Fusing Enacted and Expected Mimicry Generates a Winning Strategy that Promotes the Evolution of Cooperation

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Appendix 1

The MaRS algorithm

i. The similarity threshold:

For every PD game with the payoffs: T, R, P, S, so that T > R > P > S and $2 * R \geq T + S$

Let p_s ($0 \leq p_s \leq 1$) define the probability that the opponent will choose the same alternative, either cooperate (c) or defect (d).

The expected value for choosing c and d, respectively, is given by
E(c) = $(p_s \times R + (1-p_s) \times S)$, and E(d) = $(p_s \times P + (1-p_s) \times T)$.

Let p_s* denote the threshold level of p_s where E(c) = E(d)

Thus:

(1) $0.5 \leq p_s* = \dfrac{T-S}{T-S+R-P} \leq 1$ and

(2) E(c) ≥ E(d) if and only if $p_s \geq p_s*$

For every repeated PD game with n = 1, …, N moves, where the value of N is unknown, the variables of the MaRS algorithm are defined as follows:

ii. Variables:

Passive memory $\overrightarrow{Mp}$, $\overrightarrow{Mp} \in \{0,1\}^{M_1}$, where 1 denotes corresponding choices, and 0 denotes non-corresponding choices: $\overrightarrow{Mp}$ is a queue updated in a First In First Out (FIFO) manner following each move. The Activation Point, AP_1, of $\overrightarrow{Mp}$ can be set to obtain any value between 1 and M_1 ($1 \leq AP_1 \leq M_1$), below this activation point $\overrightarrow{Mp}$ is defined as EMPTY.

Passive similarity, $p_{sp} = \frac{1}{L}\sum_{m=1}^{L}\overrightarrow{Mp}(m)$, where L is the current size of $\overrightarrow{Mp}$ and $AP_1 \leq L \leq M_1$.

Reactive memory, $\overrightarrow{Mr}$, $\overrightarrow{Mr} \in \{0,1,-1\}^{M_2}$, where 0 denotes opponent's non-reciprocated switch towards cooperation, 1 denotes opponent's reciprocated switch towards cooperation, and -1 denotes opponent's withdrawal from a previously reciprocated switch towards cooperation, by choosing defection.

$\overrightarrow{Mr}$ is updated in a 'First In First Out' manner (FIFO) following each relevant move (the second move when mimicry is expected or following the withdrawal from previously attained cooperative reciprocation).

The Activation Point, AP_2, of $\overrightarrow{Mr}$ can be set to obtain any value between 1 and M_2 ($1 \leq AP_2 \leq M_2$), below this activation point $\overrightarrow{Mr}$ is defined as EMPTY.

Reactive similarity, $p_{sr} = \frac{1}{L}\sum_{m=1}^{L}\overrightarrow{Mr}(m)$, where L is the current size of $\overrightarrow{Mr}$ and $AP_2 \leq L \leq M_2$.

Action (a): a $\in \{cooperate, enact\ mimicry, expect\ mimicry, exclude\ mimicry\}$ where 'expect mimicry' is executed by cooperating in two subsequent moves (present and following) and 'exclude mimicry' is executed by choosing to defect.

Recent Move (r): r $\in \{mutual\ defection, non\ mutual\ defection, empty\}$

iii. Process:

In the following we reassemble the behavior of a MaRS-agent as pseudo-code

```
a := cooperate
update Mp, Mr, p_sp, p_sr

IF Mp NOT EMPTY {
    If p_sp < p_s*
        a := exclude mimicry
    ELSE {
        IF (p_sr ≥ p_s* OR Mr == EMPTY) AND r == mutual defection
            a := expect mimicry
        ELSE{
            IF r != empty
                a := enact mimicry
        }
        END
    }
}
END
RETURN a
```

Appendix 2

A proof of the advantage of the Educational over the 'Robin Hood' approach for changing similarity thresholds

We distinguish among two intervention types. The first, which we term the Robin Hood (RH) approach, and the second, which we term the Educational (Edu.) approach (Figure S3). We show that making a change of x units by applying the Edu. approach reduced the similarity threshold p_s* to a larger extent than making a change of x units by applying the RH approach.

