

Supplementary Information, Text S1

Chemical rate equations for the deterministic model.

$$\dot{c}_{luxI::gfp} = p_I c_{mRNA_{luxI::gfp}} - \left(\frac{\ln(2)}{\tau} + d_I \right) c_{luxI::gfp} \quad (1)$$

$$\dot{c}_{luxR} = -\frac{k_1^-}{K_{d1}} c_A c_{luxR} + k_1^- c_{luxR \cdot A} + p_R c_{mRNA_{luxR}} - \left(\frac{\ln(2)}{\tau} + d_R \right) c_{luxR} \quad (2)$$

$$\dot{c}_{mRNA_{luxI::gfp}} = \alpha_I k_I c_{DNA} + k_I c_{DNA \cdot (luxR \cdot A)_2} - \left(\frac{\ln(2)}{\tau} + d_{mI} \right) c_{mRNA_{luxI::gfp}} \quad (3)$$

$$\dot{c}_{mRNA_{luxR}} = \alpha_R k_R c_{DNA} + k_R c_{DNA \cdot (luxR \cdot A)_2} - \left(\frac{\ln(2)}{\tau} + d_{mR} \right) c_{mRNA_{luxR}} \quad (4)$$

$$\dot{c}_{luxR \cdot A} = -k_1^- c_{luxR \cdot A} + \frac{k_1^-}{K_{d1}} c_A c_{luxR} - 2 \frac{k_2^-}{K_{d2}} [c_{luxR \cdot A}]^2 + 2k_2^- c_{(luxR \cdot A)_2} \quad (5)$$

$$- \left(\frac{\ln(2)}{\tau} + d_C \right) c_{luxR \cdot A} \quad (6)$$

$$\dot{c}_{(luxR \cdot A)_2} = \frac{k_2^-}{K_{d2}} [c_{luxR \cdot A}]^2 - k_2^- c_{(luxR \cdot A)_2} - \frac{k_{lux}^-}{K_{dlux}} c_{(luxR \cdot A)_2} c_{DNA} + k_{lux}^- c_{DNA \cdot (luxR \cdot A)_2} \quad (7)$$

$$- \left(\frac{\ln(2)}{\tau} + d_{C_2} \right) c_{(luxR \cdot A)_2} \quad (8)$$

$$\dot{c}_{DNA} = -\frac{k_{lux}^-}{K_{dlux}} c_{(luxR \cdot A)_2} c_{DNA} + k_{lux}^- c_{DNA \cdot (luxR \cdot A)_2} + \frac{\ln(2)}{\tau} (c_{DNA} + c_{DNA \cdot (luxR \cdot A)_2}) \quad (9)$$

$$- \frac{\ln(2)}{\tau} c_{DNA} \quad (10)$$

$$\dot{c}_{DNA \cdot (luxR \cdot A)_2} = \frac{k_{lux}^-}{K_{dlux}} c_{(luxR \cdot A)_2} c_{DNA} - k_{lux}^- c_{DNA \cdot (luxR \cdot A)_2} - \frac{\ln(2)}{\tau} c_{DNA \cdot (luxR \cdot A)_2} \quad (11)$$

$$\dot{c}_A = k_1^- c_{luxR \cdot A} - \frac{k_1^-}{K_{d1}} c_A c_{luxR} + k_A c_{luxI::gfp} + D(c_{A_{ext}} - c_A) \quad (12)$$

$$- \left(\frac{\ln(2)}{\tau} + d_A \right) c_A \quad (13)$$

$$\dot{c}_{A_{ext}} = rD(c_A - c_{A_{ext}}) + \gamma \frac{V_{tot}}{V_{ext}} c_{A^*} - (\gamma + d_A) c_{A_{ext}} \quad (14)$$