Supplementary Information for

Limited communication capacity unveils strategies for human interaction

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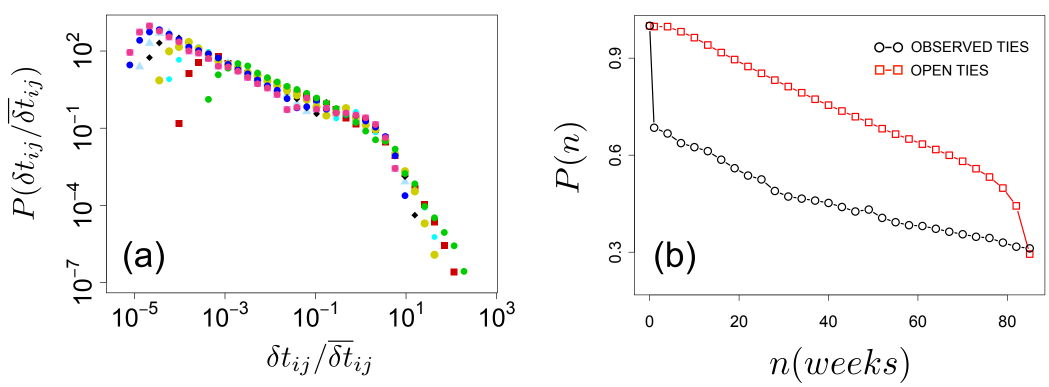


SI Figure 1: (a) Rescaled inter-event time distribution for groups of edges with different average inter-event time $\overline{\delta t}_{ij}$. Each curve is rescaled by the value of $\overline{\delta t}_{ij}$ of the correspondent bin. (b) Weekly persistence $p(n)$ of ties observed in the first week of our database as a function of the number of weeks n: while persistence drops to 70% after one month if ties are required to have activity at a given week n, it is still around 70% for one year if we consider open ties at that week, i.e. ties which where observed in the first week.

1 Entanglement between bursty activity and tie dynamics

As stressed in the main text, one of the most challenging problems in the study of the dynamics of tie creation and removal is to identify whether a tie is actually a new/old connection. Although in most social networks there are specific events for the formation of new "friends" (or followers) or the corresponding "unfriending" events, due to the cheap cost of maintaing those connections most of those ties are abandoned and thus activity between individuals is the only way to asses the existence or not of that relationship.

However, human activity is bursty, meaning that there are large periods of inactivity followed by bursts of activity [2]. This means that within a particular tie $i \leftrightarrow j$ the time between consecutive communication events δt_{ij} is heavy-tailed distributed. In our database we find that this is indeed the case and in line with [6, 7] we find that there is a universal law for the distribution of inter-event times (see Fig. 1). In particular, we find that for a particular tie $P(\delta t_{ij}) = \mathcal{P}(\delta t_{ij}/\overline{\delta t}_{ij})$ where $\mathcal{P}(x)$ is a heavy tailed universal function. Since bursty behavior seems to be universal in human activity [2], it has a deep impact in the understanding of tie dynamics and translate in a ubiquitous problem in the empirical observation of social networks: if the observation window is very short we might miss most of the ties since there is no communication in that period of time. But on the other hand, since the inter-event time distribution is heavy-tailed we might have to go to large observation windows to recover most of the ties. For example in our database we find that the average inter-event time is $\langle \delta t_{ij} \rangle = 14$ days with a standard deviation $\sigma = 18$ days, which means that the observation period must be larger than a 2-3 months only to observe (at least once) most of the ties in the social network. In our case, Ω extends over 7 months and using the previous/next 6 months intervals we calculate that 3% ties did not show activity in Ω and then could have been missed if only data within Ω was present.

Although the impact of burstiness in the observation of ties is important, it becomes critical for the problem of tie formation/decay since it is not only necessary to observe the tie but to asses its termination or formation. Thus, we need to increase substantially the observation window to identify whether the link has been formed and or decayed in our database. Short observation windows can lead to spurious effects: a tie that is present in one time window might (with large probability) do not show activity in the next time window due to a large inter-event time and thus we might incorrectly identify that event as decay of the relationship. This might be the origin of the large (30-40%) decay in persistence observed in the literature [8, 9] (and reproduced in our database, see Fig. 1(b)), since the observation windows were very short (1 month). The large probability of having a inter-event time of one month in human communication leads to the erroneous impression that 40% of the links are created/decayed in one month period and that the networks are highly volatile, since correlation between the network structure at different observation windows is very low.

