Supporting Information

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SI Text

I. Control Simulations. A series of control simulations were performed to confirm that the difference in preferred mesh size is essential to produce component segregation and blebbing. Fig. S1 presents results from systems where the thickness is constant, meaning that the A- and B-type components have equal thickness and thus, equal elastic parameters. The fraction of the components as well as the mesh size area scaling factor are varied within the same range as in Fig. 1*B*. We emphasize that the only difference between the A- and B-type components in these simulations is the preferred mesh size. For higher values of *f*, wrinkling in the A-type regions is more apparent than when $h_A = 2h_B$ in Fig. 1*B*, which is to be expected, because here, the A-type component has smaller elastic moduli. Nonetheless, the overall results here are qualitatively very similar to those results when the A-type component is thicker than the B type, both in terms of component arrangements and the formation of bleb-like structures. We can, thus, conclude that the difference in preferred mesh size is sufficient to cause component segregation as well as blebbing.

Fig. S2 presents results from systems where the A-type component is set to be two times as thick as the B type and thus, has a larger resistance to stretching and bending. However, the mesh size area scaling factor has been set to 1.0 ($M_A = M_B = 1.0$), such that neither component has a preference to develop a larger mesh size. It is clear that, at any component fraction, segregation is not induced, and no deformation occurs. From these simulations, we can conclude that a difference in the preferred mesh sizes of the two components is essential for component segregation and blebbing.

II. Derivation of Bending Energy Expression. The expressions for the elastic energy of the nuclear lamin meshwork that we use (Eqs. **1** and **2**) are standard expressions for elastic energy of thin sheets and shells introduced in the present form by Koiter (1). Under the so-called Kirchhoff–Love assumptions (2), these expressions can be derived from a fully 3D elastic theory by integrating along the shell thickness, which is discussed in detail in ref. 3. The shell is represented by its 2D neutral surface (i.e., an imaginary midsurface along which bending and stretching are decoupled). The derivation relies on the assumption of a small thickness *t*, with *t* acting as a small expansion parameter. In other words, $t \ll L$, where *L* is some characteristic lateral length scale. It is worth noting that it is necessary to keep subleading terms ($\propto t^3$) to properly capture the energetics of the isometric bending deformations. We also note that, although higher-order terms may arise in the expansion, it was shown by Koiter (4) that the terms given in Eqs. **1** and **2** are the only two terms that are compatible with the Kirchhoff–Love assumptions. The model that we use to describe the stretching and bending energies is analogous to the model used in, for example, ref. 5.

Because the bending energy expression in Eq. **2** is not in a standard form, we present here some of the steps taken to arrive at the expression. We begin from a description of the 2D bending energy in terms of the curvature tensor, b_α^β, which is symmetric ($b_\alpha^\beta = b_\beta^\alpha$):

$$\begin{aligned} E_b &= \frac{Eh^3}{24(1+\nu)}\left(\frac{\nu}{1-\nu} b_\alpha^\alpha b_\beta^\beta + b_\alpha^\beta b_\beta^\alpha\right) \\ &= \frac{Eh^3}{24(1+\nu)}\left(\frac{\nu}{1-\nu}\left(\mathrm{Tr}\left(b_\alpha^\beta\right)\right)^2 + \mathrm{Tr}\left(b_\alpha^\beta\right)^2\right) \\ &= \frac{Eh^3}{24(1-\nu^2)}\left(\nu\left(\mathrm{Tr}\left(b_\alpha^\beta\right)\right)^2 + (1-\nu)\mathrm{Tr}\left(b_\alpha^\beta\right)^2\right) \\ &= \frac{Eh^3}{24(1-\nu^2)}\left(\nu\left(\left(b_1^1\right)^2 + \left(b_2^2\right)^2 + 2b_1^1 b_2^2\right)\right. \\ &\qquad \left. + (1-\nu)\left(\left(b_1^1\right)^2 + \left(b_2^2\right)^2 + 2\left(b_1^2\right)^2\right)\right) \\ &= \frac{Eh^3}{24(1-\nu^2)}\left(\left(b_1^1 + b_2^2\right)^2 - 2(1-\nu)\left(b_1^1 b_2^2 - \left(b_1^2\right)^2\right)\right). \end{aligned} \quad \textbf{[S1]}$$

The bending energy in Eq. **S1** is nonzero when the curvature tensor is nonzero (i.e., an energy penalty is provided when the shape differs from a plane). We instead specify that our bending energy will be nonzero when the curvature tensor of the surface differs from the curvature of an arbitrary reference surface, denoted as $\bar{b}_\alpha^\beta$, as in ref. 5. Thus, we replace the curvature tensor b_α^β in the bending energy in Eq. **S1** with this difference, $(b_\alpha^\beta - \bar{b}_\alpha^\beta)$:

$$\begin{aligned} E_b &= \frac{Eh^3}{24(1-\nu^2)}\left(\left(b_1^1 - \bar{b}_1^1 + b_2^2 - \bar{b}_2^2\right)\right. \\ &\qquad \left. - 2(1-\nu)\left(\left(b_1^1 - \bar{b}_1^1\right)\left(b_2^2 - \bar{b}_2^2\right) - \left(b_1^2 - \bar{b}_1^2\right)^2\right)\right) \\ &= \frac{Eh^3}{24(1-\nu^2)}\left(\left(b_1^1 + b_2^2 - \left(\bar{b}_1^1 + \bar{b}_2^2\right)\right)^2\right. \\ &\qquad \left. -2(1-\nu)\left(b_1^1 b_2^2 - \left(b_1^2\right)^2 + \bar{b}_1^1\bar{b}_2^2 - \left(\bar{b}_1^2\right)^2 - b_1^1\bar{b}_2^2 - \bar{b}_1^1 b_2^2 + 2b_1^2\bar{b}_1^2\right)\right) \end{aligned} \quad \textbf{[S2]}$$

