

**Supplemental Materials for “Measurement Error Case  
Series Models with Application to  
Infection-Cardiovascular Risk in Older Patients on  
Dialysis”**

## Additional simulation results

Here we provide results of bias when ignoring measurement error and bias-corrected estimation under uniform, normal and gamma distributed measurement error. See the corresponding manuscript for more details.

Tables 1-3 provide bias-correction results for risk/effect patterns (a)-(f) for each measurement error distribution.

Table 4 provides bias results for risk/effect pattern (a) with uniformly distributed measurement error as a function of sample size.

Table 5 provides bias-correction results for risk/effect pattern (a) with uniformly distributed measurement error as a function of measurement error variance.

Figures 1-6 provide bias results for risk/effect patterns (a)-(f) for each measurement error distribution as a function of  $\mu_u$ .

Table 1: Bias-corrected estimates  $\hat{\beta}$  and standard deviations over 200 replications for datasets (each with  $N = 1000$  and uniformly distributed measurement error) with three 30-day risk periods and for risk/effect patterns (a)-(f).

Pattern	True ( $\beta_1$ )	Est.	SD	True ( $\beta_2$ )	Est.	SD	True ( $\beta_3$ )	Est.	SD
a	1.099	1.106	0.129	0.693	0.688	0.151	0.405	0.387	0.172
b	0.405	0.408	0.180	0.693	0.677	0.155	1.099	1.119	0.136
c	0.693	0.685	0.158	1.099	1.095	0.117	0.693	0.685	0.164
d	1.099	1.080	0.129	0.693	0.662	0.165	1.099	1.103	0.132
e	0.693	0.686	0.161	0.693	0.683	0.161	0.693	0.703	0.166
f	1.099	1.102	0.136	0.405	0.399	0.176	0.693	0.689	0.166

Table 2: Bias-corrected estimates  $\hat{\beta}$  and standard deviations over 200 replications for datasets (each with  $N = 1000$  and normally distributed measurement error) with three 30-day risk periods and for risk/effect patterns (a)-(f).

Pattern	True ( $\beta_1$ )	Est.	SD	True ( $\beta_2$ )	Est.	SD	True ( $\beta_3$ )	Est.	SD
a	1.099	1.112	0.137	0.693	0.692	0.153	0.405	0.413	0.165
b	0.405	0.391	0.189	0.693	0.692	0.142	1.099	1.084	0.128
c	0.693	0.675	0.170	1.099	1.099	0.119	0.693	0.698	0.145
d	1.099	1.117	0.144	0.693	0.708	0.162	1.099	1.110	0.135
e	0.693	0.689	0.153	0.693	0.706	0.163	0.693	0.716	0.151
f	1.099	1.117	0.130	0.405	0.395	0.184	0.693	0.695	0.166

Table 3: Bias-corrected estimates  $\hat{\beta}$  and standard deviations over 200 replications for datasets (each with  $N = 1000$  and gamma distributed measurement error) with three 30-day risk periods and for risk/effect patterns (a)-(f).

Pattern	True ( $\beta_1$ )	Est.	SD	True ( $\beta_2$ )	Est.	SD	True ( $\beta_3$ )	Est.	SD
a	1.099	1.100	0.136	0.693	0.694	0.166	0.405	0.428	0.179
b	0.405	0.395	0.157	0.693	0.690	0.140	1.099	1.096	0.133
c	0.693	0.668	0.156	1.099	1.108	0.130	0.693	0.681	0.139
d	1.099	1.094	0.142	0.693	0.689	0.162	1.099	1.088	0.130
e	0.693	0.698	0.154	0.693	0.682	0.151	0.693	0.696	0.152
f	1.099	1.114	0.126	0.405	0.389	0.199	0.693	0.692	0.152

Table 4: Bias-corrected estimates  $\hat{\beta}$  and standard deviations over 200 replications for datasets with uniformly distributed measurement error with three 30-day risk periods and for risk/effect pattern (a) and increasing sample size  $N$ .