Let $T, R, P, S \in \mathbb{R}$ where $T > R > P > S \geq 0$ as defined by a Prisoner's Dilemma game[1]. The similarity threshold (p_s*) of the game is given by:

$$p_s* = \frac{T-S}{T-S+R-P} \quad (1)$$

Defining $A = T - S,\ \ B = R - P$ and rewriting eq. (1), we get:

$$p_s^* = \frac{A}{A+B}. \quad (2)$$

Now let

$$0 < x < (A - B) \quad (3)$$

and introduce the two approaches:

1. RH, by decreasing A by x units
2. Edu, by increasing B by x units,

i.e.:

In case 1 (RH), $p_s{}^* = \frac{(A-x)}{(A-x)+B}$, and in case 2 (Edu), $p_s{}^* = \frac{A}{A+B+x}$ (4)

It follows trivially by simple algebra that (3) guarantees that $\frac{(A-x)}{(A-x)+B} > \frac{A}{A+B+x}$

[1] Formally a PD game also requires that 2 R > S + T, this constraint is not necessary for the present proof.

Appendix 3

Comparing MaRS, TFT, and WSLS while encountering a random playing agent

Table S1 A repeated game between MaRS and a random playing agent

N	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Random	**C**	**C**	**D**	**D**	**C**	**D**	**C**	**D**	**D**	**C**
MaRS	**C**	**C**	**C**	**D**	**C**	**C**	**D**	**D**	**D**	**D**
Passive Reg.	**1**	**11**	**110**	**1101**	**-**	**11010**	**110100**	**1101001**	**1010011**	**0100110**
P_{sp}	**1.000**	**1.000**	**0.667**	**0.750**	**-**	**0.600**	**0.500**	**0.571**	**0.571**	**0.428**
Reactive Reg.	**-**	**-**	**-**	**-**	**-**	**0**	**-**	**-**	**-**	**-**
P_{sr}	**-**	**-**	**-**	**-**	**-**	**0.000**	**-**	**-**	**-**	**-**
Mimicry Type for n +1		**Enacted**	**Excluded**	**Expected**	**Expected**	**Excluded**	**Excluded**	**Excluded**	**Excluded**	**Excluded**
Comments	**First step**				**1^{st}** -not registered	**2^{nd} Reactive sim. reject**				
					Gains					
Random	**3**	**3**	**5**	**1**	**3**	**5**	**0**	**1**	**1**	**0**
MaRS	**3**	**3**	**0**	**1**	**3**	**0**	**5**	**1**	**1**	**5**

N	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Random	**C**	**D**	**C**	**D**	**D**	**D**	**D**	**D**	**C**	**C**
MaRS	**D**	**D**	**D**	**D**	**D**	**D**	**D**	**C**	**C**	**C**
Passive Reg.	**1001100**	**0011001**	**0110010**	**1100101**	**1001011**	**0010111**	**0101111**	**-**	**1011111**	**0111111**
P_{sp}	**0.428**	**0.428**	**0.428**	**0.571**	**0.571**	**0.571**	**0.714**	**-**	**0.857**	**0.857**
Reactive Reg.	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**01**	
P_{sr}	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**0.5**	
Mimicry Type for n +1	**Excluded**	**Excluded**	**Excluded**	**Excluded**	**Excluded**	**Excluded**	**Expected**	**Expected**	**Enacted**	**Enacted**
Comments								**1^{st} Not** registered	**2^{nd} Reactive sim. Conf.**	
					Gains					
Random	**0**	**1**	**0**	**1**	**1**	**1**	**1**	**5**	**3**	**3**
MaRS	**5**	**1**	**5**	**1**	**1**	**1**	**1**	**0**	**3**	**3**