To cure those problems in our paper we propose a different method to asses whether a tie formed/decayed in the observation window Ω. The method is based on the *observation* of tie activity in a time window before/after Ω: if tie activity is observed in the 6 months before Ω then it is considered an old tie [cases (a) and (d) in *Main Text Fig. 1*]; on the other hand, if activity is observed in the 6 months after Ω we will assume that the tie persists [cases (b) and (d) in *Main Text Fig. 1*]. In any other case, we will consider that the tie is formed and/or decay in Ω [cases (a), (b) and (c) in *Main Text Fig. 1*]. Of course, it is possible that even if there is no communication before/after the observation window, the tie is still active after/before our database. This would require that the tie has an inter-event time δt_{ij} bigger than 7 months, i.e. case (e) in *Main Text Fig. 1*. However, in our database, only 3.5% of the links have such a long inter-event time which validates the accuracy of our definition of tie decay/formation.

On the other hand in our study a tie is considered to be opened between its formation and decay events (if they happen in Ω at all). This assumption is based on the idea that an interaction which has been observed in the past and will be observed in the future might exist at a given instant even if there is no communication by mobile phone between at that particular instant. Furthermore, our observation window is short enough to neglect safely possible formation and decays of the relationship within Ω. Our definition of relationship mitigates the excessive volatility of the social network when tie is considered only when interaction is observed at a given instant. For example, the persistence of open links is higher (70% in one year) than observed links (40% in one year) in line with off-line studies [15]. It also resembles different situations in which, although a strong relationships might exist off-line, very few calls are exchanged in time.

Finally, understanding this difference between open and observed relationships is crucial to unveil the real dynamics of social networks because it can induce also spurious effects in the observations: within a given observation window, the (revealed) aggregated connectivity $k_i(t)$ seems to grow non-trivially as a function of time (see *Main Text Fig. 2*) within Ω. Actually, it could even be fitted to a power law $k_i(t) \sim t^\gamma$ with $\gamma \simeq 1/2$ for small t. It is interesting to see that the functional form and exponent do fit those found in models of network growth [14]. But it is easy to see that this effect is (mostly) due the fact due the fact that different links have very heterogeneous number of communication events w_{ij} and within a given tie events are very bursty. Specifically, the apparent growth of $k_i(t)$ for short times is mainly due to the possibly large and highly heterogeneous time to the first event event within ties.

To understand that, suppose that a given tie is present before and after the observation window Ω and that the distribution of inter-event times within that tie is given by $P(\delta t_{ij})$. Assuming that the initial time of the observation window is random, the time to the first observation of the link is given by the waiting time equation in renewal processes [13]

$$P(\tau_{ij}) = \frac{1}{\overline{\delta t_{ij}}} \int_{\tau_{ij}}^{\infty} P(\delta t_{ij}) d\delta t_{ij} \quad (1)$$

Thus, depending on the properties of $P(\delta t_{ij})$ we could have a very large observation time (τ_{ij}) for the link. As shown in Fig. 1(a) the pdf for inter-event times depends mostly on the average inter-event time $\overline{\delta t}_{ij}$, i.e. $P(\delta t_{ij}) = \mathcal{P}(\delta t_{ij}/\overline{\delta t}_{ij})$ where $\mathcal{P}(x)$ is a universal function. Thus, for a given $\overline{\delta t}_{ij}$ we could rewrite the previous expression as

$$P(\tau_{ij}|\overline{\delta t}_{ij}) = \frac{1}{\overline{\tau_{ij}}} \int_{\tau_{ij}}^{\infty} \mathcal{P}(\delta t_{ij}/\overline{\delta t}_{ij}) d\delta t_{ij} \quad (2)$$

However, ties are very heterogeneous in the sense that they have very different $\overline{\delta t}_{ij}$. Or equivalently, they have very different weights $w_{ij} = T/\overline{\delta t}_{ij}$ [12]. Suppose that $\Pi(\overline{\delta t}_{ij})$ is the distribution of average inter-event times across links and that each user chooses her tie activities from that distribution. We assume also that no tie is form/destroy during the observation time. Then the probability to observe one of her links at time τ is given by:

$$P(\tau) = \int d\overline{\delta t}_{ij} \Pi(\overline{\delta t}_{ij}) P(\tau|\overline{\delta t}_{ij}) \quad (3)$$

Thus, the growing function of the observed connectivity as a function of time is given by the ccf of $P(\tau)$.

