

**Text S1:** Pseudocode for randomization algorithm.

### Link Permutation algorithm

The notation  $|S|$  is the cardinality (or size) of the set  $S$ .

| Pseudo-code for Link Permutation |                                                                                      |
|----------------------------------|--------------------------------------------------------------------------------------|
| 1.                               | <b>Begin</b>                                                                         |
| 2.                               | Create vector <i>linkSet</i> and add all links from <i>network</i> to <i>linkSet</i> |
| 3.                               | Create empty vector <i>testedLinks</i>                                               |
| 4.                               | <b>while</b> ( $ linkSet  \geq 2$ <b>OR</b> $ testedLinks  <  linkSet $ ):           |
| 5.                               | Pick a random link <i>A-B</i> from <i>linkSet</i>                                    |
| 6.                               | Pick a random link <i>C-D</i> from <i>linkSet</i>                                    |
| 7.                               | <b>if</b> ( <i>A-D</i> and <i>C-B</i> not in <i>network</i> ):                       |
| 8.                               | Swap <i>A-B</i> and <i>C-D</i> to <i>A-D</i> and <i>C-B</i>                          |
| 9.                               | Remove <i>A-B</i> and <i>C-D</i> from <i>linkSet</i>                                 |
| 10.                              | <b>else if</b> ( <i>A-C</i> and <i>B-D</i> not in <i>network</i> ):                  |
| 11.                              | Swap <i>A-B</i> and <i>C-D</i> to <i>A-C</i> and <i>B-D</i>                          |
| 12.                              | Remove <i>A-B</i> and <i>C-D</i> from <i>linkSet</i>                                 |
| 13.                              | <b>else</b> :                                                                        |
| 14.                              | Add link pair to <i>testedLinks</i>                                                  |
| 15.                              | <b>end while</b>                                                                     |
| 16.                              | <b>End</b>                                                                           |

### Node Permutation algorithm

The *degNodesMap* is a map from an integer to a vector of nodes that have the same equivalence class established by the by the equivalence relation:  $B[d] = \text{round}(\ln(d)+1)$ , where  $d$  is each node's degree. A log scale is used so that scale free networks (such as observed in biological networks) will have approximately the same number of nodes in each bin. The notation  $R.degree$  is the degree of node  $R$  in the original network.

| Pseudo-code for Node Permutation |                                                                              |
|----------------------------------|------------------------------------------------------------------------------|
| 1.                               | <b>Begin</b>                                                                 |
| 2.                               | Create mapping of nodes from binned degree: <i>degNodesMap</i>               |
| 3.                               | Create set of nodes paired with degree in original network: <i>recordSet</i> |
| 4.                               | Randomly shuffle <i>recordSet</i>                                            |
| 5.                               | <b>for</b> ( $R$ in <i>recordSet</i> ):                                      |
| 6.                               | Pick a random node $K$ from $degNodesMap[B[R.degree]]$                       |
| 7.                               | Swap the labels of $R$ and $K$                                               |
| 8.                               | <b>end for</b>                                                               |
| 9.                               | <b>End</b>                                                                   |

### Link Assignment algorithm

Test condition for adding a link between  $K$  and  $R$ :  $R$  is not equal to  $K$ , link  $R-K$  is not already in the network,  $R$  AND  $K$  have not recovered their original degree yet. The notation  $deg(K)$  is the current degree of node  $K$  in the *network*. The notation  $K.degree$  is the degree of node  $K$  in the original network.

| Pseudo-code for Link Assignment |                                                                                                      |
|---------------------------------|------------------------------------------------------------------------------------------------------|
| 1.                              | <b>Begin</b>                                                                                         |
| 2.                              | Create set of nodes paired with degree in original network: <i>recordSet</i>                         |
| 3.                              | Randomly shuffle <i>recordSet</i>                                                                    |
| 4.                              | Remove all links from <i>network</i>                                                                 |
| 5.                              | <b>for</b> ( $k = 1$ to $ recordSet $ ):                                                             |
| 6.                              | $K = recordSet[k]$                                                                                   |
| 7.                              | Generate list of indices of <i>recordSet</i> : <i>recordIndexSet</i>                                 |
| 8.                              | <b>while</b> ( $deg(K) < K.degree$ <b>OR</b> $ recordIndexSet  == 0$ <b>OR</b> $ recordSet  == 0$ ): |
| 9.                              | Pick a random <i>index</i> from <i>recordIndexSet</i>                                                |
| 10.                             | $R = recordSet[index]$                                                                               |
| 11.                             | <b>if</b> ( $K$ and $R$ can be linked in the <i>network</i> ):                                       |
| 12.                             | Create $R-K$ in <i>network</i>                                                                       |
| 13.                             | <b>if</b> ( $deg(K) == K.degree$ ):                                                                  |
| 14.                             | Remove $K$ from <i>recordSet</i>                                                                     |
| 15.                             | $k = 1$                                                                                              |
| 16.                             | <b>break while</b>                                                                                   |
| 17.                             | <b>if</b> ( $deg(R) == R.degree$ ):                                                                  |
| 18.                             | Remove $R$ from <i>recordSet</i>                                                                     |
| 19.                             | $k = 1$                                                                                              |
| 20.                             | <b>break while</b>                                                                                   |
| 21.                             | <b>else</b> :                                                                                        |
| 22.                             | Remove <i>index</i> from <i>recordIndexSet</i>                                                       |
| 23.                             | <b>end while</b>                                                                                     |
| 24.                             | <b>end for</b>                                                                                       |
| 25.                             | <b>End</b>                                                                                           |

