APPENDIX 1

Derivation of accuracy of breeding values including F

The accuracy of evaluation (*r*) for a breeding value is the correlation between the true value (*TBV*) and its estimator (*EBV*). A property of an *EBV* calculated by the process of BLUP for a normally distributed random variable is that it is a best linear unbiased estimate. Therefore:

$$TBV = EBV + \epsilon$$

The value of 1 for the slope of the regression of TBV on EBV is an expression of the unbiased property. The error term ε is the prediction error, which has no covariance with the EBV, otherwise the linear prediction of TBV by EBV could be improved, contradicting EBV itself being the best predictor. The $var(\varepsilon)$ is the prediction error variance or PEV. Therefore it follows var(TBV) = var(EBV) + PEV, and cov(TBV, EBV) = var(EBV). So following the standard expression for the correlation $r = \sqrt{(var(EBV)/Var(TBV))}$ and $r = \sqrt{(1 - PEV/var(TBV))}$ (for example, see Bijma (2012)). The PEV is obtained directly from the ASREML analysis used here, whereas var(TBV) is obtained from the leading diagonal of Wright's Numerator Relationship Matrix (A) that is used within the BLUP evaluation. For an animal i the leading diagonal of A is given by i0 where i1 is the inbreeding coefficient of animal i2 with respect to the same base as is used to calculate A and $var(TBV) = (1 + F_i)V_A$ where V_A is the additive genetic variance in the base generation which is directly estimated by REML methods. Therefore:

$$r = \sqrt{(1 - PEV_i/[(1 + F_i)V_A])}$$

This is used as an approximation when the assumption of normality fails, as typically the TBV is unknown, and empirical estimates are then impossible.