrombomodulin. This movie demonstrates the deceleration and cessation of the thrombin wave in the presence of thrombomodulin (TF density is 4 pmol/m²). Upper row: When the thrombomodulin concentration is high enough (1.5 nM), cessation of the thrombin wave can be observed. Light scattering intensity profile (left) and thrombin concentration profile (right) are shown. Clot growth rate decreases up to the full stop, and spreading of the thrombin wave stops. Lower row: In the case of a lower thrombomodulin concentration, the thrombin peak decelerates gradually, with a decrease in height (right). Light scattering profiles are shown on the left.

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