## Link Assignment + Second-order modification

The algorithm for link assignment + second order is similar to the link assignment algorithm, but instead of choosing a random node from the whole network (represented by *recordSet*) for node *K* to connect to, it picks from a set of nodes that fall into the same log degree bin [established by the equivalence relation:  $B[d] = \text{round}(\ln(d)+1)$ ], as the original neighbors of *K*. For example, say that node *K* was linked to a set of nodes that have the following degree sequence: {1, 30, 100, 400}. This can be termed the neighbor degree sequence of node *K*. By choosing nodes from the same *binned* neighbor degree sequence, the algorithm assures that node *K* has approximately the same neighbor degree sequence before and after randomization.

## Fixing Degree Sequence Errors

For the two link assignment methods, a few nodes may not be able to conserve their degree due to conflicts in the test cases once the network becomes more densely connected. In that case the following procedure can be applied:

| Pseudo-code for Fixing Degree Sequence Errors |                                                                                                    |
|-----------------------------------------------|----------------------------------------------------------------------------------------------------|
| 1.                                            | <b>Begin</b>                                                                                       |
| 2.                                            | #Fix odd errors first                                                                              |
| 3.                                            | Create a set of nodes that did not conserve degree: <i>errNodes</i>                                |
| 4.                                            | Sort <i>errNodes</i> in increasing order of $\text{deltaDeg}(N) = N.\text{degree} - \text{deg}(N)$ |
| 5.                                            | <b>for</b> ( <i>N1</i> <b>in</b> <i>errNodes</i> ):                                                |
| 6.                                            | <b>if</b> ( $\text{deltaDeg}(N1)$ odd):                                                            |
| 7.                                            | <i>N2</i> = next node in <i>errNodes</i> with $\text{deltaDeg}(N2)$ odd                            |
| 8.                                            | <b>for</b> (link <i>L1-L2</i> <b>in</b> <i>network</i> ):                                          |
| 9.                                            | <b>if</b> ( <i>testCase1</i> ):                                                                    |
| 10.                                           | Remove link <i>L1-L2</i> from <i>network</i>                                                       |
| 11.                                           | Create links (either form <i>N1-L1</i> and <i>N2-L2</i> or <i>N1-L2</i> and <i>N2-L1</i> )         |
| 12.                                           | <b>break for</b>                                                                                   |
| 13.                                           | <b>end for</b>                                                                                     |
| 14.                                           | <b>end for</b>                                                                                     |
| 15.                                           | #Fix even errors next ( <i>errNodes</i> should only have even errors at this point)                |
| 16.                                           | <b>for</b> ( <i>N1</i> <b>in</b> <i>errNodes</i> ):                                                |
| 17.                                           | <b>for</b> ( <i>k</i> =0 <b>to</b> $\text{deltaDeg}(N1)/2$ ):                                      |
| 18.                                           | <b>for</b> (link <i>L1-L2</i> <b>in</b> <i>network</i> ):                                          |
| 19.                                           | <b>if</b> ( <i>testCase2</i> ):                                                                    |
| 20.                                           | Remove link <i>L1-L2</i> from <i>network</i>                                                       |
| 21.                                           | Create links <i>N1-L1</i> and <i>N1-L2</i>                                                         |
| 22.                                           | <b>end for</b>                                                                                     |
| 23.                                           | <b>end for</b>                                                                                     |
| 24.                                           | <b>end for</b>                                                                                     |
| 25.                                           | <b>End</b>                                                                                         |

The following are the test cases used above.

*testCase1*: the nodes *L1* OR *L2* have an odd degree, AND [the nodes *L1* AND *L2* are not nodes *N1* OR *N2*, AND [the network doesn't already have links *N1-L1* AND *N2-L2* OR links *N1-L2* AND *N2-L1*]].

*testCase2*: the nodes *L1* AND *L2* are not *N1*, AND the network doesn't already have links *N1-L1* AND *N1-L2*.