Supporting Information

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Fig. S1. The logistic model was fit with values for parameter *r* ranging from 2.5 to 3.9. For each value of *r*, we generated a 50-y time series using the observation and process noise levels described in the main text. The Markov chain Monte Carlo procedure was performed in JAGS (1) via R (2). Convergence was checked using batch-mean plots and the Gelman–Rubin statistic ($\hat{R} < 1.2$). As the true model becomes more unstable (represented by a higher Lyapunov exponent), the estimation error increases. Estimation error for parameter *r* is defined as $|r_{fit} - r_{true}|$.

1. Plummer M (2003) JAGS: A program for analysis of Bayesian graphical models using Gibbs sampling. Proceedings of the Third International Workshop on Distributed Statistical Computing. Available at www.r-project.org/conferences/DSC-2003/Proceedings/index.html.

2. R Development Core Team (2011) R: A Language and Environment for Statistical Computing (R Foundation for Statistical Computing, Vienna).



Fig. 52. Estimated Lyapunov exponent vs. true Lyapunov exponent for the logistic model fit with parameter *r* ranging from 2.5 to 3.9. For each value of *r*, we generated 50 points using the observation and process noise levels described in the main text. The red line is one to one. The failure of the Bayesian fitting routine is demonstrated for unstable values of *r*. The estimated model is always stable despite the true model being unstable. The Lyapunov exponent is always poorly estimated when the true Lyapunov exponent is positive.



Fig. S3. Example trace plot (A) and normal-normal plot (B) of 500-point batch means for the LPA model illustrating that the Markov chain Monte Carlo routine converged. The red points denote the range of the data and the end of the first and third quantiles.



Fig. 54. Time series expected values (100 y, 100 replicates) for the stochastic vs. deterministic version of each model illustrating the similarity between the expected value of the stochastic model and deterministic models. A–D show results for the logistic, two-species, age-structured, and spatial models, respectively.



Fig. S5. Example histograms of accepted Markov chain Monte Carlo draws (15,000 iterations) and true parameter values (red dots) for each parameter in the logistic model. X_0 is the initial condition, and V_{obs} is the variance of the observation error.



Fig. S6. Example trace plots (*Left*) and normal–normal plots of 500-point batch means (*Right*) for the logistic (*A* and *B*), two-species (*C* and *D*), age-structured (*E* and *F*), and spatial (*G* and *H*) illustrating the convergence of the Markov chain Monte Carlo procedure. Despite initialization on the correct parameters, the models often converged on incorrect best-fit parameters.

Table S1.	Model structure	and model	parameter	values	used in	the	simulations
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Model name	Process model structure*	Parameter values			
Logistic	$x_{t+1} = \left[x_t r \left(1 - \frac{x_t}{\kappa}\right)\right] \exp(\epsilon_t)$	x ₀ = 0.5, r = 3.7, K = 1			
Two-species	$x_{t+1} = \left[x_t r_1 \left(1 - \frac{x_t}{K_1}\right) - c x_t y_t\right] \exp(\epsilon_t)$	$x_0 = 0.4, y_0 = 0.4, r_1 = 3.8, r_2 = 3.7$			
	$y_{t+1} = \left[y_t r_2 \left(1 - \frac{y_t}{K_2} \right) + c x_t y_t \right] \exp(\epsilon_t)$	$K_1 = 1, K_2 = 1, c = 0.1$			
Age-structured	$\begin{aligned} x_{0,t} &= (x_{2,t} + x_{3,t})f \\ x_{1,t+1} &= x_{0,t} \exp(r - rx_{0,t} + \varepsilon_t) \\ x_{2,t+1} &= s_1 x_{1,t} \\ x_{3,t+1} &= s_2 (x_{2,t} + x_{3,t}) \end{aligned}$	$x_{1,0} = 0.5, x_{2,0} = 0.1, x_{3,0} = 0.1, f = 20$ $r = 3.0, s_1 = 0.05, s_2 = 0.04$			
Spatial	$\tilde{x}_{i,t} = x_{i,t} \exp(r - rx_{i,t} + \epsilon_t)$	$R_0 = 5.1$			
	$x_{i,t+1} = \sum_{j=1}^4 d_{ij} \tilde{x}_{j,t}$	$d = \begin{bmatrix} 0.015 & 0.019 & 0.055 & 0.041 \\ 0.042 & 0.021 & 0.012 & 0.026 \\ 0.038 & 0.066 & 0.052 & 0.031 \\ 0.037 & 0.047 & 0.002 & 0.069 \end{bmatrix}$			

*Process noise ε_t is normally distributed with $\mu = 0$ and $\sigma = 0.005$.

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Table S2. Parameter values and descriptions of nonmodel parameters

Parameter	Value	Description			
т	100	Total number of time-steps for model simulations			
L	50,000	Number of iterations in the Markov chain Monte Carlo (MCMC) routine			
В	5,000	Number of iterations in the MCMC burn-in			
С	3	Number of MCMC chains in Geyer's algorithm			
CV	0.005	Initial coefficient of variation of the adaptive MCMC proposal distribution			
I	500	Number of iterations between adjustment of proposal coefficient of variation and covariance			
CV _P	0.5	Coefficient of variation of the prior probability distribution for each parameter (independent truncated normal distribution with mean equal to the true parameter value)			