N	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Random	**C**	**D**	**D**	**C**	**C**	**C**	**D**	**D**	**D**	**C**
MaRS	**C**	**C**	**D**	**D**	**C**	**C**	**C**	**D**	**D**	**C**
Passive Reg.	**1111111**	**1111110**	**1111101**	**1111010**	**1110101**	**1101011**	**1010110**	**0101101**	**1011011**	**-**
P_{sp}	**1**	**0.857**	**0.857**	**0.714**	**0.714**	**0.714**	**0.571**	**0.571**	**0.714**	**-**
Reactive Reg.	**-**	**01-1**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**
P_{sr}	**-**	**0**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	
Mimicry Type for n +1	**Enacted**	**Enacted**	**Enacted**	**Enacted**	**Enacted**	**Enacted**	**Excluded**	**Excluded**	**Expected**	**Expected**
Comments		**Reactive sim. reduced**								**1^{st} Not** registered
					Gains					
Random	**3**	**5**	**1**	**0**	**3**	**3**	**5**	**1**	**1**	**3**
MaRS	**3**	**0**	**1**	**5**	**3**	**3**	**0**	**1**	**1**	**3**

N	31	32	33	34	35	36	37	38	39	40
Random	C	D	C	D	C	D	C	C	C	C
MaRS	C	C	D	D	D	D	D	D	D	D
Passive Reg.	0110111	1101110	1011100	0111001	1110010	1100101	1001010	0010100	101000	010000
P_{sp}	0.714	0.714	0.571	0.571	0.571	0.571	0.429	0.286	0.286	0.143
Reactive Reg.	01-11	01-11-1	-	-	-	-	-	-	-	-
P_{sr}	0.25	0	0	0	0	0	0	0	0	0
Mimicry Type for n +1	Enacted	Enacted	Excluded	Excluded	Excluded	Excluded	Excluded	Excluded	Excluded	Excluded
Comments	2nd Reactive sim. conf.	Reactive sim. reduced								
Gains										
Random	3	5	0	1	0	1	0	0	0	0
MaRS	3	0	5	1	5	1	5	5	5	5

N	41	42	43	44	45	46	47	48	49	50
Random	C	D	C	C	C	D	C	C	C	D
MaRS	D	D	D	D	D	D	D	D	D	D
Passive Reg.	0000000	0000001	0000010	0000100	0001000	0010001	0100010	1000100	0001000	0010001
P_{sp}	0	0.143	0.143	0.143	0.143	0.286	0.286	0.286	0.143	0.286
Reactive Reg.	-	-	-	-	-	-	-	-	-	-
P_{sr}	0	0	0	0	0	0	0	0	0	0
Mimicry Type for n +1	Excluded	Excluded	Excluded	Excluded	Excluded	Excluded	Excluded	Excluded	Excluded	Excluded
Comments										
Gains										
Random	3	5	0	3	3	5	0	3	3	5
MaRS	3	0	5	3	3	0	5	3	3	0
Total Random	76									
Total MaRS	136									
MaRS's Advantage	60									