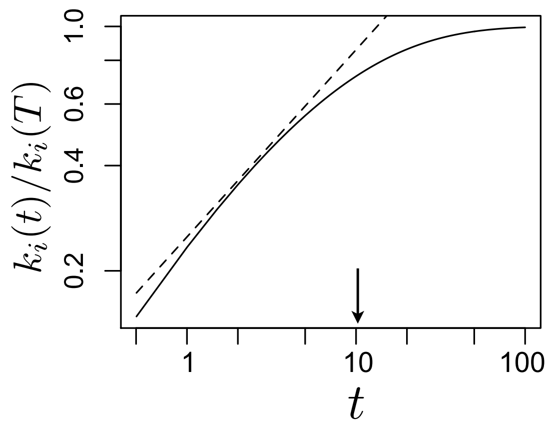
$$k_i(t) = k_i(T) \int_0^t P(\tau) d\tau \quad (4)$$

where $k_i(T)$ is the total connectivity of node i in the observation window Ω. Note that since $\mathcal{P}(x)$ and $\Pi(\overline{\delta t}_{ij})$ are heavy tailed, then $P(\tau)$ is heavy tailed too and thus the $k_i(t)$ can show an apparent non-trivial time dependence even if all links are open during Ω. Expression (4) shows that one should be careful to consider the observed aggregate connectivity $k_i(t)$ as a proxy for social connectivity at any time t, since it is profoundly affected by the bursty and heterogeneous activity of human behavior encoded in $P(\tau)$. Note to mention the effect of tie formation/destruction which is not included in (4).

Strikingly, an apparent $k_i(t) \sim t^\gamma$ growth can be observed even in the case in which both tie activity and weights are severely bounded: if we assume that the distribution of inter-event times is given by the exponential pdf $P(\delta t|\overline{\delta t}) = e^{-\delta t/\overline{\delta t}}/\overline{\delta t}$ and also that the pdf for the average inter-event time is an exponential $\Pi(\overline{\delta t}) = e^{-\overline{\delta t}/a}/a$ we get exactly from Eq. (4) that

$$k_i(t) = k_i(T) \left\{ 1 - 2\sqrt{\frac{t}{a}} K_1 \left(2\sqrt{\frac{t}{a}} \right) \right\}, \qquad 0 \le t \le T \quad (5)$$

where $K_1(x)$ is the Modified Bessel Function of the second kind [11]. Thus, for a single user the number of observed ties grows in a non trivial way as a function of time even for this homogeneous

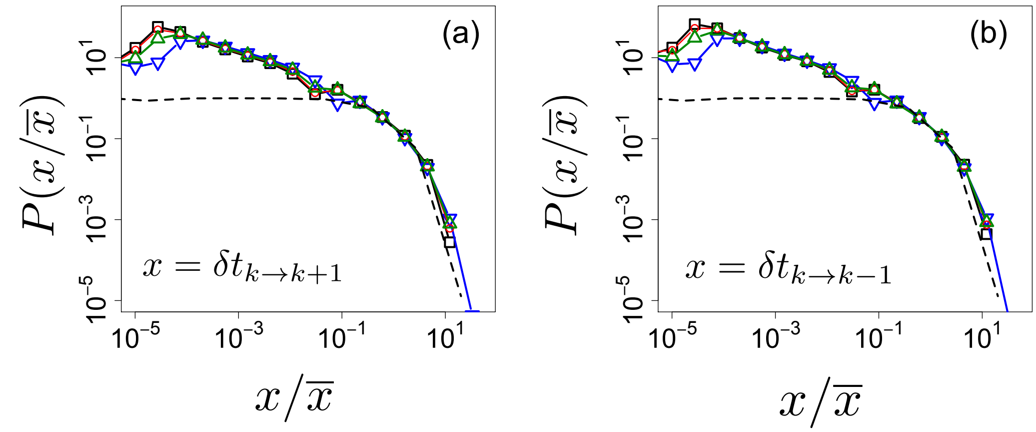


SI Figure 2: Apparent growth in the connectivity given by equation (4) as a function of time for an exponential distributions of average inter-event time with $a = 10$ days (marked by the arrow). Dashed line is the fit to a power-law growth for the initial growth (up to 5 days) that yields $k_i(t) \sim t^\gamma$ with $\gamma = 0.53 \pm 0.02$.