<b>N</b>	<b>True (<math>\beta_1</math>)</b>	<b>Est.</b>	<b>SD</b>	<b>True (<math>\beta_2</math>)</b>	<b>Est.</b>	<b>SD</b>	<b>True (<math>\beta_3</math>)</b>	<b>Est.</b>	<b>SD</b>
100	1.099	1.121	0.440	0.693	0.601	0.527	0.405	0.388	0.674
200	1.099	1.101	0.299	0.693	0.686	0.357	0.405	0.345	0.406
400	1.099	1.115	0.227	0.693	0.715	0.235	0.405	0.414	0.292
600	1.099	1.113	0.163	0.693	0.674	0.219	0.405	0.462	0.222
800	1.099	1.107	0.147	0.693	0.697	0.170	0.405	0.416	0.204
1000	1.099	1.106	0.129	0.693	0.688	0.151	0.405	0.387	0.172

Table 5: Bias-correction for increasing variance of the distribution of exposure onset measurement error (Uniform) with  $\mu_u = 8$ . Values are averages of estimates  $\hat{\beta}$  and mean square error (MSE) from 200 replications for datasets (each with  $N = 1000$ ) with three 30-day risk periods and for risk/effect patterns (a).

$\sigma_u^2$	$\beta_1$ (1.0986)		$\beta_2$ (0.6931)		$\beta_3$ (0.4055)	
	Estimate	MSE	Estimate	MSE	Estimate	MSE
0.33	1.1298	0.0553	0.6671	0.0748	0.4013	0.1157
3.00	1.0927	0.0686	0.7007	0.0836	0.4251	0.1234
8.33	1.1041	0.0557	0.7081	0.0587	0.4104	0.1103
16.33	1.1020	0.0605	0.6925	0.0684	0.4243	0.1149