A repeated PD game of MaRS vs. a random playing strategy (T = 5, R = 3, P = 1, S = 0; and the similarity threshold used by MaRS is $p_s^* = 0.714$). Cooperation and Defection are indicated as C and D. MaRS is equipped with two memory registries, a passive registry (Passive Reg.) and a reactive registry (Reactive Reg.), each containing 7 items, which are updated in a First In First Out (FIFO) manner. The passive registry accumulates values of 1and 0, for games where both opponents chose an identical or a different move, respectively. It is used to compute the passive similarity index, p_{sp}, by dividing the sum of the inputs by the overall number of input units (obtaining values of 1 to 6 for the first 6 games and being fixed on 7 for the rest of the game). The reactive registry accumulates values of 1, 0, and -1, the first two are derived from instances in which the opponent either reciprocated or avoided reciprocating cooperation after it was initiated by MaRS. The value of -1 is used only for instances where the opponent first responded by reciprocating cooperation, yet deviated from the achieved mutual cooperation in one of the following games (hence reveling the fact that he does not really deserve the 1 he got for reciprocating cooperation). In the present example MaRS consults p_{sp} from the second game, yet consults p_{sr} only after it has recorded 5 instances (see game 31), hence attempting to avoid hasty conclusions, that may block MaRS's attempts to propose cooperation and expect mimicry. MaRS can of course have various mutants that function with a number of memory spans and activation points, each being optimal in a specific environment. After calculating the similarity threshold of the game, $p_s^* = (T - S) / (T – S + R - P)$, and making a first move of cooperation, MaRS makes three types of decisions: 1) if $p_{sp} < p_s^*$ it *excludes* mimicry and defects (for example, decisions made on games: 3,7, 8,9, or 33 to 50) ; 2) if $p_{sp} \geq p_s^*$ **and** both opponents have defected in the previous game, **and** $p_{sr} \geq p_s^*$ **or** p_{sr} is not yet being used for making decisions (before game 32), it *expects* mimicry by cooperating twice (the first instance signals cooperation and its similarity outcomes are not registered, while the second instance enables the opponent to respond to the expectation by choosing to cooperate, hence confirming (conf.) or rejecting (reject) the expectation for similarity, as in games: 5,6, 18,19, 30,31); 3) in all other cases (where $p_{sp} \geq p_s*$) MaRS *enacts* mimicry by copying the last move of the opponent. Note that the decision for all types of mimicry is made following the results of game n, yet it is implemented in game n + 1 (and also n + 2 in the case of *expected* mimicry).

Table S2 A repeated game between TFT and a random playing agent

N	1	2	3	4	5	6	7	8	9	10
Random	C	C	D	D	C	D	C	D	D	C
TFT	C	C	C	D	D	C	D	C	D	D
Gains										
Random	3	3	5	1	0	5	0	5	1	0
TFT	3	3	0	1	5	0	5	0	1	5

N	11	12	13	14	15	16	17	18	19	20
Random	C	D	C	D	D	D	D	D	C	C
TFT	C	C	D	C	D	D	D	D	D	C
Gains										
Random	3	5	0	5	1	1	1	1	0	3
TFT	3	0	5	0	1	1	1	1	5	3

N	21	22	23	24	25	26	27	28	29	30
Random	C	D	D	C	C	C	D	D	D	C
TFT	C	C	D	D	C	C	C	D	D	D
Gains										
Random	3	5	1	0	3	3	5	1	1	0
TFT	3	0	1	5	3	3	0	1	1	5

N	31	32	33	34	35	36	37	38	39	40
Random	C	D	C	D	C	D	C	C	C	C
TFT	C	C	D	C	D	C	D	C	C	C
Gains										
Random	3	5	0	5	0	5	0	3	3	3
TFT	3	0	5	0	5	0	5	3	3	3

N	41	42	43	44	45	46	47	48	49	50
Random	C	D	C	C	C	D	C	C	C	D
TFT	C	C	D	C	C	C	D	C	C	C
Gains										
Random	3	5	0	3	3	5	0	3	3	5
TFT	3	0	5	3	3	0	5	3	3	0
Total Random	122									
Total TFT	117									
TFT's Advantage	-5									

A repeated PD game of Tit For Tat (TFT) vs. a random playing strategy with the same sequence of random moves shown in Table S1 (T = 5, R = 3, P = 1, S = 0). TFT cooperates in the first move and continues by mimicking the last move of the opponent.

Table S3 A repeated game between WSLS and a random playing agent

N	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Random	**C**	**C**	**D**	**D**	**C**	**D**	**C**	**D**	**D**	**C**
WSLS	**C**	**C**	**C**	**D**	**C**	**C**	**D**	**D**	**C**	**D**
Gains										
Random	**3**	**3**	**5**	**1**	**3**	**5**	**0**	**1**	**5**	**0**
WSLS	**3**	**3**	**0**	**1**	**3**	**0**	**5**	**1**	**0**	**5**

N	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Random	**C**	**D**	**C**	**D**	**D**	**D**	**D**	**D**	**C**	**C**
WSLS	**D**	**D**	**C**	**C**	**D**	**C**	**D**	**C**	**D**	**D**
Gains										
Random	**0**	**1**	**3**	**5**	**1**	**5**	**1**	**5**	**0**	**0**
WSLS	**5**	**1**	**3**	**0**	**1**	**0**	**1**	**0**	**5**	**5**

