# Supporting Information

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### SI Text

**Geographic Separation.** We say that a graph is geographically separated if the influence zones of every node of a given hierarchical level do not overlap and if they are included in the influence zone of the nodes of the previous level in the hierarchy. Formally, if we designate by $\mathcal{I}_l^i$ the influence zone of the node $i$ located at level $l$ in the hierarchy, $\mathcal{I}_l = \cup_{i\in l}\mathcal{I}_l^i$ represents the reunion of all the influence zones for nodes belonging to the level $n$. We say that the graph is geographically separated if:

$$\mathcal{I}_l \subset \mathcal{I}_{l+1}\ \forall l, \quad [16]$$

$$\mathcal{I}_l^i \cap \mathcal{I}_l^j = \emptyset \ \text{ if } \ j \neq i, \forall l. \quad [17]$$

The degree of geographical separability of a graph strongly depends on the definition of the influence zone of a node. For instance, if we take the influence zone of a node $i$ to be the surface of smallest area containing all the nodes connected to $i$, it follows that all planar graphs are totally separated. In the context of transportation networks, we expect hubs to radiate up to a certain distance around them, that is, to connect to all the nodes located in a convex shape. We simply define the influence zone of a node $i$ as the circle centered on the barycenter of $i$'s neighbors that belong to the next level, with a radius of the maximum distance between the barycenter and those points. Fig. S1 is intended to help the reader visualize these influence zones in an example: The green circle represents the influence zone of the root, and the red circles represent the influence zones of the hubs connected to it. One can see that the graph is geographically separated up to a good approximation.

To quantify this notion of geographical separability, we define the separation index of the level $l$ as the average over all the nodes belonging to $l$ of the separation function. The separation function is equal to 1 if the distance $d(i,j)$ between the centers of the influence zones of $i$ and $j$ is larger than their respective radius (no overlap), and equal to:

$$S(i,j) = 1 - \frac{\text{Area of the overlap between } \mathcal{I}_l^i \text{ and } \mathcal{I}_l^j}{\min(\text{Area of } \mathcal{I}_l^i, \text{Area of } \mathcal{I}_l^j)}. \quad [18]$$

One can see that the separation function is equal to 1 if the nodes' influence zones do not overlap at all and 0 if they perfectly overlap (all the influence zones overlapping, like Russian dolls). Therefore, the separation index is equal to 1 if the level $S$ is perfectly separated and equal to 0 if the influence zones are completely mixed. Fig. S2 illustrates the value of the separation index for different situations.

### Understanding the Scaling with a Toy Model

We consider the toy model defined by the fractal tree depicted in Fig. S4, for which the distance between the levels $n$ and $n+1$ is given by:

$$\ell_n = \ell_0 b^n, \quad [S1]$$

where $b \in [0,1]$ is the scaling factor. Each node at the level $n$ is connected to $z$ nodes at the level $n+1$, which implies that:

$$N_n = z^n, \quad [S2]$$

where $z > 0$ is an integer. A simple calculation on this graph shows that in the limit $z^g \gg 1$, the total length of the graph with $g$ levels scales as:

$$L_{tot} \sim N^{\frac{\ln(b)}{\ln(z)}+1}, \quad [S3]$$

where $\frac{\ln(b)}{\ln(z)}+1 \leq 1$ because $b \leq 1$ and $z > 1$. This simple model thus provides a simple mechanism accounting for continuous values of $\tau$, whose value depends on the scaling factor $b$. It provides a simplified picture of the graphs in the intermediate regime $\beta \simeq \beta^*$ and exhibits the key features of the graphs in this regime: the hub structure reminiscent of the star graph and where the nodes connected to each hub form geographically distinct regions organized in a hierarchical fashion.