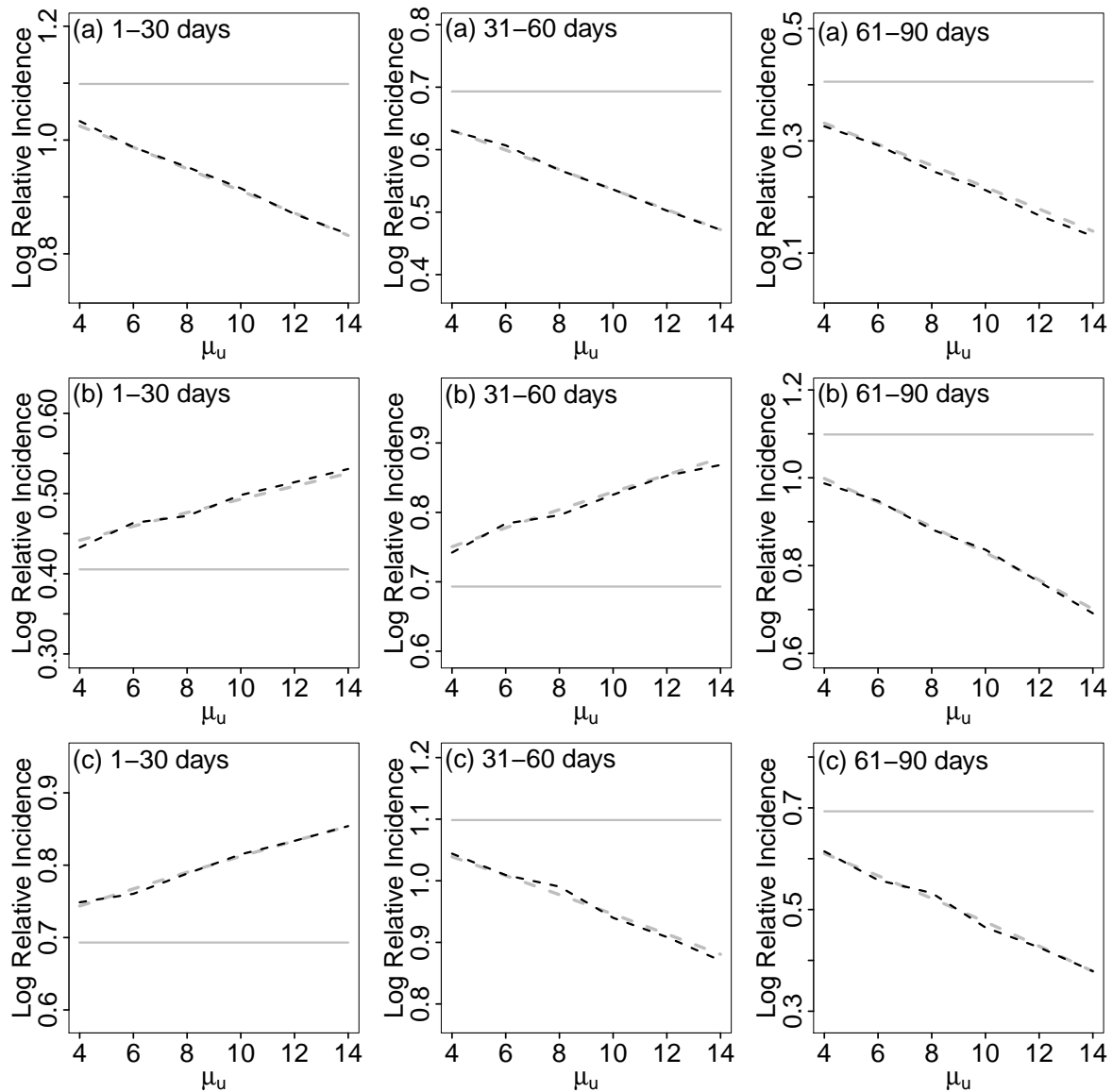


Figure 1: Characterization of bias for uniformly distributed measurement error for risk/effect patterns (a)-(c). Bias for true patterns of log relative incidence  $\beta$  (indicated by horizontal lines) over three risk periods. Dashed black curves denote naive case series estimates ignoring measurement error, which targets  $\beta^*$  (dashed gray curves) instead of  $\beta$ . Given are averages over 200 simulated data sets.

