Supplementary Information

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Experimental setup

The experimental setup used in this work is schematically depicted in Fig. 1. All parts used are commercially available except for the measurement chamber, shown in Fig. 2. The aluminium chamber comprises the aerosol in and outlets, electrical connections to the chip and magnets for magnetic actuation, and a window for the optical readout by means of a laser-Doppler vibrometer (Polytech MSA-500). The measurements were performed with two sets of chips with dimension of 0.7×1 cm^2 and 0.7×0.7 cm^2, respectively. The aluminium chambers were adapted accordingly to fit the chip dimensions. An air-tight connection between sensor chip and chamber was guaranteed by an O-ring or silicone grease. A tight connection is crucial in order to have a reproducible air flow to velocity relation. The air velocity was then calculated from the air flow and chip orifice size through which the aerosol has to pass.

In Fig. 3 the measured damping due to the airflow passing the resonator was measured. The damping is increasing linearly with air velocity. The velocity related damping limits the applicable velocity range. The oscillation of the resonator stopped at velocities of roughly $u > 150$ m/s, when the phase-locked loop (PLL) could not sustain a stable oscillation anymore. The PLL was adjusted to the particular resonators by means of the PLL-Adviser which is part of the Zurich Instrument PLL software package.

Single filter-fiber collection efficiency

All models of air filters are based on calculations and assumptions made for single filter-fibers. The first reliable models describing the collection efficiency of single filter fibers were developed in the 50's among others most famously by Irving Langmuir. Most models are only valid for a limited range of boundary conditions, in which they were empirically obtained by fitting with experimental data. This makes it difficult to use them for discussing quantitative experimental

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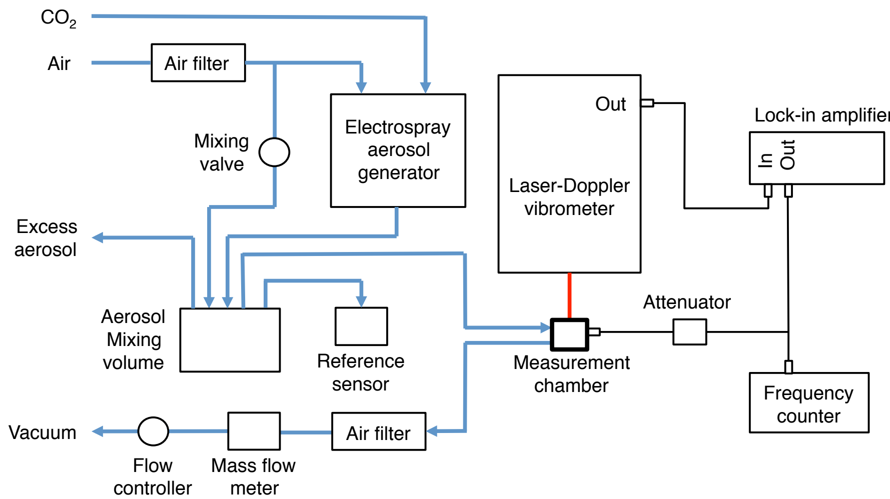

Figure 1: Schematic overview of experimental setup.

data. But there are some fundamental physical relationships in all models which allow a qualitative discussion. In the following section we describe accepted models for diffusive and inertial precipitation on filter-fibers [1]. We are going to use these models as references to discuss the measured data with the resonant single filter-fibers.

A solution for the single filter-fiber collection efficiency due to Brownian diffusion was first developed by Langmuir in 1942 [2, 7, 3]

$$E_D = \beta Pe^{-2/3} = \beta \left(\frac{d_f u}{D} \right)^{-2/3} \tag{1}$$

where β is an model dependent factor and D is the particle diffusion coefficient

$$D = \frac{k_B T C_c}{3\pi \nu d_p} \tag{2}$$

with the Boltzmann constant k_B and temperature T. The value for β approximately calculated by Langmuir was later improved by a more precise model by Natanson in 1957 [4, 3]. Since then more models have been developed, most of them taking into account the flow field disturbance due to the dense proximity of fibers in an air filter. All models describe the collection efficiency based on the Peclet number Pe [5, 6, 7]. It has been shown that Natanson's model is accurate for low Reynolds numbers [5, 3] with