N	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Random	**C**	**D**	**D**	**C**	**C**	**C**	**D**	**D**	**D**	**C**
WSLS	**D**	**D**	**C**	**D**	**D**	**D**	**D**	**C**	**D**	**C**
Gains										
Random	**0**	**1**	**5**	**0**	**0**	**0**	**1**	**5**	**1**	**3**
WSLS	**5**	**1**	**0**	**5**	**5**	**5**	**1**	**0**	**1**	**3**

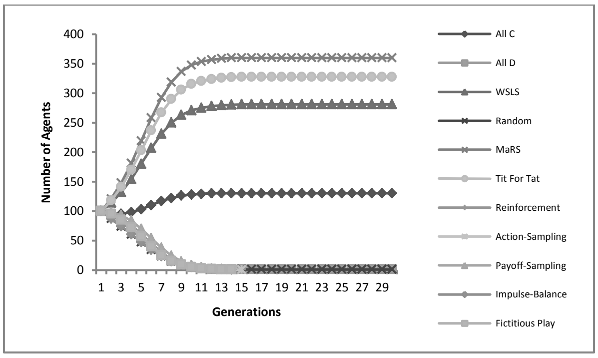
N	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Random	**C**	**D**	**C**	**D**	**C**	**D**	**C**	**C**	**C**	**C**
WSLS	**C**	**C**	**D**	**D**	**C**	**C**	**D**	**D**	**D**	**D**
Gains										
Random	**3**	**5**	**0**	**1**	**3**	**5**	**0**	**0**	**0**	**0**
WSLS	**3**	**0**	**5**	**1**	**3**	**0**	**5**	**5**	**5**	**5**

N	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
Random	**C**	**D**	**C**	**C**	**C**	**D**	**C**	**C**	**C**	**D**
WSLS	**D**	**D**	**C**	**C**	**C**	**C**	**D**	**D**	**D**	**D**
Gains										
Random	**0**	**1**	**3**	**3**	**3**	**5**	**0**	**0**	**0**	**1**
WSLS	**5**	**1**	**3**	**3**	**3**	**0**	**5**	**5**	**5**	**1**
Total Random	**96**									
Total WSLS	**131**									
WSLS's Advantage	**35**									

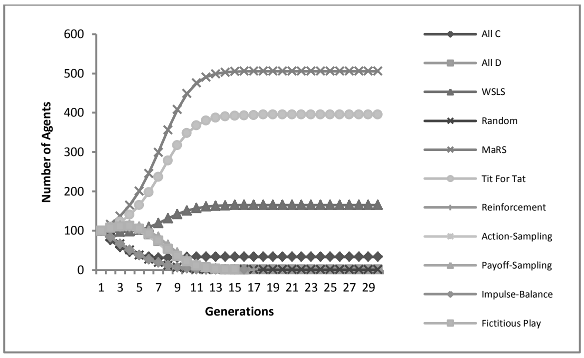
A repeated PD game of Win-Stay, Lose - Shift (WSLS) vs. a random playing strategy with the same sequence of random moves shown in Tables S1 and S2 (T = 5, R = 3, P = 1, S = 0). WSLS cooperates in the first move and proceeds by repeating its previous move if it resulted in the t or the r payoffs; otherwise, if it obtained the p or the s payoff it switches its previous choice.