(both in the events and in the links properties) case, a behavior which extends further from $t = a$, the average $\overline{\delta t}$ (see Fig. 2). This result for a single user based on the universal bursty and heterogeneous activity in ties, together with the large heterogeneity found in social connectivity (which is related to $k_i(T)$) could explain the apparent non-trivial growth of the aggregate $k_i(t)$ observed in social networks [1] and highlights the importance of taking into consideration the heterogeneity of activity of humans to define properly the way we measure and observe their social networks. Finally these results emphasize the goodness of our method to detect open ties, since in this simple example all ties are open at any time and then $\kappa_i(t)$ is constant throughout the observation window Ω.

2 Bursty dynamics of tie activation or deactivation

We observe that most people form and destroy edges almost constantly in time (see *Main Text Fig. 3*). However, despite the linear growth of the number of added and removed connections, the distribution of the time gap between creation/removing of ties is not Poissonian (Fig.3), which is in line with recent results [4]. Fig. 3 (a) and (b) show respectively the pdf of the time it takes for the node i with degree k_i to create one more connection ($\delta t_{k,k+1}$) and to loose one connection $\delta t_{k,k-1}$. Specifically, we divide the whole population of users in four groups depending on their value of α_i and plot the distribution for each group. Despite the exponential cut-off, the results indicate some bursty patterns of activity for sort times. In addition all distributions collapse into a single curve suggesting that a universal form for the burstiness in the tie activation/deactivation.



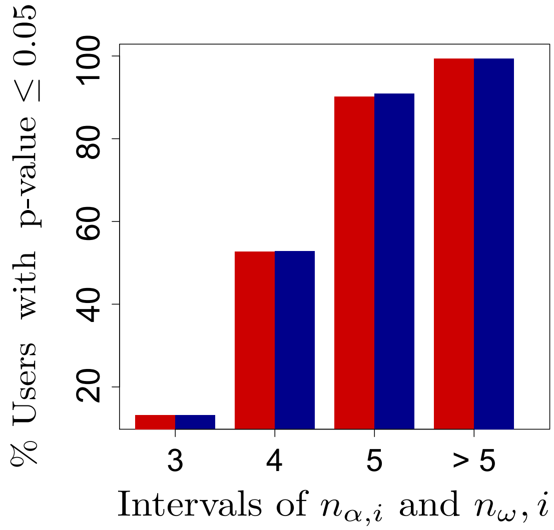
SI Figure 3: (Rescaled) Distribution of the time gap between edge creation (a) and edge removal (b) for groups of nodes with different activity rate α_i, where groups have been obtained according to the quartiles of α_i for the whole population.

3 Linear growth of tie activation or deactivation

Although tie activation/deactivation events do not happen homogeneously in time, the strong cut off in the bursty inter-event time found in the previous section suggests that there exists a typical time scale in which those events happen and thus, for a larger enough observation time, we should expect linear growth for the accumulated number of events $n_{\alpha,i}(t)$ and $n_{\omega,i}(t)$. Indeed, by taking these time series and fitting them to linear models we get the rates α_i and ω_i explained in the main text. The statistical significance of the fit of those to each individual dynamics is shown in Fig. 4 where we can see that the linear fit is statistically significant for a majority of users with $n_{\alpha,i}(T) = 5$ and for most users with $n_{\alpha,i}(T) > 5$ (same results for $n_{\omega,i}$). On the other hand, for those selected individuals for which p-value < 0.05 the goodness of fit is on average $R^2 \simeq 0.91$ with 93% of them with $R^2 > 0.8$. Thus, the results presented in the following section and in the main text for α_i and ω_i are only for those with $n_{\alpha,i}(T) \geq 5$ (same for $n_{\omega,i}(T)$) for which the goodness of fit is around $R^2 \simeq 0.91$ and the percentage of those with a p-value smaller than 0.05 is around 100%. They amount up to 75% of the total number of users.

4 Statistical evidence for the conservation of social capacity

One of the key findings in our study is the fact that for a given individual i the rate at which ties are formed α_i equals that at which ties decay ω_i. This implies that social capacity, i.e. the number of open connections at a given instant is more or less constant in time. In this section we describe the analysis performed to reach this statement and the null model used to asses the statistical significance of $\alpha_i \simeq \omega_i$. The basic problem is the fact that for most of our users in the database, the number of events $n_{\alpha,i}$ and $n_{\omega,i}$ is very small and then we get large differences between the values of α_i and ω_i obtained.