$$\beta = 2.9(2 - lnRe_f)^{-1/3}. \tag{3}$$

The models for single filter-fiber collection efficiency by inertial impaction

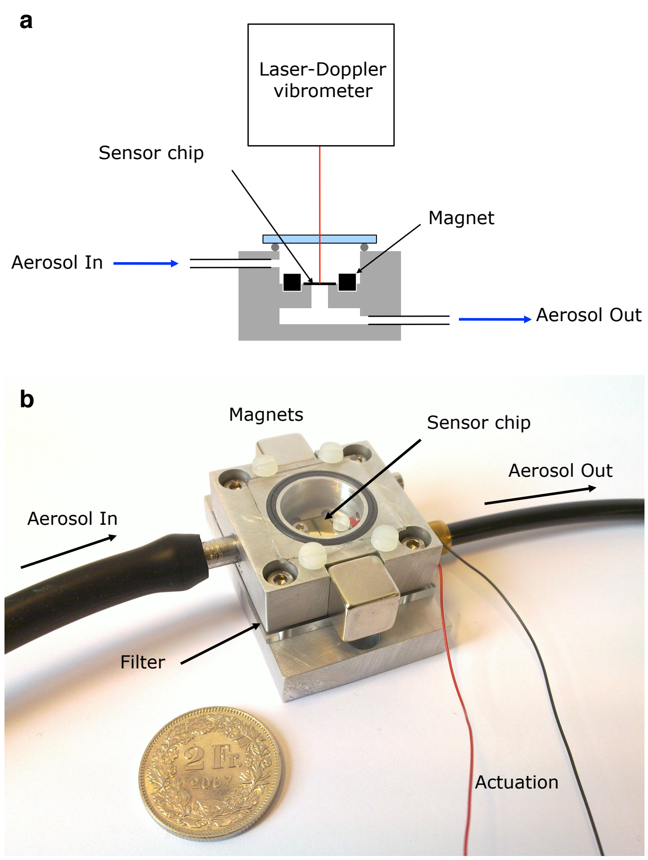

Figure 2: a) Schematic drawing of aerosol measurement chamber. b) Photograph image of an aerosol measurement chamber.

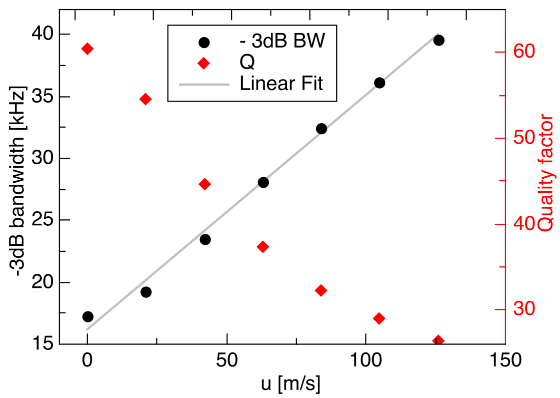

Figure 3: Measured -3dB bandwidth (-3dB BW) and quality factor (Q) of a resonant filter-fiber (138 μm long, 3 μm wide, 220 nm thick silicon nitride string with 50 nm Al top layer with a resonance frequency of 1.0 MHz) for varying air velocities.

are of empirical nature. All of them are based on the Stokes number

$$Stk = \frac{\rho_p d_p^2 C_c u}{18 \nu d_f} \tag{4}$$

where ρ_p is the particle mass density, d_p the particle diameter, d_f the fiber diameter, u the air velocity, ν the air viscosity, and C_c the Cunningham factor correcting Stokes' law for small particles ($d_p < \lambda$, mean free path of air molecules)

$$C_c = 1 + \frac{\lambda}{d_p}[2.34 + 1.05 \exp(-0.39 \frac{\lambda}{d_p})] \tag{5}$$

The collection efficiency can be described by [8, 1]

$$E_I = \frac{2(Stk)J}{Ku^2} \tag{6}$$

where J is an empirical dimensionless term

$$J = 29.6 \left(\frac{d_p}{d_f}\right)^2 - 27.5 \left(\frac{d_p}{d_f}\right)^{2.8} \tag{7}$$

which is valid if $d_p/d_f < 0.4$. Ku is the dimensionless Kuwabara hydrodynamic factor correcting for the flow field distortion caused by neighboring fibers. The factor is based on the fiber volume fraction α in the filter

$$Ku = -\frac{ln\alpha)}{2} - 0.75 + \alpha - \frac{\alpha^2}{4}. \tag{8}$$

The total collection efficiency is the sum of the individual collection efficiencies. For a qualitative discussion, the collection efficiencies are simplified to

$$E_c = E_D + E_I = a_1(d_f u)^{-2/3} + a_2 u/d_f. \tag{9}$$

References

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