Appendix

The nerve model uses a local, differential formalism which describes the excitation process without propagation. The model structure and parameter values are based on (65; 66; 67) with updates from (68; 69; 70), and are given below. The first element of the state vector ϕ denotes the membrane potential v; the remaining elements describe the channel dynamics m, h, p, s which correspond to the states of first-order systems with nonlinear time constants. Variable e_f denotes the excitation function, which is directly proportional to the electric field waveform.

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{pmatrix} \equiv \begin{pmatrix} v \\ m \\ h \\ p \\ s \end{pmatrix}$$

$$\frac{\partial}{\partial t}\phi = \begin{pmatrix} \frac{1}{c_m}\left(e_f(t) + \left(g_{Na}\phi_2^3\phi_3 + g_{Na,p}\phi_4^3\right)(V - E_{Na}) - g_K\phi_5(V - E_K) - g_l(V - E_l)\right) \\ \frac{m_\infty - \phi_2}{\tau_m(\phi_1)} \\ \frac{h_\infty - \phi_3}{\tau_h(\phi_1)} \\ \frac{p_\infty - \phi_4}{\tau_p(\phi_1)} \\ \frac{s_\infty - \phi_5}{\tau_s(\phi_1)} \end{pmatrix}$$

$c_m = 2.0\,\mu\mathrm{F/cm^2}$

$g_{Na} = 290\,\mathrm{mS/cm^2}$
$g_{Na,p} = 25\,\mathrm{mS/cm^2}$
$g_K = 80\,\mathrm{mS/cm^2}$
$g_l = 7\,\mathrm{mS/cm^2}$
$E_{Na} = 50.0\,\mathrm{mV}$
$E_K = -90.0\,\mathrm{mV}$
$E_l = -90.0\,\mathrm{mV}$
$T = 310\,\mathrm{K}$

$\delta_m = 1.86\,(\mathrm{mV\,ms})^{-1}$
$\epsilon_m = 21.4\,\mathrm{mV}$
$\zeta_m = 10.3\,\mathrm{mV}$
$\eta_m = 0.086\,(\mathrm{mV\,ms})^{-1}$
$\theta_m = 25.7\,\mathrm{mV}$
$\iota_m = 9.16\,\mathrm{mV}$
$\delta_h = 0.062\,(\mathrm{mV\,ms})^{-1}$
$\epsilon_h = 114.0\,\mathrm{mV}$
$\zeta_h = 11.0\,\mathrm{mV}$
$\eta_h = 2.3\,\mathrm{ms^{-1}}$
$\theta_h = 31.8\,\mathrm{mV}$
$\iota_h = 13.4\,\mathrm{mV}$
$\delta_p = 0.01\,(\mathrm{mV\,ms})^{-1}$
$\epsilon_p = 27\,\mathrm{mV}$

$\zeta_p = 10.2\,\mathrm{mV}$
$\eta_p = 2.5 \cdot 10^{-4}\,(\mathrm{mV\,ms})^{-1}$
$\theta_p = 34\,\mathrm{mV}$
$\iota_p = 10\,\mathrm{mV}$
$\delta_s = 0.3\,\mathrm{ms^{-1}}$
$\epsilon_s = 53\,\mathrm{mV}$
$\zeta_s = -5\,\mathrm{mV}$
$\eta_s = 0.03\,\mathrm{ms^{-1}}$
$\theta_s = 90\,\mathrm{mV}$
$\iota_s = -1\,\mathrm{mV}$

$T_{r,act} = 293\,\mathrm{K}$
$T_{r,dea} = 293\,\mathrm{K}$
$T_{r,K} = 309\,\mathrm{K}$

$\kappa_{act} = \exp\left(\frac{(T - T_{r,act})\ln 2.2}{10\,\mathrm{K}}\right)$
$\kappa_{dea} = \exp\left(\frac{(T - T_{r,dea})\ln 2.9}{10\,\mathrm{K}}\right)$
$\kappa_K = \exp\left(\frac{(T - T_{r,K})\ln 3.0}{10\,\mathrm{K}}\right)$

$\alpha_m = \kappa_{act}\psi_{m\alpha}(\phi_1)$
$\beta_m = \kappa_{act}\psi_{m\beta}(\phi_1)$
$\tau_m = \frac{1}{\alpha_m + \beta_m}$
$m_\infty = \frac{\alpha_m}{\alpha_m + \beta_m}$

$$\alpha_h = \kappa_{dea}\psi_{h\alpha}(\phi_1)$$
$$\beta_h = \frac{\kappa_{dea}\eta_h}{1+\exp\left(-\frac{v+\theta_h}{\iota_h}\right)}$$
$$\tau_h = \frac{1}{\alpha_h+\beta_h}$$
$$h_\infty = \frac{\alpha_h}{(\alpha_h+\beta_h)}$$

$$\alpha_p = \kappa_{act}\psi_{p\alpha}(\phi_1)$$
$$\beta_p = \kappa_{act}\psi_{p\beta}(\phi_1)$$
$$\tau_p = \frac{1}{\alpha_p+\beta_p}$$
$$p_\infty = \frac{\alpha_p}{\alpha_p+\beta_p}$$

$$\alpha_s = \frac{\kappa_K\delta_s}{1+\exp\left(\frac{\phi_1+\epsilon_s}{\zeta_s}\right)}$$
$$\beta_s = \frac{\kappa_K\eta_s}{1+\exp\left(\frac{\phi_1+\theta_s}{\iota_s}\right)}$$

$$\tau_s = \frac{1}{\alpha_s+\beta_s}$$

$$s_\infty = \frac{\alpha_s}{\alpha_s+\beta_s}$$

$$\psi_{m\alpha} = \begin{cases} \delta_m\zeta_m & \text{for } \phi_1 = -\epsilon_m \\ \frac{\delta_m(\phi_1+\epsilon_m)}{1-\exp\left(-\frac{\phi_1+\epsilon_m}{\zeta_m}\right)} & \text{else} \end{cases}$$

$$\psi_{m\beta} = \begin{cases} -\eta_m\iota_m & \text{for } \phi_1 = -\theta_m \\ -\frac{\eta_m(\phi_1+\theta_m)}{1-\exp\left(\frac{\phi_1+\theta_m}{\iota_m}\right)} & \text{else} \end{cases}$$

$$\psi_{h\alpha} = \begin{cases} -\delta_h\zeta_h & \text{for } \phi_1 = -\epsilon_h \\ -\frac{\delta_h(\phi_1+\epsilon_h)}{1-\exp\left(\frac{\phi_1+\epsilon_h}{\zeta_h}\right)} & \text{else} \end{cases}$$

$$\psi_{p\alpha} = \begin{cases} \delta_p\zeta_p & \text{for } \phi_1 = -\epsilon_p \\ \frac{\delta_p(\phi_1+\epsilon_p)}{1-\exp\left(-\frac{\phi_1+\epsilon_p}{\zeta_p}\right)} & \text{else} \end{cases}$$

$$\psi_{p\beta} = \begin{cases} -\eta_p\iota_p & \text{for } \phi_1 = -\theta_p \\ -\frac{\eta_p(\phi_1+\theta_p)}{1-\exp\left(\frac{\phi_1+\theta_p}{\iota_p}\right)} & \text{else} \end{cases}$$

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