Appendix 3: Iterative Correlation Computation

This appendix describes in detail the computation of equation 4 from the text. Specifically, we will show that

$$r(n) = \left| \overline{P}^{+} - \overline{P}^{-} \right| \tag{B.1}$$

can be written by using recursive definitions for $\overline{P}^{+}$ and $\overline{P}^{-}$ as follows:

$$r(n) = \left| \frac{P_n^{+}}{\Omega_n^{+}} + \frac{\Omega_n^{+} - I_n^{+}}{\Omega_n^{+}} \overline{P}_{n-1}^{+} - \frac{P_n^{-}}{\Omega_n^{-}} - \frac{\Omega_n^{-} - I_n^{-}}{\Omega_n^{-}} \overline{P}_{n-1}^{-} \right| . \tag{B.2}$$

where

$$\overline{P}_n^{+} = \frac{1}{\Omega_n^{+}} \sum_{i=1}^{n} \theta^{n-i} P_i^{+}, \tag{B.3}$$

$$\overline{P}_n^{-} = \frac{1}{\Omega_n^{-}} \sum_{i=1}^{n} \theta^{n-i} P_i^{-}, \tag{B.4}$$

$$\Omega_n^{+} = \sum_{i=1}^{n} I_i^{+} \theta^{n-i}, \tag{B.5}$$

$$\Omega_n^{-} = \sum_{i=1}^{n} I_i^{-} \theta^{n-i}. \tag{B.6}$$

Here, θ is a constant subject to the constraint $\theta < 1$. I_n^{+} and I_n^{-} are indicator functions defined by,

$$I_n^{+} = \begin{cases} 1 & \text{if positive feedback was received on trial } n \\ 0 & \text{otherwise} \end{cases} \tag{B.7}$$

and

$$I_n^{-} = \begin{cases} 1 & \text{if negative feedback was received on trial } n \\ 0 & \text{otherwise} \end{cases} . \tag{B.8}$$

P_n^{+} and P_n^{-} are the response confidence on trial n depending on whether positive or negative feedback was received on that trial. Specifically,

$$P_n^{+} = \begin{cases} P_n & \text{if positive feedback was received on trial } n \\ 0 & \text{otherwise} \end{cases} \tag{B.9}$$

and

$$P_n^{-} = \begin{cases} P_n & \text{if negative feedback was received on trial } n \\ 0 & \text{otherwise} \end{cases} . \tag{B.10}$$

To begin, we show that Ω_n^+ and Ω_n^- can be defined recursively as,

$$\Omega_n^+ = \theta\Omega_{n-1}^+ + I_n^+ \tag{B.11}$$

and

$$\Omega_n^- = \theta\Omega_{n-1}^- + I_n^-. \tag{B.12}$$

Proof:

By definition we can write,

$$\Omega_n^+ = \sum_{i=1}^{n} I_i^+ \theta^{n-i} \tag{B.13}$$

and

$$\Omega_{n-1}^+ = \sum_{i=1}^{n-1} I_i^+ \theta^{n-1-i}. \tag{B.14}$$

Next, we factor a θ out of equation B.13,

$$\Omega_n^+ = \theta \sum_{i=1}^{n} I_i^+ \theta^{n-1-i} \tag{B.15}$$

and pull the nth term out of the sum,

$$\Omega_n^+ = \theta \left[\frac{I_n^+}{\theta} + \sum_{i=1}^{n-1} I_i^+ \theta^{n-1-i} \right]. \tag{B.16}$$

Note that the second term inside the parentheses of equation B.16 is exactly Ω_{n-1}^+, as defined in equation B.14. Thus, we find that

$$\Omega_n^+ = \theta\Omega_{n-1}^+ + I_n^+. \tag{B.17}$$

The proof for equation B.12 follows identical steps.

We now show that $\overline{P}_n^+$ and $\overline{P}_n^-$ can be defined recursively as,

$$\overline{P}_n^+ = \frac{P_n^+}{\Omega_n^+} + \frac{\Omega_n^+ - I_n^+}{\Omega_n^+} \overline{P}_{n-1}^+ \tag{B.18}$$

and

$$\overline{P}_n^- = \frac{P_n^-}{\Omega_n^-} + \frac{\Omega_n^- - I_n^+}{\Omega_n^-} \overline{P}_{n-1}^-. \tag{B.19}$$

Proof: By definition we write,

$$\overline{P}_n^+ = \frac{1}{\Omega_n^+} \sum_{i=1}^{n} \theta^{n-i} P_i^+ \tag{B.20}$$

and

$$\overline{P}_{n-1}^+ = \frac{1}{\Omega_{n-1}^+} \sum_{i=1}^{n-1} \theta^{n-1-i} P_i^+. \tag{B.21}$$

Next, we factor a θ out of equation B.20,

$$\overline{P}_n^+ = \frac{\theta}{\Omega_n^+} \sum_{i=1}^{n} \theta^{n-1-i} P_i^+ \tag{B.22}$$

and then pull the nth term out of the sum,

$$\overline{P}_n^+ = \frac{P_n^+}{\Omega_n^+} + \frac{\theta}{\Omega_n^+} \sum_{i=1}^{n-1} \theta^{n-1-i} P_i^+. \tag{B.23}$$

By multiplying the numerator and denominator of the second term in equation B.23 by Ω_{n-1}^+, we get

$$\overline{P}_n^+ = \frac{P_n^+}{\Omega_n^+} + \frac{\theta \Omega_{n-1}^+}{\Omega_n^+} \left[\frac{1}{\Omega_{n-1}^+} \sum_{i=1}^{n-1} \theta^{n-1-i} P_i^+ \right]. \tag{B.24}$$

Note that the term inside the parentheses in equation B.24 is exactly $\overline{P}_{n-1}^+$. Thus,

$$\overline{P}_n^+ = \frac{P_n^+}{\Omega_n^+} + \frac{\theta \Omega_{n-1}^+}{\Omega_n^+} \overline{P}_{n-1}^+. \tag{B.25}$$

Finally, solving equation B.11 for Ω_{n-1}^+, and substituting into equation B.25 we get,

$$\overline{P}_n^+ = \frac{P_n^+}{\Omega_n^+} + \frac{\Omega_n^+ - I_n^+}{\Omega_n^+} \overline{P}_{n-1}^+. \tag{B.26}$$

Thus, equation B.1 becomes,

$$r(n) = \left| \frac{P_n^+}{\Omega_n^+} + \frac{\Omega_n^+ - I_n^+}{\Omega_n^+} \overline{P}_{n-1}^+ - \frac{P_n^-}{\Omega_n^-} - \frac{\Omega_n^- - I_n^-}{\Omega_n^-} \overline{P}_{n-1}^- \right|. \tag{B.27}$$