It is also interesting to note that the parameter $z$ can be easily determined from the average degree $\langle k \rangle$ of the network:

$$z = \langle k \rangle - 1, \quad [S4]$$

and that the parameter $b$ of the toy model can be related to our model by measuring the decrease of the mean distance between different levels of the hierarchy, as in Fig. 3. By plotting these curves for different values of $\beta/\beta^*$, we find that the coefficient of the exponential decays decreases linearly with $\beta/\beta^*$, and therefore that $b \sim e^{\beta/\beta^*}$ (however, the comparison only makes sense in the regime $\beta \sim \beta^*$ because, otherwise, the graphs do not exhibit spatial hierarchy).

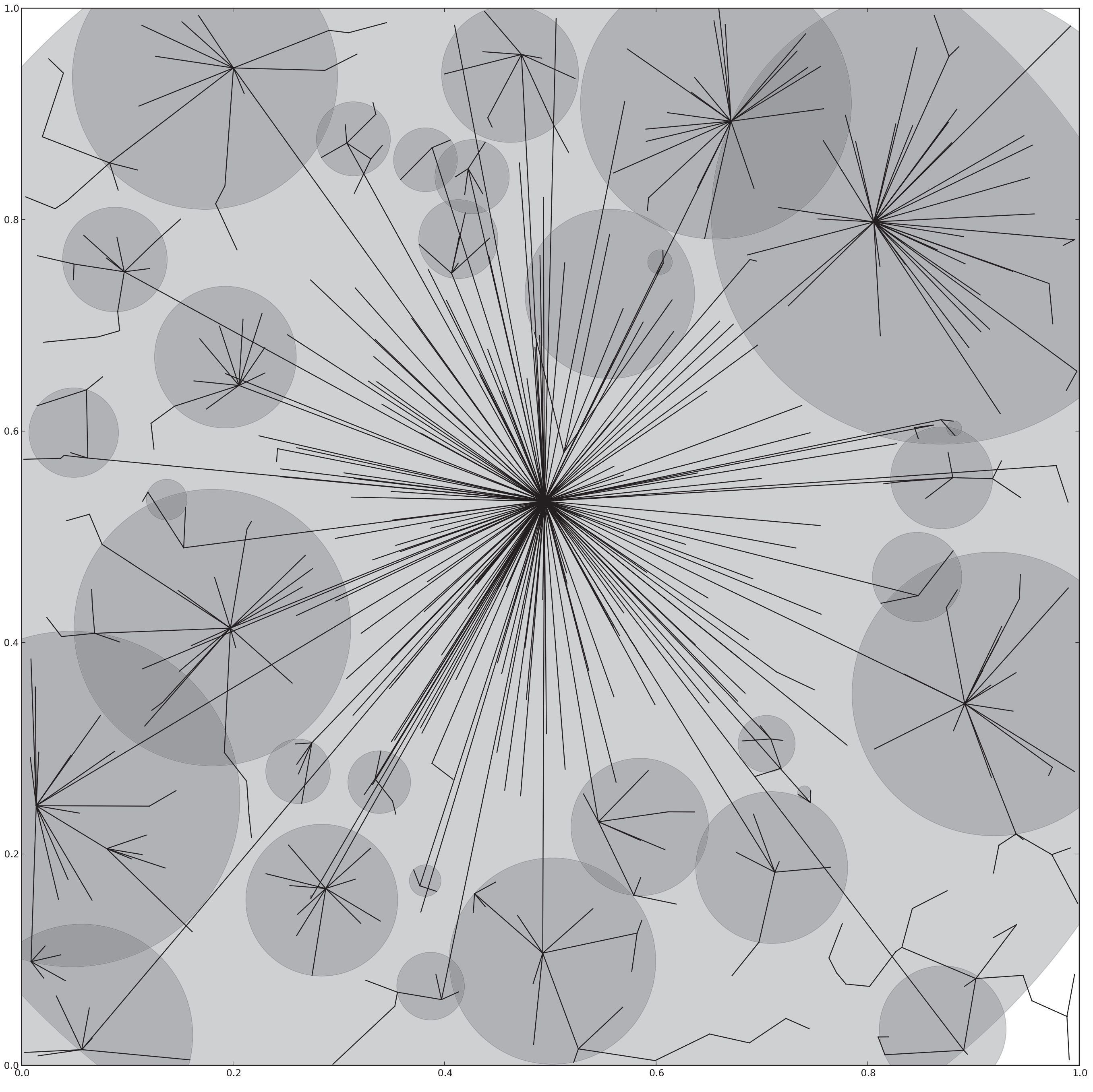


**Fig. S1.** Example of a graph in which we represent the influence zones for the first two hierarchical levels.

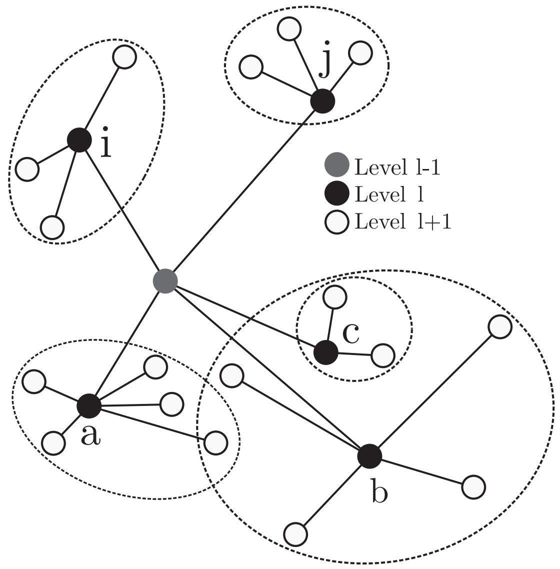


**Fig. S2.** Illustration of the influence zones (dotted lines) around several hubs. We have, according to the definition of the separation index, $S(i,j)=0$, $0<S(a,b)<1$, and $S(b,c)=1$.

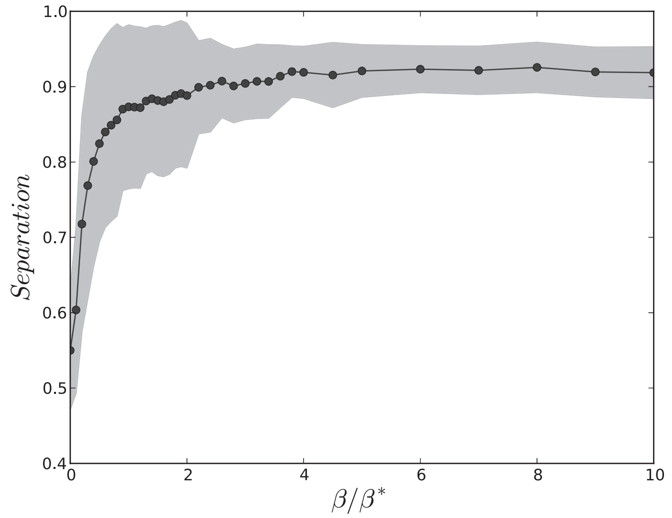


**Fig. S3.** Separation index averaged over all the graph's level vs. $\beta/\beta^*$. The shaded area represents the SD.

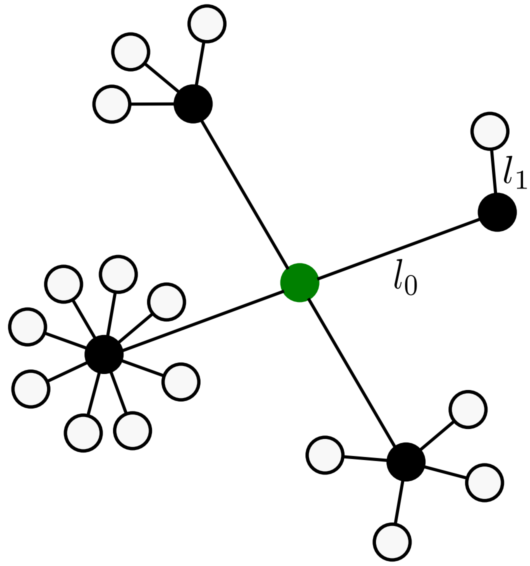


**Fig. S4.** Schematic representation of the hierarchical fractal network used as a toy model.