We next substitute the definitions for the mean and Gaussian curvatures, $H = (b_1^1 + b_2^2)/2$ and $K = b_1^1 b_2^2 - (b_1^2)^2$, respectively. We also substitute the reference mean and Gaussian curvatures, $H_0 = (\bar{b}_1^1 + \bar{b}_2^2)/2$ and $K_0 = \bar{b}_1^1\bar{b}_2^2 - (\bar{b}_1^2)^2$, respectively, to arrive at

$$\begin{aligned} E_b = \frac{Eh^3}{24(1-\nu^2)}&\left(4(H-H_0)^2 - 2(1-\nu)\right. \\ &\left.\times\left(K + K_0 - b_1^1\bar{b}_2^2 - \bar{b}_1^1 b_2^2 + 2b_1^2\bar{b}_1^2\right)\right) \end{aligned} \quad \textbf{[S3]}$$

Lastly, although the reference metric can be chosen to have any arbitrary form, we specify it to be a sphere, and thus, $\bar{b}_1^1 = \bar{b}_2^2 = 1/R_0 = H_0$ and $\bar{b}_1^2 = 0$, allowing additional simplification:

$$E_b = \frac{Eh^3}{12(1-\nu^2)}\left(2(H-H_0)^2 - (1-\nu)(K + K_0 - 2HH_0)\right), \quad \textbf{[S4]}$$

from which Eq. **2** follows.

1. Koiter WT (1966) On the nonlinear theory of thin elastic shells. *Proc K Ned Akad Wet B* 69(1):1–54.
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3. Efrati E, Sharon E, Kupferman R (2009) Elastic theory of unconstrained non-Euclidean plates. *J Mech Phys Solids* 57(4):762–775.
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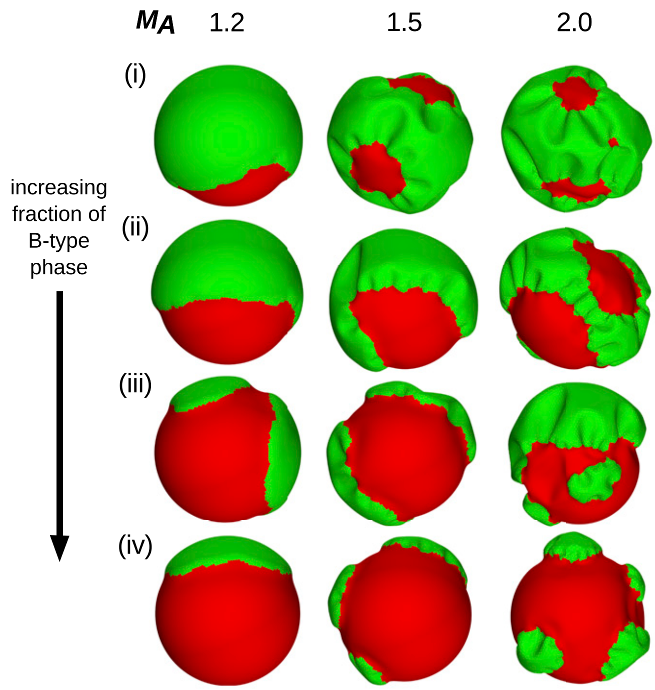

Fig. S1. Low-energy configurations for systems with $h_A = h_B$, with component fractions (*i*) $f = 0.2$, (*ii*) $f = 0.4$, (*iii*) $f = 0.6$, and (*iv*) $f = 0.8$. From left to right, the columns show systems with mesh area scaling factors of $M_A = 1.2$, 1.5, and 2.0, respectively. The results here are similar to the results in Fig. 1*B*, where the thickness of the A-type component is two times as large as the B-type component. Here, the only difference between the components is the preferred mesh size, and segregation and bleb-like deformations are observed in these systems as well.

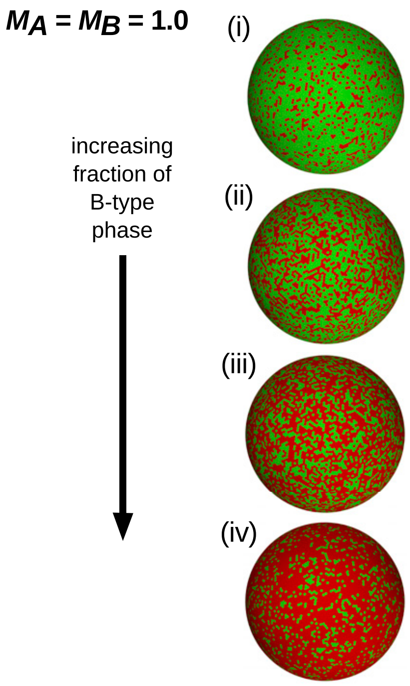

Fig. S2. Low-energy configurations for systems with $M_A = M_B = 1$ and $h_A = 2h_B$ for component fractions (*i*) $f = 0.2$, (*ii*) $f = 0.4$, (*iii*) $f = 0.6$, and (*iv*) $f = 0.8$. With no difference in the preferred mesh sizes of the components, no segregation is observed, and no deformations from a sphere are observed.