Quantum-state transfer from an ion to a photon

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Supplementary Information

Cavity parameters The coherent ion-cavity coupling rate is $g = 2\pi \times 1.4$ MHz. The cavity field decay rate is $\kappa = 2\pi \times 0.05$ MHz, and the atomic polarization decay rate from the $P_{3/2}$ state is $\gamma = 2\pi \times 11.5$ MHz.The effective coupling strength of the two Raman transitions $i = 1, 2$ is given by $\Omega_i^{\text{eff}} = 2\pi \times \frac{gG_i\Omega_i}{2\Delta_i}$, where G_i is a geometric factor that takes into account both the relevant Clebsch-Gordon coefficients and the projection of the vacuum-mode polarization onto the atomic dipole moment[1]. The detunings $\Delta_{1,2}$ are approximately 400 MHz. In order to preserve the amplitudes of the initial state during mapping, we ensure equal transition probabilities for the two Raman transitions by setting the ratio of the Rabi frequencies $\Omega_1/\Omega_2 = G_2/G_1 = 2$, with absolute values $\Omega_1 = 2\pi \times 17.5$ MHz and $\Omega_2 = 2\pi \times 8.75$ MHz.

The polarization modes of the cavity are degenerate, that is, we observe no birefringent splitting in the cavity linewidth κ, with measurements sensitive to a tenth of the linewidth. We hypothesize that this degeneracy is due to the mounting of the cavity mirrors in circular frames[2], which distribute the mechanical stress of the mounting homogeneously across the mirror. In other experiments using high-finesse cavities, a similar mounting strategy has minimized birefringence[3,4]. This degeneracy avoids entanglement of the polarization with the frequency degree of freedom of the photon.

Data analysis In order to characterize the mapping process, we carry out process tomography. For this purpose, we perform state tomography of the photonic output state for four orthogonal atomic input states[5]. Each state tomography of a single photonic polarization qubit consists of measurements in the three bases H/V, D/A, and R/L. Note that H and V correspond to the two cavity modes, where V is parallel to the $\hat{z}$ axis of Fig. 1a in the main text and H to the $\hat{x}$ axis. (In contrast, in Ref. [6], we defined H rather than V to be parallel to the magnetic field axis.) In each basis, we perform two measurements of equal duration in which the output paths to avalanche photodiodes APD1 and APD2 are swapped by rotating waveplates L/2 and L/4. Summing these measurements allows us to compensate for unequal detection efficiencies in the two paths[6].

The process and density matrices plotted in Fig. 2a and b in the main text are reconstructed from the data using a maximum likelihood fit[7]. We extract fidelities and their statistical uncertainties via non-parametric bootstrapping assuming a multinomial distribution[8]. Statistical uncertain-

ties are stated as one standard deviation. The data consist of $32,368$ single-photon detection events for the detection time window of 55 μs. Within the same time window, we detect 52 two-photon events, consistent with dark counts and afterpulsing of the detectors.

Time independence If the atomic qubit is comprised of nondegenerate states, Larmor precession will change the qubit's phase over time. For a monochromatic mapping protocol in which the photonic qubit is encoded solely in polarization, the phase of the photonic qubit thus depends on the time of photon generation[9]. In contrast, for the two Raman fields $\Omega_1 e^{i\omega_{l_1} t}$ and $\Omega_2 e^{i\omega_{l_2} t}$ at frequencies ω_{l_1} and ω_{l_2}, the mapping pulse can be applied at any time for the correct choice of frequency difference between the two Raman fields $\omega_{l_1} - \omega_{l_2} = \Delta E_{S,S'}/\hbar$, where $\Delta E_{S,S'}$ is the Zeeman splitting between the two qubit states S and S'. In this case, the atomic qubit is always mapped to the same photonic qubit state, independent of the photon generation time.

To show this, we define a model system consisting of initial states $|S,n\rangle$, $|S',n\rangle$, intermediate states $|P,n\rangle$, $|P',n\rangle$ and target state $|D,n\rangle$ with energies $E_{S,S',P,P',D} = \hbar\omega_{S,S',P,P',D}$. Here, $n = 0,1$ denotes the number of photons in either of the two degenerate cavity modes at energy $\hbar\omega_C$. A similar model system was used to explain the time independence of the bichromatic entanglement protocol that we recently demonstrated[6]. The $|S,n\rangle \leftrightarrow |P,n\rangle$ transition is driven by the field $\Omega_1 e^{i\omega_{l_1} t}$ with detuning $\Delta_{l_1} = \omega_S - \omega_P - \omega_{l_1}$, while the $|S',n\rangle \leftrightarrow |P',n\rangle$ transition is driven by the field $\Omega_2 e^{i\omega_{l_2} t}$ with detuning $\Delta_{l_2} = \omega_{S'} - \omega_{P'} - \omega_{l_2}$. We choose a unitary transformation into a rotating frame that takes into account the atomic precession at frequency $\omega_S - \omega_{S'}$:

$U = e^{-i\omega_{l_1} t\,|S\rangle\langle S|}\; e^{-i\omega_{l_2} t\,|S'\rangle\langle S'|}$. After this transformation and adiabatic elimination of the state $|P, n\rangle$, the Hamiltonian reads

$$\begin{aligned}\mathcal{H}/\hbar =&(\omega_S + \omega_{l_1})\,|S\rangle\langle S| + (\omega_{S'} + \omega_{l_2})\,|S'\rangle\langle S'| + \omega_{P'}\,|P'\rangle\langle P'| + \omega_D\,|D\rangle\langle D| \\ &+ \omega_C\,|1\rangle\langle 1| + \left(g_1^{\text{eff}}\,|D,1\rangle\langle S,0| + g_2^{\text{eff}}\,|D,1\rangle\langle S',0| + \text{h.c.}\right), \end{aligned} \tag{1}$$

where the energy reference is the $|P, 0\rangle$ state ($\omega_P = 0$). Both couplings $g_i^{\text{eff}} = \frac{\Omega_i \cdot g}{2\Delta_{l_i}}$ are time-independent. Choosing the frequencies of the two fields to match the two Raman conditions $\omega_S + \omega_{l_1} = \omega_D + \omega_C = \omega_{S'} + \omega_{l_2}$ corresponds to a frequency difference $|\omega_{l_1} - \omega_{l_2}| = |\omega_S - \omega_{S'}|$. The two states $|S, 0\rangle$ and $|S', 0\rangle$ are degenerate in this frame, resulting in a constant phase of the atomic state. As the couplings are also time-independent, the phase φ of the atomic state does not change during the transfer to the photonic state (equation 1 of the main text). As both modes of the cavity are degenerate, the phase φ of the photonic state remains constant after the transfer.

We initially proposed a mapping scheme for calcium ions in Ref. [1] in which a σ^--polarized drive beam would couple both $|S\rangle$ and $|S'\rangle$ to the state $|3^2D_{5/2}, m_J = -3/2\rangle$. The drive beam of that scheme, however, would introduce a differential AC-Stark shift between the two qubit states. This shift would have to be compensated in order to preserve the time-independence of the mapping process. As such compensation appears challenging in the experiment, we developed the present scheme. Here, the π-polarized drive beam avoids differential AC-Stark shifts between the qubit states.

So far, we have neglected off-resonant Raman transitions, i.e., Ω_1 coupling $|S', n\rangle$ to $|P', n\rangle$

and Ω_2 coupling $|S,n\rangle$ to $|P,n\rangle$. Taking these couplings into account, the terms g_i^{eff} in the Hamiltonian are proportional to $\Omega_i + \Omega_j e^{i(\omega_{l_j}-\omega_{l_i})t}$ after transformation into the rotating frame and adiabatic elimination of $|P,n\rangle$. Here, the second term, oscillating at $|\omega_{l_i} - \omega_{l_j}|$, corresponds to off-resonant Raman transitions in which a photon with unwanted polarization is generated. These terms are neglected in the rotating wave approximation because $|g_i^{\text{eff}}| \ll |\omega_{l_1} - \omega_{l_2}|$. These off-resonant coupling terms, however, explain why the fidelity of the mapping process only reaches its maximum after about 3 μs (Fig. 3 of the main text). As confirmed by our simulations, the off-resonant Raman processes generate photons with unwanted polarization at the timescale of 100 ns after turning on the drive laser pulse. However, the probability for this process is very low due to the large detuning from Raman resonance, and the effect is quickly overcome by the much higher probability of generating photons with the desired polarization thereafter.

Simulations Numerical simulations of the state-mapping process are based on the Quantum Optics and Computation Toolbox for MATLAB[10]. We formulate the master equation for the 18-level $^{40}Ca^{+}$ system interacting with two orthogonal modes of an optical cavity. We then numerically integrate the master equation to obtain the system's density matrix as a function of time. The simulation includes atomic and cavity decay and the laser linewidth. Relative motion of the ion with respect to the cavity mode is taken into account by introducing an effective atom-cavity coupling g_{motion} smaller than g. This motion results from the (presumably mechanical) oscillation of the ion trap with respect to the cavity[1]. Furthermore, small effects such as finite switching time of the laser, laser-amplitude noise and relative phase noise are neglected in the model.

The simulations require us to specify the input parameters: magnetic field B, Raman-laser frequencies ω_{l_1} and ω_{l_2}, photon detection path efficiency, and Rabi frequencies Ω_1 and Ω_2, as well as system parameters g, κ, and γ. B and the laser frequencies are determined from spectroscopy of the quadrupole transition to within 3 kHz. A detection path efficiency of 6.8% is used to scale the simulation results, consistent with previous measurements[1]. Ω_1 and Ω_2 are determined experimentally via Stark-shift measurements with an uncertainty on the order of 20%. However, the temporal shape of the photons is highly dependent on Ω_1 and Ω_2 and on the atom-cavity coupling g. In the simulation, we therefore adjust Ω_1 and Ω_2 within the experimental uncertainty range and find that the values $\Omega_1 = 2\pi \times 17.5$ MHz and $\Omega_2 = 2\pi \times 8.75$ MHz generate photon shapes that have the best agreement with data. In order to improve this agreement, we adjust g to the effective value $g_{\mathrm{motion}} = 0.6g$, consistent with calibration measurements that indicate the ion was not optimally coupled to the cavity field.

As discussed in the main text and presented in Fig. 2c, there are eight combinations of initial state and detection basis for which the temporal photon shapes on both detectors are identical. However, in two of these eight cases, the simulated photon shapes in the two polarization modes do not overlap perfectly with one another. This small discrepancy occurs for the $|S + iS'\rangle$ initial state and the D/A basis as well as for the $|S - S'\rangle$ state and the R/L basis, and it is due to errors that accumulate during the numerical integration routine.

For each detection time window in Fig. 3 in the main text, we estimate the relative contribu-

tions of APD dark counts and data, and the simulated density matrices are weighted accordingly. Additionally, off-diagonal matrix terms are scaled by a factor of 0.99 representing imperfect state initialization and by the exponential $e^{-2t/\tau}$, where $\tau = 110$ μs is the atomic coherence time.

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