SI Figure 4: Ratio of the number users for who a linear fit to the $n_{\alpha,i}(t) \sim \alpha_i t$ (red) and $n_{\omega,i}(t) \sim \omega_i t$ (blue) time series has a p-value smaller than 0.05 for the F-test. Different columns refer to different groups of users according to their total number of activated/deactivated ties in the observation period Ω.

We will test that our results are comparable statistically to a null model in which ties are formed and destroyed in the observation window Ω according to two different realizations of a Poisson process with the same rate $\alpha = \omega$. The choice of Poisson process as the renewal process that describes the formation and decay process is supported by the bounded probability distribution for the inter-event times between formation/decay of events seen in previous section. Of course this is an approximation, because there is a large probability of bursts of formation/decay events than predicted by the exponential distribution of the Poisson process. The approximation works better for large times or number of events, since in that limit the strong decay of the inter-event time distribution for large values makes the process to converge to the behavior of a Poisson process very quickly by means of the Central Limit Theorem [16].

Since there is a large heterogeneity of social activity in our database we take as input for our null model the actual values of $n_{\omega,i}$ to incorporate that heterogeneity in our null model. We have also done simulations taking $n_{\alpha,i}$ and the results are the same. Thus, our Monte Carlo simulations of the null model are as follows: for every individual i we take $\lambda_i = n_{\omega,i}/212$ as the rate for tie formation and decay of ties per day and simulate two Poisson processes in the observation window Ω with the same rate, one for the formation of ties and the other for the decay of ties. We then calculate the times series of the aggregate number of events $\hat{n}_{\alpha,i}(t)$ and $\hat{n}_{\omega,i}(t)$ and fit them to linear models to obtained the simulated $\hat{\alpha}_i$ and $\hat{\omega}_i$. In line with the results of previous section, we only consider for the fit those simulations for which the $\hat{n}_{\alpha,i}(T) \geq 5$ and $\hat{n}_{\omega,i}(T) \geq 5$ in the fit.

As shown in the caption of *Main Text Fig. 3b* (see details there), the observed values for $n_{\alpha,i}(T)$ and $n_{\omega,i}(T)$ in our database can be well explained by our simulations, suggesting that our model works well at that particular time scale. We also find a good agreement between the measured values of α_i and ω_i and the results of our null model as shown in *Main Text Fig. 3c*, although there a small amount of outliers that cannot be explained by our model.