Figure S1

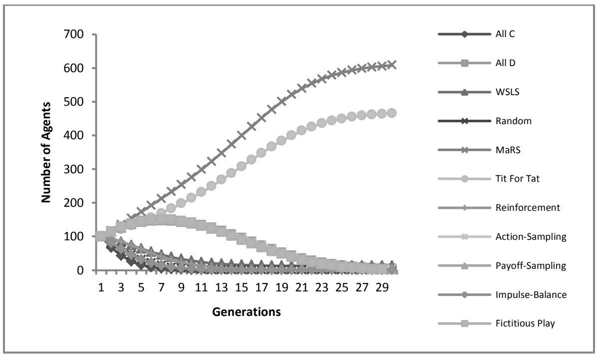
A



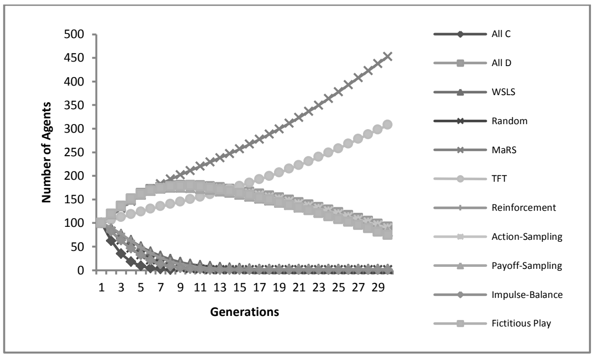
B



C

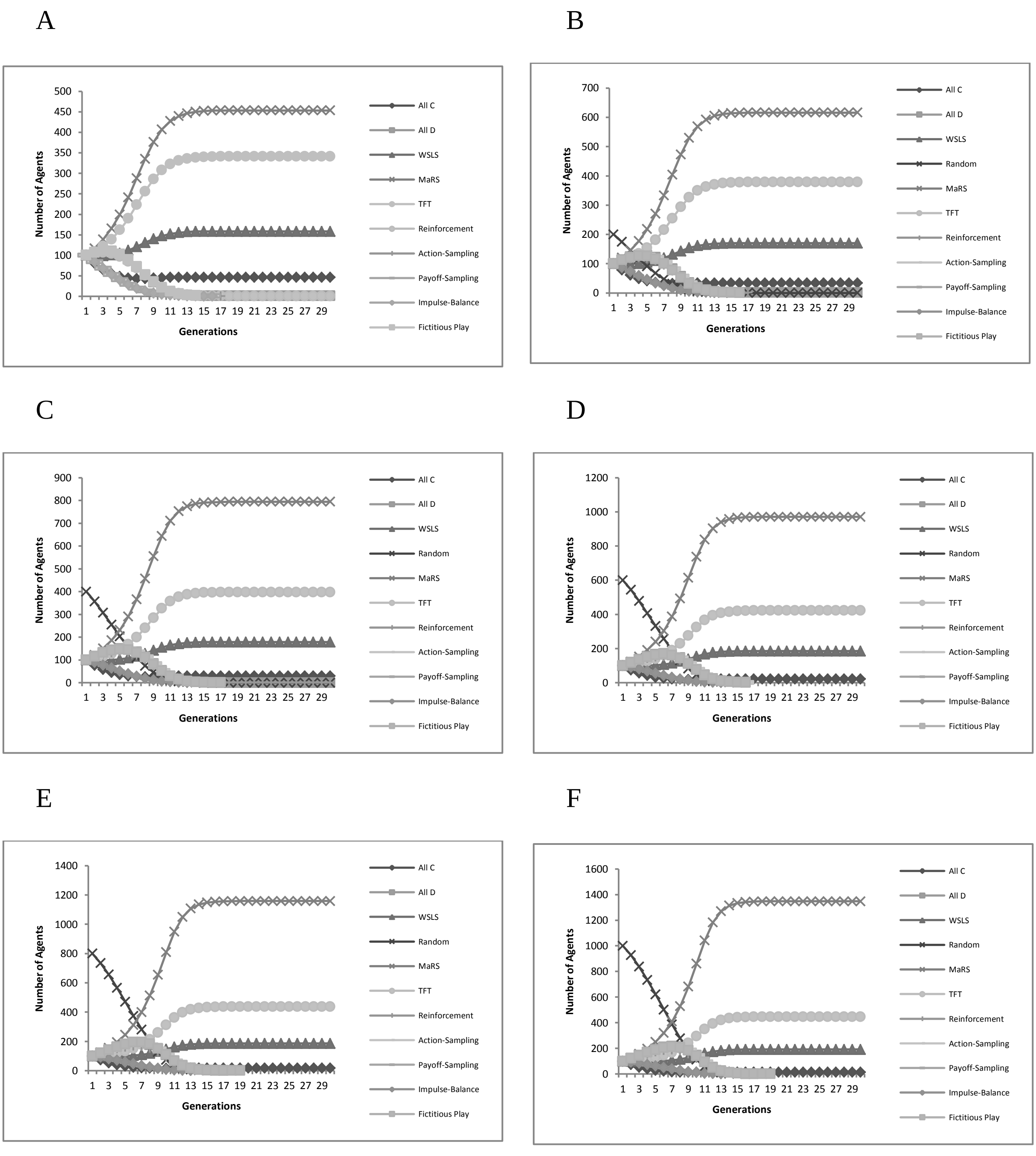


D



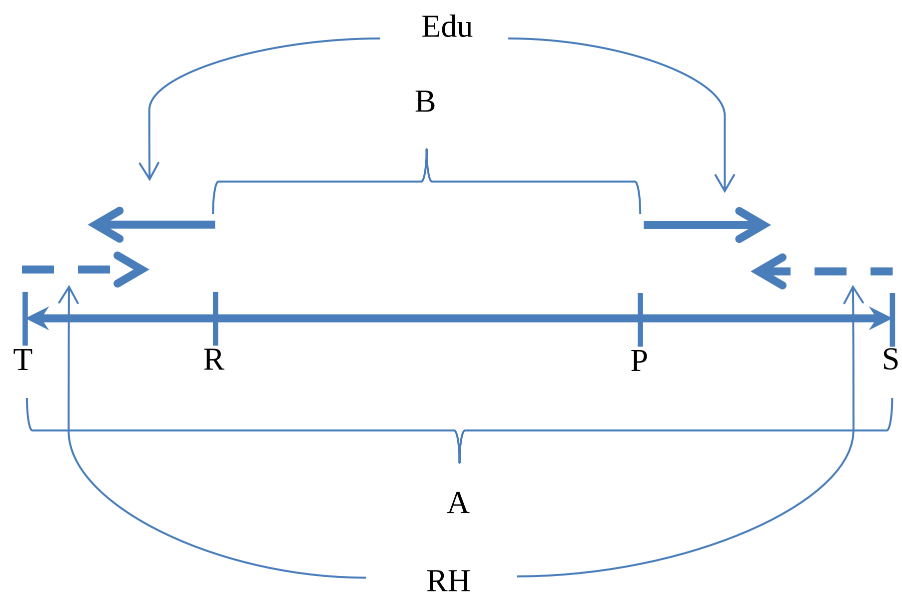
Evolutionary simulations of four behavioral niches initially populated with eleven behavioral strategies. The simulations are conducted with the same parameters as those depicted in Figure 2, but each simulation is run under a different PD matrix and its respective similarity threshold, p_s*. The payoffs and similarity thresholds for the four panels are as follows: (A) T = 20, R = 16, P = 4, S = 1, p_s = 0.61; (B) T = 5, R = 3, P = 1, S = 0, p_s = 0.71;(C) T = 20, R = 16, P = 12, S = 1, p_s = 0.83; and (D) T = 20, R = 14, P = 12, S = 1, p_s = 0.90. Each panel consists of the averages over 100 evolutionary runs with identical initial conditions.

Figure S2



Evolutionary simulations of six behavioral niches initially populated with eleven behavioral strategies (apart from panel a, which comprises ten strategies). The simulations are conducted with the same parameters as those described in Figure 3. All niches are initialized with a population that contains 100 agents from each behavioral strategy; however the initial amount of random playing agents differs. Panels A to F comprise: 0, 200, 400, 600, 800, and 1000 randomly behaving agents, respectively. Each panel consists of the averages over 100 evolutionary runs with identical initial conditions.

Figure S3



The four variables of the PD game, $T > R > P > S$, described by two intervals, A = T – S and B = R – P. The RH approach aims to reduce the similarity threshold of the game ($p_s^* = (T - S) / (T - S + R - P)$) by reducing A, while the Edu approach aims to reduce the similarity threshold by extending B.