# Supporting Information<br>Backholm et al. 10.1073/pnas.1219965110

## Area Moment of Inertia for a Cylinder<br>Area Moment of Inertia for a Cylinder

The definition of the area moment of inertia for a symmetrical cross-section is (1)

$$
I = I_x = I_y = \int_A y^2 dA.
$$
 [S1]

This can be rewritten in polar coordinates  $(dA = rdrd\theta, y = r \sin \theta)$ and solved for the case of a cylindrical shell as

$$
I = \int_{0}^{2\pi} \sin^2 \theta \, d\theta \int_{d/2}^{D/2} r^3 dr = \frac{\pi}{64} \left[ D^4 - d^4 \right],
$$
 [S2]

where  $D$  and  $d$  are the outer and inner diameters of the cylinder, respectively.

### Maxwell Model

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We model the worm as a system with a purely viscous damper (damping coefficient  $c$ ) connected in series with a purely elastic spring (spring constant  $k_w$ ), as shown in Fig. 2C in the main text. In this system, both of the components will be affected by the same force, but will deflect in different ways. According to theory, one then gets the differential equation (Eq. 2 in the main text)

$$
\dot{y} = \frac{F}{c} + \frac{\dot{F}}{k_w},
$$
 [S3]

where y is the bending of the worm, and the dot indicates a time derivative. The force applied to the system can, in our case, be written as  $F = k_p x$ , where  $k_p$  and x are the stiffness and the deflection of the pipette, respectively. Furthermore, the pipette deflection can be written as  $x = x_u - y$ , where  $x_u = v_u t$  is the motion of the U-shaped pipette, moving at a constant speed  $v_u$ . This gives us  $F = k_p(v_u t - y)$  and

$$
y = v_u t - \frac{F}{k_p} \tag{S4}
$$

as well as

$$
\dot{y} = v_u - \frac{\dot{F}}{k_p}.
$$
 [S5]

By plugging Eq. S5 into S3, we get

$$
v_u - \frac{\dot{F}}{k_p} = \frac{F}{c} + \frac{\dot{F}}{k_w},
$$

and after reordering

$$
\dot{F}\left[\frac{1}{k_p} + \frac{1}{k_w}\right] + \frac{F}{c} = v_u
$$
\n
$$
\dot{F} + \frac{k_p k_w}{c(k_p + k_w)} F = \frac{v_u k_p k_w}{k_p + k_w}
$$

 $\dot{F} + AF = B$ ,

where A and B are constants  $(B = Acv_u)$ . This linear nonhomogeneous ordinary differential equation can be analytically solved (2) as

$$
F(t) = \frac{B}{A} \left[ 1 + C_1 e^{-At} \right],
$$

where  $C_1$  is a constant of integration. With the initial condition  $F(t = 0) = 0$ , we get  $C_1 = -1$  and

$$
F(t) = v_{u}c \left[ 1 - e^{-k_{p}k_{w}/(c(k_{p} + k_{w}))t} \right].
$$
 [S6]

A combination of Eqs. S4 and S6 results in

$$
y(t) = v_u \left( t - \frac{c}{k_p} \left[ 1 - e^{-k_p k_w / (c(k_p + k_w))t} \right] \right),
$$
 [S7]

giving us an expression for how the bending of the worm varies as a function of time (this is the same as Eq. 3 in the main text).

To get an expression for the bending as a function of the force, Eq.  $S6$  is solved for t, giving

$$
t(F) = -\frac{c(k_p + k_w)}{k_p k_w} \ln\left(1 - \frac{F}{c v_u}\right),
$$
 [S8]

resulting in

$$
y(F) = -\frac{v_u c}{k_p} \left[ \frac{k_w + k_p}{k_w} \ln \left( 1 - \frac{F}{v_u c} \right) + \frac{F}{v_u c} \right],
$$
 [S9]

when plugging Eq. S8 into Eq. S7. This is the exact deformation– force solution for the Maxwell model. The initial slope of Eq. S9 can be calculated as

$$
\lim_{F \to 0} \frac{\mathrm{d}y}{\mathrm{d}F} = \frac{1}{k_w},
$$

and corresponds to that expected in the EBT.

To get the force–deformation expression, we need to rewrite Eq.  $S9$  as  $F(y)$ . This equation is not, however, analytically solvable for  $F$ , and the natural logarithm in Eq.  $S9$  thus needs to be Taylor expanded (to the second order), giving

$$
y(F) = -\frac{v_u c}{k_p} \left[ \frac{k_w + k_p}{k_w} \left( -\frac{F}{v_u c} \left[ 1 + \frac{F}{2v_u c} \right] \right) + \frac{F}{v_u c} \right].
$$

Reordering and solving the quadratic equation of  $F$  as a function of y finally gives (Eq. 4 in the main text)

$$
F(y) = \frac{k_p v_u c}{k_p + k_w} \left( -1 + \sqrt{1 + \frac{2k_w (k_p + k_w)}{k_p v_u c}} y \right).
$$
 [S10]

This approximate force–deformation solution was shown to give very similar values for  $k_w$  and c as the exact deformation-force solution in Eq. S9, and is thus valid to use when describing the data.

#### Kelvin–Voigt Model

The differential equation characterizing a spring and a dashpot connected in parallel can be written as

$$
F = k_w y + c\dot{y}.
$$
 [S11]

This equation is solved in the same way as described above, resulting in an expression for the bending as a function of time

$$
y(t) = \frac{k_p v_u c}{(k_p + k_w)^2} \left[ \frac{k_p + k_w}{c} t - 1 + e^{-\frac{k_p + k_w}{c} t} \right].
$$
 [S12]

This is the functional form used for the Kelvin–Voigt fit in Fig. 2 in the main text.

#### Reproducibility of Experimental Results

Varying Bending Speeds. Results from bending measurements performed with different speeds on different worms are shown in Fig. S1.

The difference in the constant stiffness values is due to different diameters of the studied worms. The damping coefficient is inversely proportional to bending speed.

1. Young WC, Budynas RG (2002) Roark's Formulas for Stress and Strain (McGraw-Hill, New York).

Along the Body Measurements. Results from micropipette deflection experiments performed along the body of three different young adults are shown in Fig. S2. The stiffness has been normalized by the stiffness at the vulva to make it easier to compare results between different worms. The head is stiffer than the tail in all cases, and the dashed lines act to guide the eye.

#### Viscous Relaxation of the Worm

In Fig. S3 all the force–deformation data from a bending experiment on a young adult worm are shown.

Before contact between the support and the worm, there is no deflection of the pipette and the negative bending values are thus an artifact from the definition of  $y = x_u - x$  (defined as 0 at the contact point). After the bending was performed, the support was stopped and the worm was left to relax. The force decreased as a function of time (0.5 s between each data point), which is a strong implication of a viscous relaxation.

2. Kreyszig E (2006) Advanced Engineering Mathematics (Wiley, New York), p 27.



Fig. S1. (Upper) Stiffness and (Lower) damping coefficient as function of bending speed for three different young adult worms.



Fig. S2. The effective Young's modulus as a function of position along the body of three different young adult worms. The modulus has been normalized by the value measured at the vulva for each worm.



**Fig. S3.** Entire force–deformation from a bending experiment of a young adult worm. Bending starts at the contact point between the worm and the support<br>and the material clearly relaxes after the motion of the support has

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