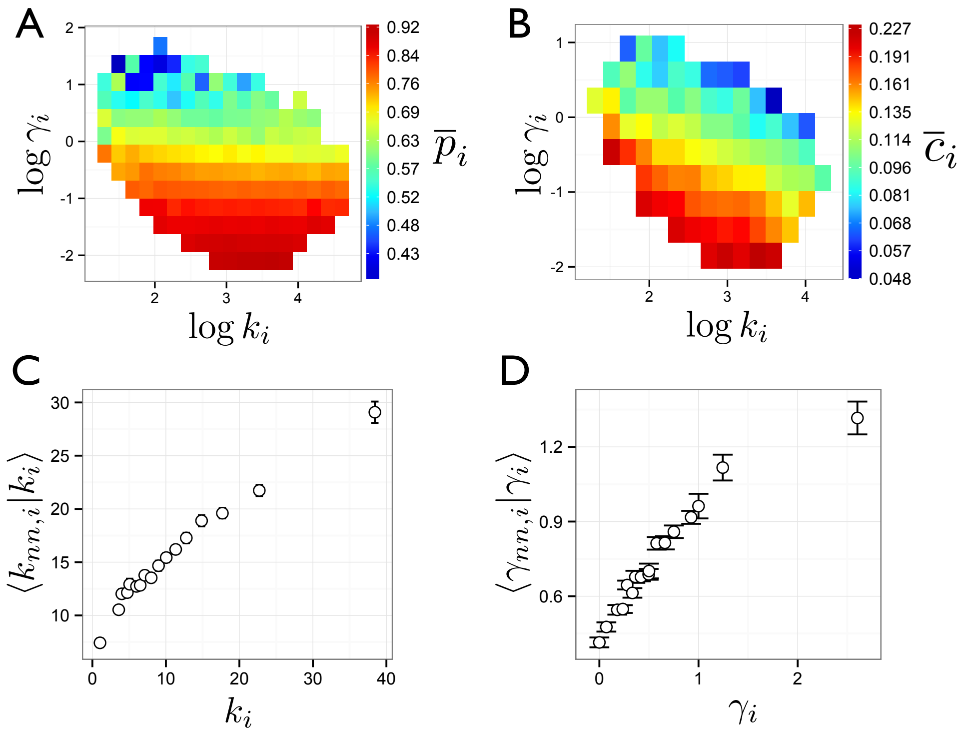
5 Measuring neighborhood persistence

We measured the persistence p_i of a user i as the fraction of his neighbors present at the beginning of the observation period Ω that are maintained until the end of Ω. Specifically, $p_i = (\mathcal{E}_i(0) \cap \mathcal{E}_i(T))/\mathcal{E}_i(0)$, where $\mathcal{E}_i(0)$ and $\mathcal{E}_i(T)$ are respectively the set of ties that user i has at time 0 and time T (see *Main Text Fig. 1*). Once measured p_i for all users in our dataset, we find that the average persistence $\overline{p}_i$ is 0.75. As discussed in the main text, this suggests that although in a given time period users activate and deactivate many connections (on average half of their social connectivity), after a period of 7 months they maintain on average the 75% of their initial social network.

We also mentioned that this value is much larger than the one obtained in a model in which each tie is activated and deactivated with the same probability, suggesting that as expected individuals do not establish or remove social connections randomly. To address the latter, we simulated the following process: for a given user we preserve *(i)* all the properties of his measured social strategy (k_i, κ_i, $n_{\alpha,i}$, $n_{\omega,i}$) and *(ii)* the real sequence of both his tie activation and deactivation time instants. Thus, following the order of such sequences, at each activation (deactivation) time we allow the user to add (remove) one of his neighbors randomly chosen among all his neighbors. Note that in the random model we maintain all the properties of the individual social network and strategy and the only thing that we destroy is the selection of neighbors added and/or removed within the observation period Ω. We then repeat this process for all users in our dataset and for each of them we measure the new network persistence p'_i. As discussed in the main text, we found that $\overline{p'_i} = 50\%$, against the $\overline{p}_i = 75\%$ measured for the real case, which suggests that the way in which people activate, maintain and deactivate social relationships is, as expected, not random and some ties are more probable to be destroyed than others.

6 Relation of the social strategy with topological properties

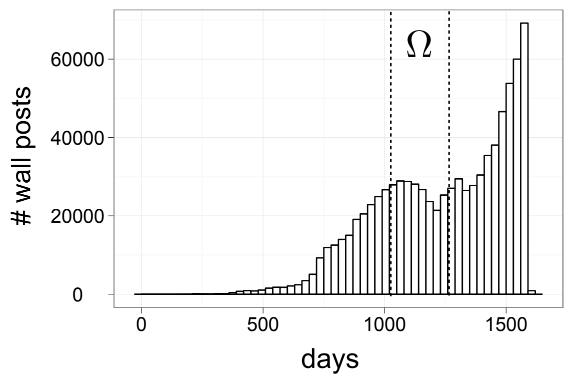
We find a significant dependence between the social strategy for an individual (encoded through the parameter γ_i) and the topological properties around that individual. Specifically, figure 5 shows how the persistence defined in the previous section depends heavily on γ_i but shows a large independence with the total connectivity of individuals in the period of observation. Specifically social keepers (those with $\gamma < 0.2$) do show a large persistence in their social neighborhood (even up to 90%), while social explorers ($\gamma > 2$) only keep a small fraction of their initial ties at the end of the 7 months period, even down to 40%. On the other hand, the aggregated clustering coefficient also depends on the social strategy: social keepers tend to have more clustered neighborhoods than social explorers. Specifically, we find that $\overline{c}_i$ can be up to 0.22 for social keepers, while it decreases to 0.05 for social explorers. Note that in the case of the clustering coefficient we also observe that it decreases with increasing average connectivity, a effect well known in social networks [14]: $c(k_i)$ is typically a decreasing function with k_i reflecting the fact that for largely