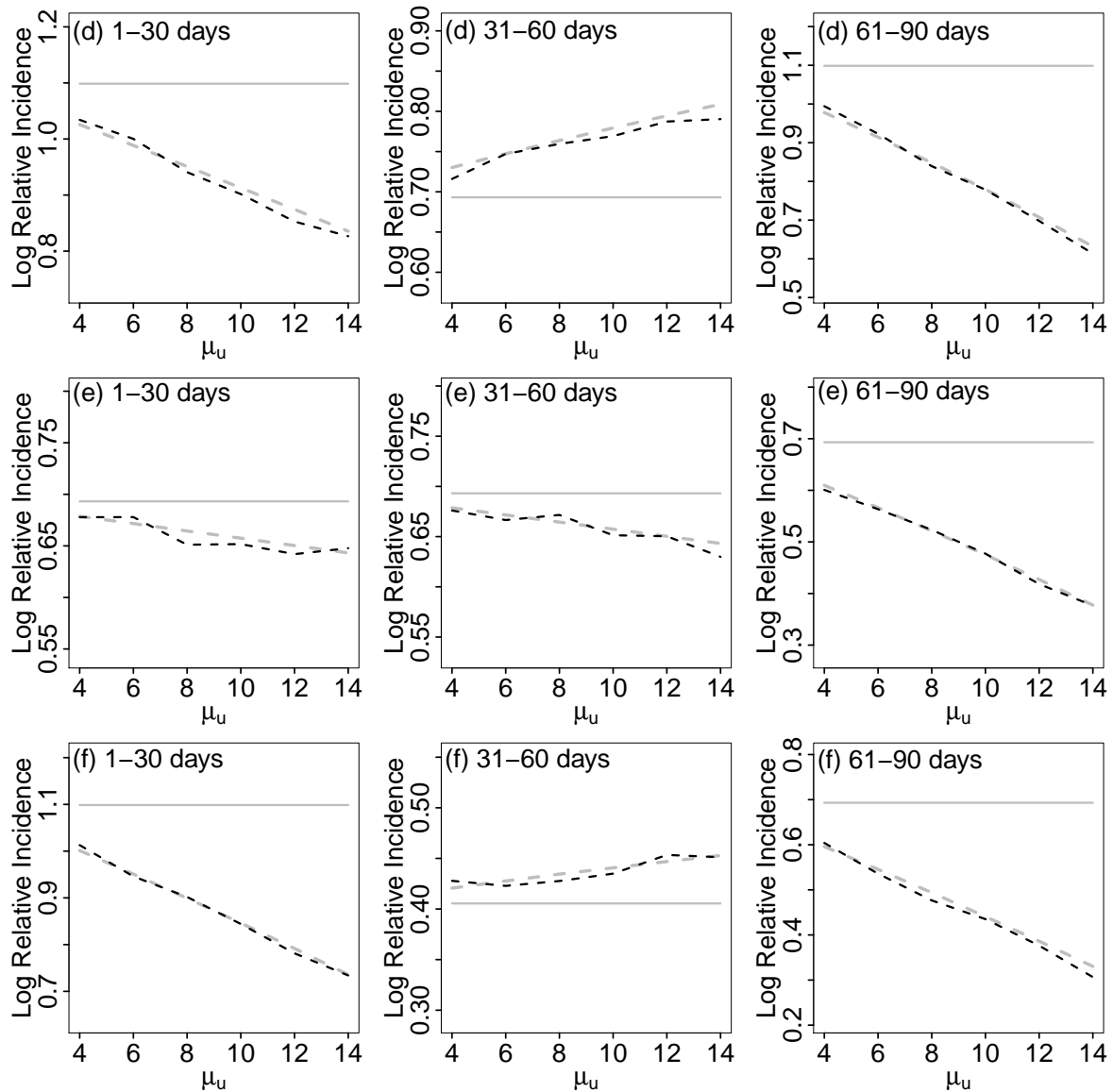


Figure 2: Characterization of bias for uniformly distributed measurement error for risk/effect patterns (d)-(f). Bias for true pattern of log relative incidence  $\beta$  (indicated by horizontal lines) over three risk periods. Dashed black curves denote naive case series estimates ignoring measurement error, which targets  $\beta^*$  (dashed gray curves) instead of  $\beta$ . Given are averages over 200 simulated data sets.

