$$\begin{bmatrix} X^1 \\ \vdots \\ X^m \end{bmatrix} = \alpha \begin{bmatrix} A_1 \times B^1 \\ \vdots \\ A_n \times B^1 \\ \vdots \\ A_1 \times B^m \\ \vdots \\ A_n \times B^m \end{bmatrix} \begin{bmatrix} X^1 \\ \vdots \\ X^m \end{bmatrix} + (1-\alpha) \begin{bmatrix} X^{10} \\ \vdots \\ X^{m0} \end{bmatrix} \quad (4)$$

Let *C* denote $\begin{bmatrix} A_1 \times B^1 & \cdots & A_n \times B^1 & \cdots & A_1 \times B^n & \cdots & A_n \times B^m \end{bmatrix}^T$ and

$i = sn + t$, $j = rn + \theta$, $s = sI\{t > 0\} + (s-1)I\{t = 0\}$, $t = tI\{t > 0\} + nI\{t = 0\}$,

$r = rI\{\theta > 0\} + (r-1)I\{\theta = 0\}$, $\theta = \theta I\{\theta > 0\} + nI\{\theta = 0\}$, $0 \le t, \theta < n$.

Then we get: $c_{i,j} = a_{t,\theta} b_{r+1,s+1}$ and $c_{j,i} = a_{\theta,t} b_{s+1,r+1}$.

By comparing the above two equations, we can easily find that *C* is a symmetrical matrix with row and column number *n×m*. If we use X^* to represents $\begin{bmatrix} X^1 & \cdots & X^n \end{bmatrix}^T$, then equation (4) can also be written as:

$$X^* = \alpha C X^* + (1-\alpha) X^{*0} \quad (5)$$

According to (Vanunu, et al. 2010)[29], in order to get a converged solution for equation (5), *C* is normalized as: $C^{norm} = D^{-1/2} C D^{-1/2}$, where *D* is diagonal matrix with $d_{i,i}$ equals to the sum of the *i*-th row of *C*. Therefore, we also have $c_{i,j}^{norm} = \frac{c_{i,j}}{\sqrt{d_{i,i} d_{j,j}}}$ and $d_{i,i} = \sum_{u=1}^{nm} c_{i,u} = \sum_{u=1}^{nm} a_{t,\theta_u} b_{r_u+1,s+1} = \sum_{p=1}^{n} a_{t,p} \sum_{q=1}^{m} b_{q,s+1}$

where $u = r_u n + \theta_u$. By incorporating the above equation into $c_{i,j}^{norm}$, we can get

$$c_{i,j}^{norm} = \frac{a_{t,\theta}, b_{r+1,s+1}}{\sqrt{\sum_{p=1}^{n} a_{t,p} \sum_{q=1}^{m} b_{q,s+1}} \sqrt{\sum_{p=1}^{n} a_{\theta,p} \sum_{q=1}^{m} b_{q,r+1}}} = \frac{a_{t,\theta}}{\sqrt{\sum_{p=1}^{n} a_{t,p} \sum_{p=1}^{n} a_{\theta,p}}} \frac{b_{r+1,s+1}}{\sqrt{\sum_{q=1}^{m} b_{q,s+1} \sum_{q=1}^{m} b_{q,r+1}}}$$

Therefore, if we normalize *A* and *B* as $a_{i,j}^{norm} = \frac{a_{i,j}}{\sqrt{\sum_{p=1}^{n} a_{i,p} \sum_{p=1}^{n} a_{j,p}}}$ and $b_{i,j}^{norm} = \frac{b_{i,j}}{\sqrt{\sum_{q=1}^{n} b_{q,i} \sum_{q=1}^{n} b_{q,j}}}$

We can get $c_{i,j}^{norm} = a_{t,\theta}^{norm} b_{r+1,s+1}^{norm}$.

With this equation, we can rewrite equation (5) as $X^* = \alpha C^{norm} X^* + (1-\alpha) X^{*0}$

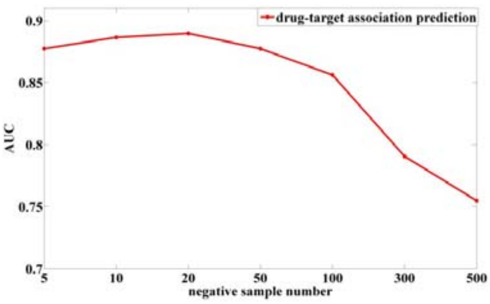

Figure S1 Cross-validation results of BLM using different numbers of negative samples.