SI Figure 5: Relation of the social strategy with topological properties: dependence of the average persistence of ties (A) and aggregated clustering (B) as a function of the total connectivity k_i and social strategy γ_i. (C) Average value of next neighbor connectivity $k_{nn,i}$ of a node as a function of its own connectivity k_i. The Pearson correlation coefficient between the two quantities is $\rho(k_i, k_{nn,i}) = 0.342$ with a confidence range of [0.278,0.316] (D) Average value of the parameter $\gamma_{nn,i}$ for the neighbors of an individual as a function of her own value of γ_i, $\rho(\gamma_i, \gamma_{nn,i}) = 0.412$ with confidence interval [0.394,0.429]. A clear growth can be seen in both cases, indicating a strong assortativity.

connected people it is increasingly more difficult to have a moderate clustering. However, in Fig. 5 we see that the clustering not only depends on the connectivity, but also on the social strategy. Since both factors have opposite effect on clustering we find, for example, that social keepers with large connectivity might have the same clustering as social explorers with small connectivity. Thus the aggregated clustering found in social networks is a function of both connectivity and social strategy, suggesting that its value is determined dynamically by the tie formation/destruction processes around a given individual.

Finally, in our database we observe that social connectivity is assortative, in line with other studies [14]. More interestingly, we find that also social strategies of communication are assortative, as it is shown in Fig. 5. As mentioned in the main text, this result indicates that people that establish and remove many connections from their network at a high rate (social explorers) are more likely to interact with people that also change their network quickly. Analogously, those



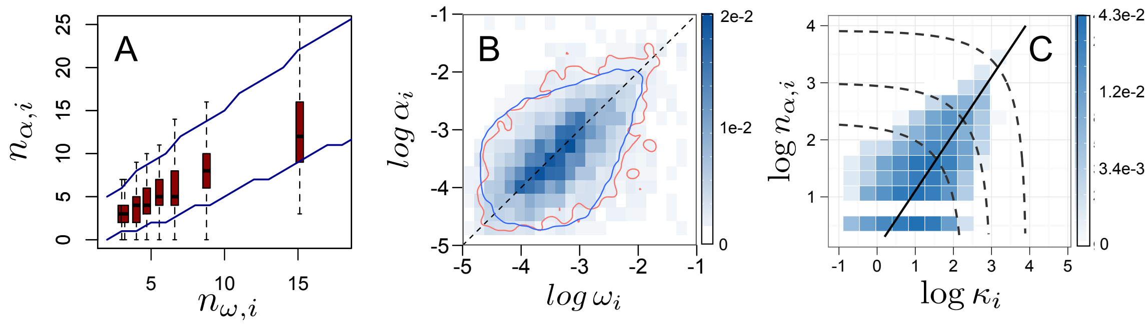
SI Figure 6: Activity in the Facebook database. Number of communications through the wall in our database for periods of 30 days. Dashed lines show the limits of the observation time window Ω.

individuals that maintain a more stable social network (social keepers) also interact with people with the same strategy. As a consequence, the large volatility observed in the neighborhood of social explorers also extends to large proportions of the network around them and the same applies for social keepers. The global network thus consists of almost static zones of social keepers and high volatile clusters of social explorers that, as discussed in the main text, also have important implications in terms of information diffusion.

7 Facebook data set

We have also analyzed other communication data to test our results. In particular, we have studied the 90,269 users of the New Orleans Network crawled during by Viswanath *et al.* [10]. The data consists of communication events between users through Facebook wall from September 26th, 2006 to January 22nd, 2009. Contrary to the mobile phone data, the Facebook data is not steady in time, since the database extends over the early days of Facebook growth and thus it shows a growth in the activity over years, which translates in more wall posts and also more users as a function of time (see Fig. 6).

To minimize this effect we have chosen only communication events between users that did show any activity in the observation window Ω (the time interval between 1000 and 1212 days in the database) and also which were present 20 days before and after Ω. We do not consider the links to be reciprocated in order to have more data accessible for our analysis. With this filter our database contains 125×10^3 communication events of $\sim 10^4$ users and 69×10^3 ties. On average, users interact with $\langle k_i(T) \rangle = 6.15$ users in 7 months and the social activity is $\langle n_{\alpha,i}(T) \rangle = 3.01$, $\langle n_{\omega,i}(T) \rangle = 3.02$ ties formed and decayed respectively. Our results are very similar to the ones observed for mobile phone data, namely that social activity is roughly half of the social connectivity in 7 months. However, users show a lower level of wall activity: for example, 40% of the users are involved in less than 10 communication events through the wall in seven months (while in the mobile phone data the average number of calls exchanged per user was ~ 700 in seven months).