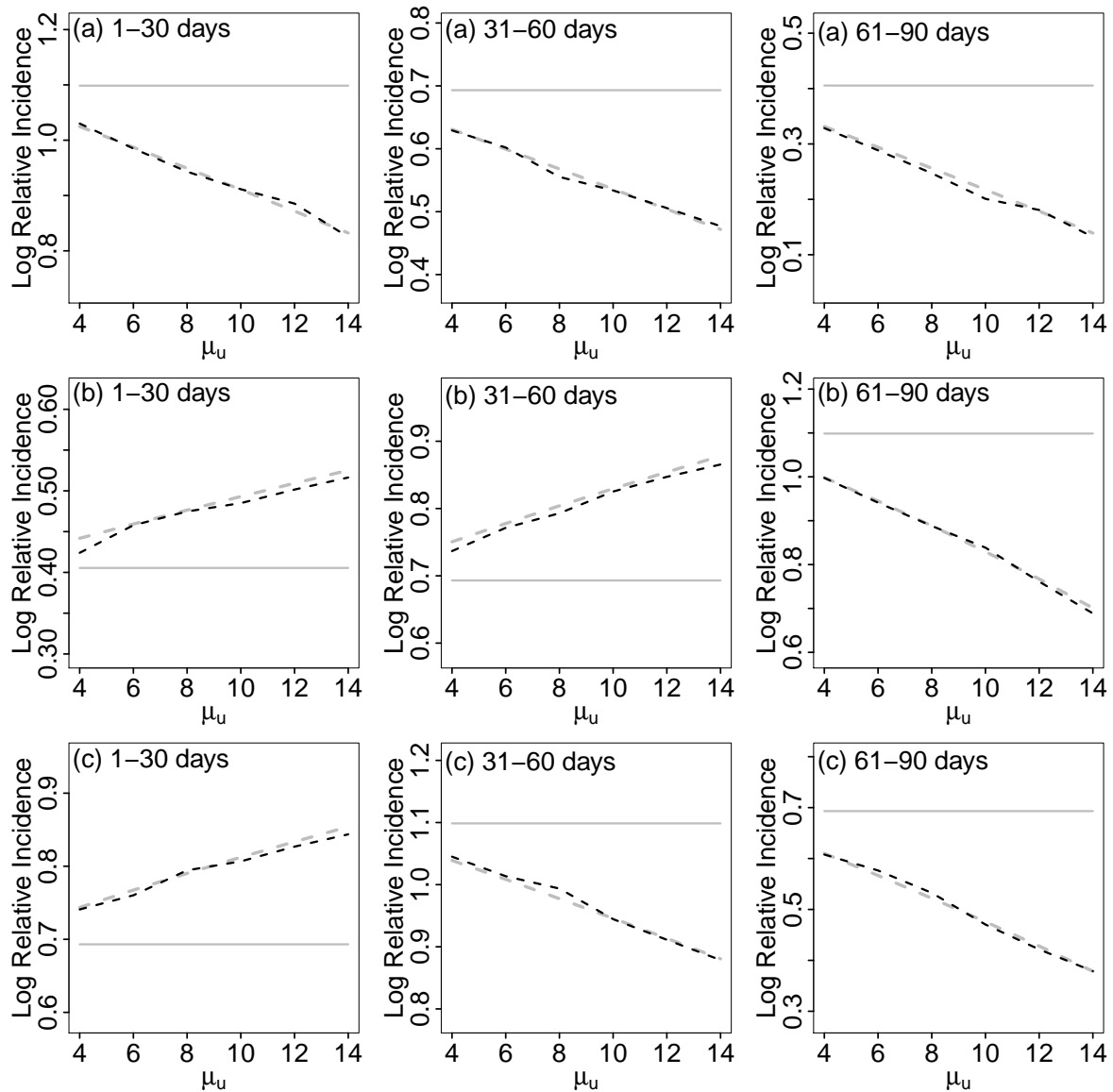


Figure 3: Characterization of bias for normally distributed measurement error for risk/effect patterns (a)-(c). Bias for true patterns of log relative incidence  $\beta$  (indicated by horizontal lines) over three risk periods. Dashed black curves denote naive case series estimates ignoring measurement error, which targets  $\beta^*$  (dashed gray curves) instead of  $\beta$ . Given are averages over 200 simulated data sets.

