Appendix A

In this appendix we describe the boundary conditions implemented in the planar finite element models. Both the fibers and the tendon undergo stretch along the line of pull (y direction), while fibers also undergo in-plane shear (y -z plane in Fig. 3) due to pennation. We therefore represent the average or macroscopic deformation of the tissue with the following isochoric deformation tensor:

$$\mathbf{F} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \Lambda_y & k \\ 0 & 0 & \Lambda_z \end{pmatrix}, \tag{A.1}$$

where Λ_y is stretch in the y direction, k is shear in the y-z plane, and $\Lambda_z = 1/\Lambda_y$ is the stretch in the z direction, so that the deformation is isochoric. This deformation tensor represents plane strain conditions. The values of Λ_y, Λ_z and k were defined to simulate stretching of pennate fibers and are agreement with our observations of the histological sections of muscles fixed in the stretched position.

The muscle fiber direction can be defined as $a_0 = \begin{pmatrix} 0 & -\cos\theta & \sin\theta \end{pmatrix}^T$, where θ is the pennation angle (Fig. 3A). The average stretch in the fiber due to a deformation defined by $\mathbf{F}$ is $\Lambda = \sqrt{a_0 \mathbf{C} a_0}$, where $\mathbf{C} = \mathbf{F}^T\mathbf{F}$ is the right Cauchy-Green deformation tensor (Holzapfel, 2000) and Λ is the ratio of the final length of the fiber to its initial length. Λ can be expanded as:

$$\Lambda = \sqrt{\Lambda_y^2 \cos^2\theta - 2k\Lambda_y \cos\theta \sin\theta + \left(k^2 + \frac{1}{\Lambda_y^2}\right)\sin\theta^2}, \tag{A.2}$$

and is the macroscopic stretch of the fiber. The macroscopic tensile strain is defined as $\Lambda - 1$. For a stretch of Λ= 1.24 we used k = -0.4 and $\Lambda_y = 1.08$. We used k = -0.18 and $\Lambda_y = 1.037$ to produce an overall stretch of Λ= 1.10.

The periodicity assumption is based on the idealization that a macroscopic region of muscle can be assumed to be composed of a large number of fibers with the exact same geometry. All neighboring fibers therefore undergo the exact same deformation. This assumption requires that the opposite edges of the model have the same shape and maintain the same shape throughout the deformation (Fig. 3A) (Drago and Pindera, 2007; Smit et al., 1998). This is referred to as a periodic boundary condition and is enforced via (Sharafi and Blemker, 2010):

$$\mathbf{x}\left(\mathbf{X}_0+\mathbf{d}\right)=\mathbf{x}\left(\mathbf{X}_0\right)+\mathbf{F}\cdot\mathbf{d}, \tag{A.3}$$

where $\mathbf{X}_0$ and $\mathbf{X}_0+\mathbf{d}$ are the initial positions of two mirror points on two opposite edges (Fig. 9) whose coordinates in the final configuration are $\mathbf{x}\left(\mathbf{X}_0\right)$ and $\mathbf{x}\left(\mathbf{X}_0+\mathbf{d}\right)$.

The periodicity assumption does not hold in the z direction, therefore homogenous boundary conditions (Drago and Pindera, 2007) are imposed on the two edges that are orthogonal to the z direction, i.e. the tapered end of the fiber attaching to the tendon as well the opposite end:

$$\mathbf{x}=\mathbf{F}\cdot\mathbf{X}, \tag{A.4}$$

where $\mathbf{X}$ is the position of each point on these two edges in the undeformed configuration and $\mathbf{x}$ defines the position vector in the deformed configuration.

The plane defined by $x = 0$ is the plane of symmetry. For all the nodes on this plane $u_x = 0$, where u_x is the displacement in the x direction. The model is one element thick in the x direction and all the nodes on the two planes orthogonal to the x axis are constrained to have the same displacements in the y and z directions. The thickness of the elements in the x direction, t, was chosen to optimize the mesh and did not affect the finite element solution. The plane orthogonal to the x axis at $x = t$ was additionally constrained to remain flat during the deformation.

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