Supporting Information

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SI Text

This note provides an explanation of the simulation procedure for obtaining numerical estimates of the three descriptive statistics $[\mathbb{E}D_1, \ell, p(1)]$ for the first SIRSN construction.

The key conceptual point is that the mathematical model under study is defined in the continuum, whereas simulations rely on approximating by a discrete $n \times n$ square grid. For given n, one could make the simulation results arbitrarily accurate. However, error bars would be very misleading because there is no statistical theory of how close the true values for finite n are to the desired $n \to \infty$ limit values.

The parameter that is most difficult to study by simulation is $p(1)$ (the others are $n \to \infty$ limits of directly analogous parameters for the $n \times n$ grid, estimated directly by sample averages), and here are some details of the procedure used to estimate $p(1)$.

The size of lattice square used for simulation was $[0, 127] \times [0, 127]$ and the random translations used to get away from the x and y axes were independently chosen uniformly from the integers in the range $[128, 256]$. We analyzed spanning subnetworks on 64 points sampled uniformly at random. The number 64 was chosen based on 75 independent preliminary simulations, which showed that the normalized network length per unit area remained roughly constant as the number of sampled points was varied from 64 to 256 at the different values of γ.

In the continuum limit, theory shows that the length-per-unit area $p(r)$ of $\mathcal{E}(r)$ scales according to $p(r) = p(1)/r$. In simulations on a discrete grid we expected, and observed, deviations from this precise relation for several reasons. When r is comparable to the unit edge length in the grid, the facts that we are sampling discretely from regularly spaced (rather than dense) points in a non-isotropic context all create differences from the continuum model. When r is comparable to the side length of the square, boundary effects (missed edges in routes between points far outside the square) are hard to control. In both cases, these effects tend to underestimate the true continuum value of $p(r)$.

To decide on an appropriate value of r to use for the numerical estimate of $p(1)$, we performed 50 independent simulations of the construction with parameter $\gamma = 0.65$, where we computed $p(r)$ of the network as we successively deleted edges with r ranging from 10 to 120. From these simulations, we found the maximum value of $r \times p(r)$ occurred at $r = 30$, so we selected this value of r. The plotted value of $p(1)$ at each value of γ is an average taken over 100 independent simulations of $30 \times p(30)$.