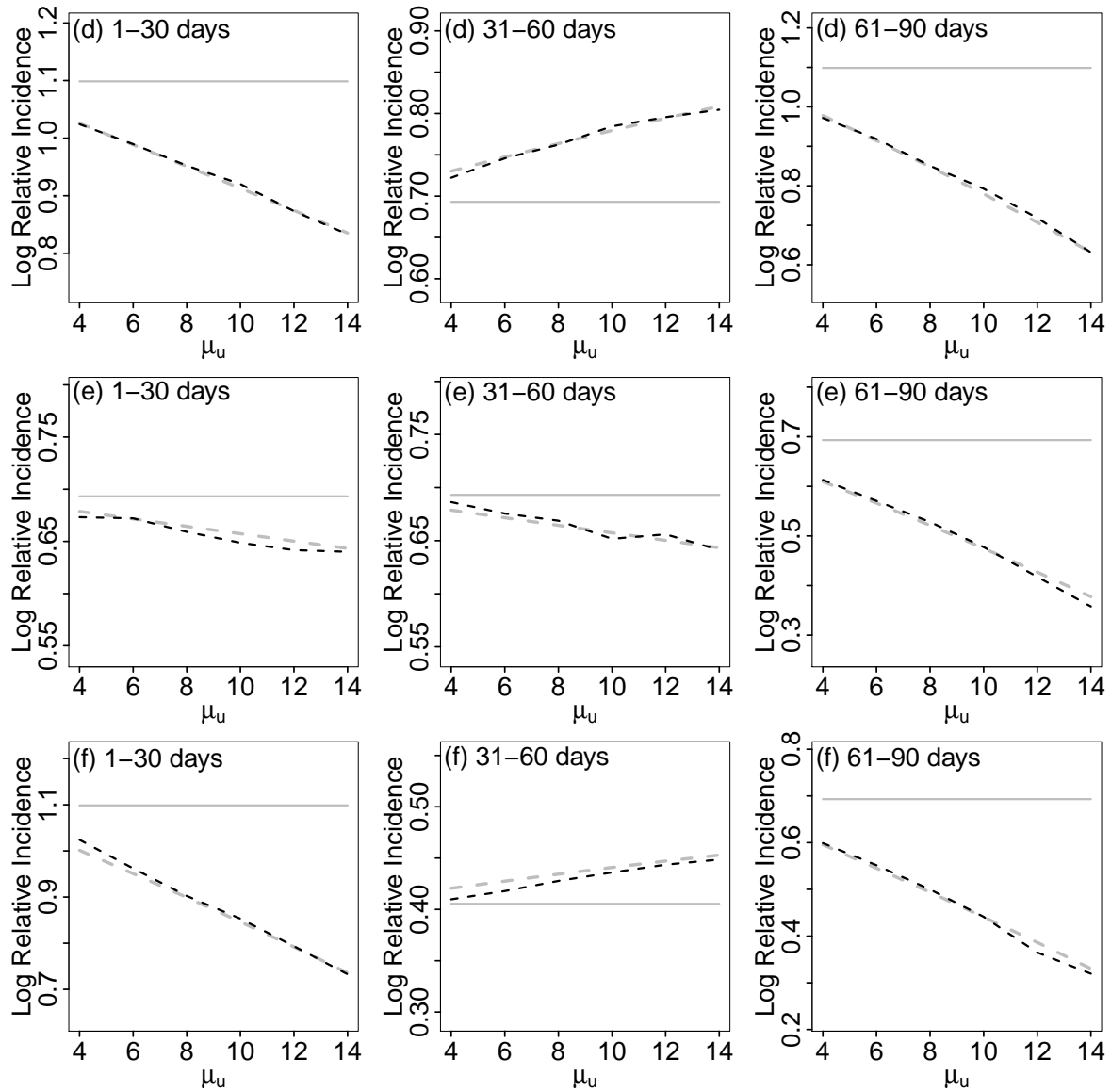


Figure 4: Characterization of bias for normally distributed measurement error for risk/effect patterns (d)-(f). Bias for true patterns of log relative incidence  $\beta$  (indicated by horizontal lines) over three risk periods. Dashed black curves denote naive case series estimates ignoring measurement error, which targets  $\beta^*$  (dashed gray curves) instead of  $\beta$ . Given are averages over 200 simulated data sets.



