Supplementary Information

# Description and first application of a new technique to measure the gravitational mass of antihydrogen

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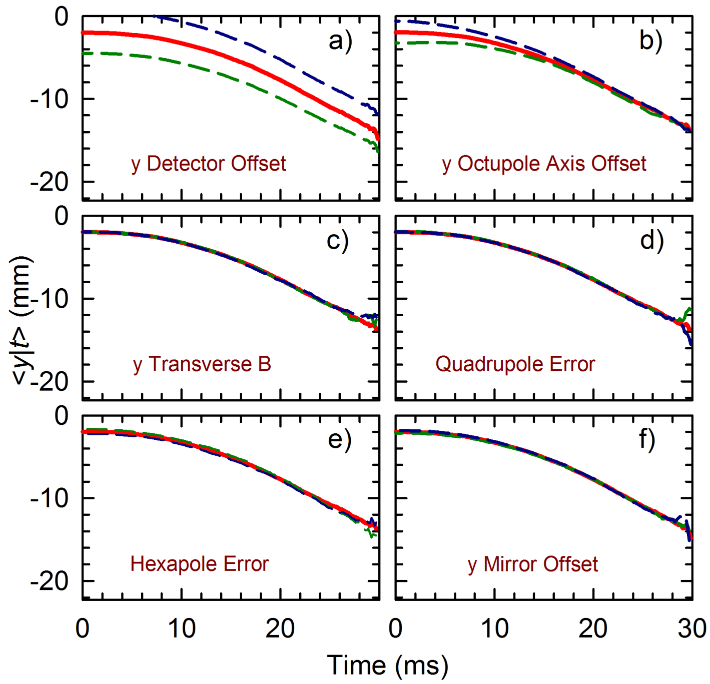
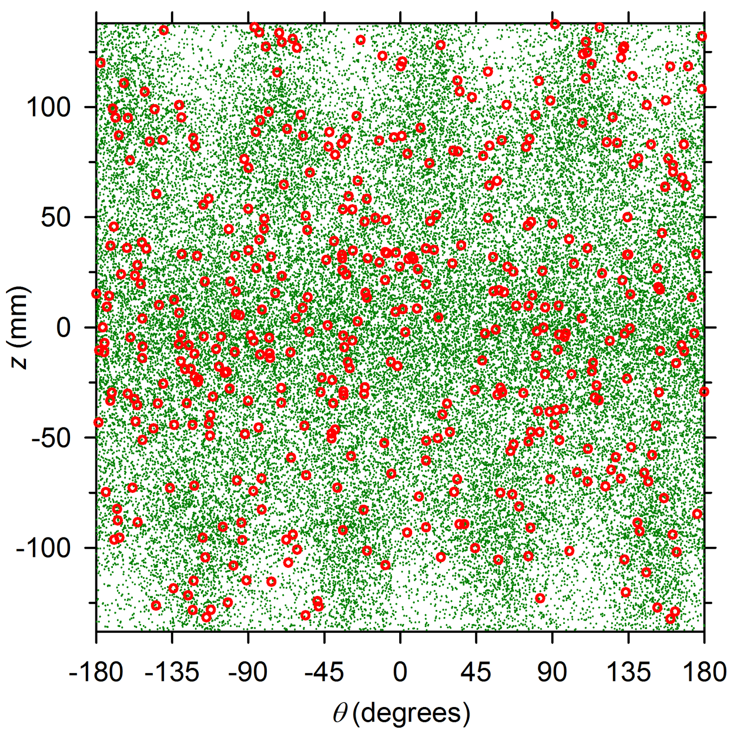
## SUPPLEMENTARY FIGURE




**Supplementary Figure S1: Systematic error analysis.** Effects on the reverse cumulative average $\langle y|t\rangle$ of representative systematic errors for $F = 80$. a) Detector axis offsets of $\pm 5$ mm in the $\hat{\mathbf{y}}$ direction. b) Octupole axis offsets of $\pm 0.05$ mm in the $\hat{\mathbf{y}}$ direction. c) Transverse magnetic fields resulting from $\pm 2$ milliradian tilts of the solenoidal axis. d) Quadrupolar components of the octupole field of $\pm 0.001$ T. e) Hexapolar components of the octupole field of $\pm 0.001$ T. f) $\hat{\mathbf{y}}$ directed $\pm 1$ mm offsets of the right mirror coil. In all cases the reverse cumulative average without systematic errors is given by the red solid line, and with the errors by the two dashed lines. The errors analyzed here are the maximum errors allowed by mechanical constraints.