SI Figure 7: Social dynamics in the Facebook database. (a) Relationship between the number of formed $n_{\alpha,i}$ and decayed $n_{\omega,i}$ ties in the observation window for the users in our database. The box plot shows the 25% and 75% percentiles (filled box) and 5% and 95% percentiles (whiskers), the solid black line is the relationship $n_{\alpha,i} = n_{\omega,i}$ and the blue curves correspond to the 5% and 95% percentiles of the corresponding Poisson null model in SI section E for our data. (b) Density plot $\rho(\omega_i, \alpha_i)$ for the users with more than 2 ties formed and decayed. Dashed line is the $\alpha_i = \omega_i$ and the curves correspond to the contour lines $\rho = 0.03$ for the density of actual values of the rates (red) and the ones obtained in the Poissonian model in SI section E (blue). (c) Log-density plot of the social activity $n_{\alpha,i}$ and the social capacity $\overline{\kappa}_i$. Dashed lines correspond to the iso-connectivity lines $k_i(T) = 10, 20, 50$ and the solid line is the relationship $n_{\alpha,i} = 1.04\overline{\kappa}_i$ obtained through PCA that explains 81% of the variance.

Thus, to determine the social dynamical strategies in Facebook data we concentrate on those users that show a moderate level of communication, i.e. those that have more than 10 events in the 7 months of Ω. For those users in our database we find that $n_{\alpha,i}(T) \simeq n_{\omega,i}(T)$ and $\alpha_i \simeq \omega_i$, signaling that users in Facebook tend also to conserve the number of open connections $\kappa_i(t)$ in time (see Fig. 7 (a) and (b)) . On average we find that $\langle \kappa_i(t) \rangle = 3.23$. Finally, as in the mobile phone data we find also a relationship between the capacity and the activity of users: in particular, 81% of the variance can be explain by the relationship $n_{\alpha,i} = 1.04\overline{\kappa}_i$ [see Fig. 7 (c)].

In addition, as in the mobile phone network, we find a large assortativity not only in the social connectivity, but also and more importantly in social dynamical strategies, i.e. individuals with low γ (social keepers) tend to gather in the social network, while social explorers tend to interact between them (see Fig. 8). Our results show that the dynamical strategies of communication between users through Facebook wall also follow the same pattern as in mobile phone.

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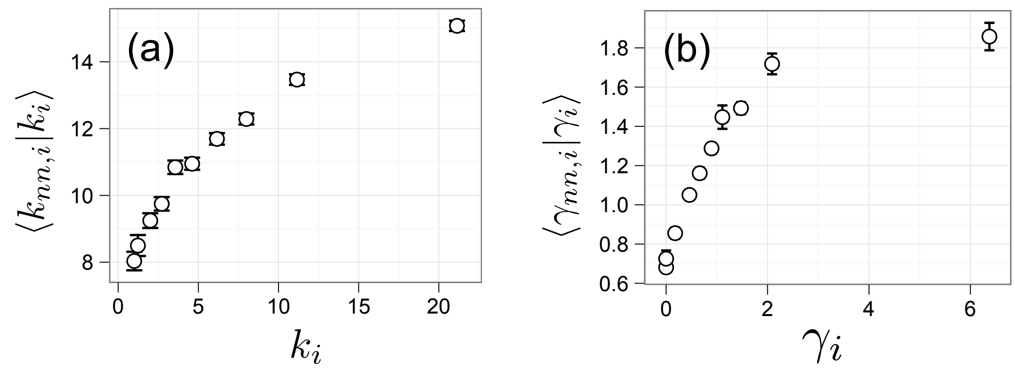

SI Figure 8: Assortativity of connectivity and social strategy in Facebook social network. (a) Average next neighbor connectivity of a node $k_{nn,i}$ as a function her own connectivity k_i for the 10^4 users in the Facebook datase. A clear grown can be seen, signaling an assortativity in this social network, with a Pearson correlation coefficient $\rho(k_i, k_{nn,i}) = 0.257$ with confidence range [0.238,0.275]. (b) Average value of the parameter γ_i for the neighbors of an individual as a function of her own value of γ_i. Similarly to k_i we observe a clear growth and a Pearson correlation coefficient $\rho(\gamma_i, \gamma_{nn,i}) = 0.197$ with confidence range [0.177,0.217]. which indicates a strong assortativity of the social dynamical strategies in the Facebook database.

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