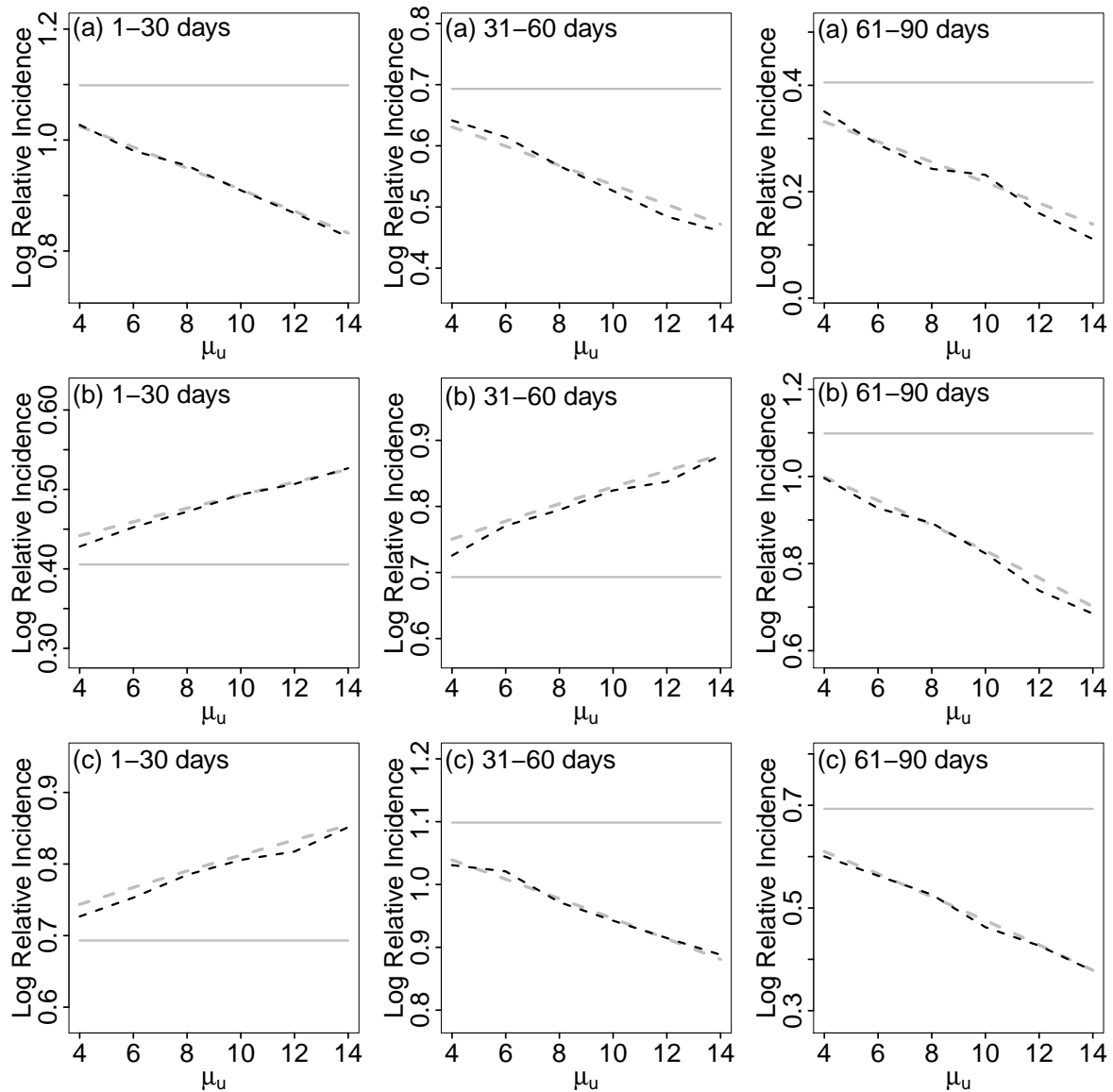


Figure 5: Characterization of bias for gamma distributed measurement error for risk/effect patterns (a)-(c). Bias for true patterns of log relative incidence  $\beta$  (indicated by horizontal lines) over three risk periods. Dashed black curves denote naive case series estimates ignoring measurement error, which targets  $\beta^*$  (dashed gray curves) instead of  $\beta$ . Given are averages over 200 simulated data sets.