**Supplementary Figure S2: $\theta$-$z$ Distributions.** The distribution in $\theta$ and $z$ of 50,000 $F = 1$ simulation points (green dots) and the experimental events (red circles). Note the periodicity in $\theta$ at high $z$.

## SUPPLEMENTARY NOTE 1

### Systematic effects on the gravitational measurement from magnetic field errors

We investigated a thorough set of potential magnetic field errors, and of these field errors, the octupole displacement error discussed in the main body of the paper has the largest potential effect on the gravity measurement. In some cases, it was obvious that the errors would not be significant. In others cases (2, 3, 4, and 6 below), we found the impact of these field errors by examining their effect on the reverse cumulative average $\langle y|t\rangle$. Some representative results are plotted in Sup. Fig. S1.

1. Large-scale wire placement errors in the octupole. The octupole was manufactured by ultrasonically bonding the octupole wires into place with a precision, multi-axis, numerically controlled machine. The machine places the wires with a large-scale accuracy of better than 0.03 mm. These large scale errors could result in several sorts of magnetic field errors:

   (a) Errors in the wire turn-around locations at the octupole axial ends. As only the most energetic antihydrogen atoms approach the ends, these anti-atoms escape early and do not influence the late annihilating subset of the data that is sensitive to gravity.

   (b) An overall twist to the octupole angular orientation. This would have no effect on the measurement since the experiment is not sensitive to the small angular deviations allowed by this mechanism.

   (c) Azimuthal variations in the octupole current density that deviate from the ideal $\sin 4\theta$ dependency. The resulting field errors would be small and very high order; the principal correction comes in at 24-pole (twelve-fold symmetric) order, and, consequently, scale with radius as $(r/R_{\mathrm{Wall}})^{11}$. Note that only hexapole (three-fold symmetric) and decapole (five-fold symmetric) fields have the correct symmetry to interact strongly with the four-fold symmetric octupole. Other field errors do not result in the $\sin\theta$ components necessary to strongly influence $\langle y|t\rangle$.

2. Wire by wire placement errors in the octupole. The six-around-one geometry of the magnet cable[34] causes individual wires to deviate from their intended positions by no more than 0.1 mm. We estimate the resulting field errors by randomly perturbing each wire from its programmed location, and then performing a multipolar expansion of the resulting field. After repeating this procedure many times, we find that the error field components are about 0.06% of the intended octupole field, or 0.001 T when the octupole is fully on, decaying to about 0.00004 T at $t = 30$ ms. (These are upper bounds since we assume that the postulated errors are constant in $z$. Variations in $z$ would cause these errors to partially cancel.) As discussed above, only the hexapole and decapole fields are significant, and these errors are less than the allowed errors from an octupole displacement.

3. Magnetic errors from the magnet leads. These errors are comparable to the possible hexapole/decapole errors, and are less than the allowed errors from an octupole displacement.

4. Tilts and displacement of the mirror coils. These errors are limited by mechanical constraints, and are smaller than the octupole axis displacement.

5. The magnet shutdown eddy currents. To leading order, these fields simply mimic the desired mirror and octupole fields, and their effects are included in our simulations[38]. The octupole eddy currents

do, however, generate additional $z$-dependent fields that do not closely mimic the intended octupole field. These fields, which are a few percent of the total octupole field, closely preserve the eight-fold symmetry of the octupole and, thus, cannot change $\langle y|t \rangle$ significantly.

6. Misalignments between the trap axis and the external solenoid. Measurements with trapped plasmas limit this misalignment to less than $\pm 2$ milliradians, corresponding to a transverse field error of about $\pm 0.002$ T. As the octupole field only decays to 0.06 T during the experiment, it is not surprising that error fields thirty times smaller have little effect. Moreover, these fields do not have the correct symmetry to interact strongly with the four-fold symmetric octupole.

7. External fields from other sources including the Earth's magnetic field. These are much smaller than the solenoid field, and generally originate from sources located much further away. They do not possess the gradients required to generate higher-order fields, and, consequently, do not have the symmetry to produce a significant effect. As our trap is made entirely from non-magnetic materials, there is nothing to convert a lower-order field into a higher-order field.

## SUPPLEMENTARY NOTE 2

### Simulation verification

In addition to the simulation benchmarking tests with antiprotons described in ref. 38, and the temporal and $z$ spatial agreement between the code and antihydrogen atoms described in refs. 30,31,38, Sup. Fig. S2 compares the $\theta$-$z$ distributions of the experimental events with the predictions of the simulation. There are not enough experimental events to closely compare the fine structure predicted by the simulation to the events, but the events do appear to largely respect the high $|z|$ keep-out regions in $\theta$.