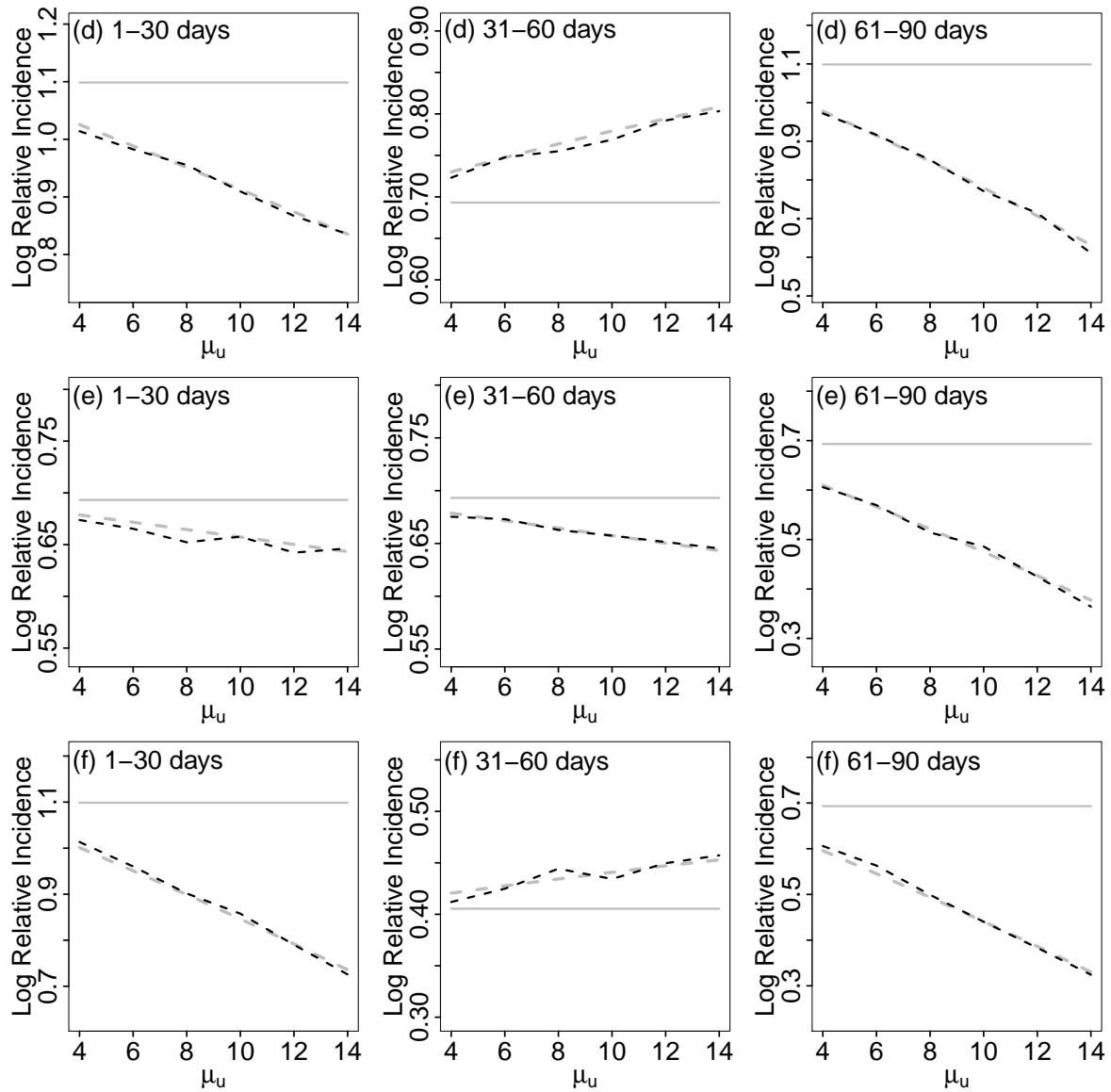


Figure 6: Characterization of bias for gamma distributed measurement error for risk/effect patterns (d)-(f). Bias for true patterns of log relative incidence  $\beta$  (indicated by horizontal lines) over three risk periods. Dashed black curves denote naive case series estimates ignoring measurement error, which targets  $\beta^*$  (dashed gray curves) instead of  $\beta$ . Given are averages over 200 simulated data sets.