

Table 1. Mean values of  $PCF$  and  $PNF$  estimated using observed risks  $R$  in a population assuming that the model is well calibrated; risk estimates in a case-control sampling when disease prevalence  $\mu$  is known, and based on observations of  $(R, Y)$  in the population. Results are based on 500 simulations for each set of parameters  $(\alpha, \beta)$  for the beta distribution and values of  $q$  and  $p$ . Each simulation has  $N = 10000$  samples with  $\mu = 0.3$ .  $ARE$ s are computed as the ratio of the influence function based variances.

$p$	$PCF$	$\widehat{PCF}$			$var(\widehat{PCF})$			$ARE^{\S}$		
	true	$R$	$CC$	$(R, Y)$	$R$	$CC$	$(R, Y)$	$CC/R$	$(R, Y)/R$	$(R, Y)/CC$
$\alpha = 6.55, \beta = 15.28^*$										
0.10	0.16	0.16	0.16	0.16	0	0.24	0.25	71.02	71.80	1.01
0.20	0.29	0.29	0.29	0.29	0.01	0.42	0.42	63.48	64.34	1.01
0.30	0.42	0.42	0.42	0.42	0.01	0.52	0.53	61.40	62.26	1.01
0.40	0.53	0.53	0.53	0.53	0.01	0.56	0.57	61.20	62.03	1.01
$\alpha = 1, \beta = 2.33^{**}$										
0.10	0.25	0.25	0.25	0.25	0.02	0.20	0.24	8.82	10.63	1.21
0.20	0.43	0.43	0.43	0.43	0.04	0.38	0.44	8.70	10.15	1.17
0.30	0.58	0.58	0.58	0.58	0.05	0.47	0.54	9.21	10.41	1.13
0.40	0.70	0.70	0.70	0.70	0.05	0.48	0.52	10.00	10.95	1.09
$\alpha = 0.3, \beta = 0.701^{***}$										
0.10	0.31	0.31	0.31	0.31	0.08	0.06	0.22	0.77	2.63	3.43
0.20	0.56	0.56	0.56	0.56	0.17	0.24	0.50	1.45	3.03	2.10
0.30	0.75	0.74	0.75	0.74	0.15	0.34	0.53	2.31	3.57	1.55
0.40	0.87	0.87	0.87	0.87	0.09	0.30	0.38	3.50	4.46	1.27
$q$	$PNF$	$\widehat{PNF}$			$var(\widehat{PNF})$			$ARE$		
	true	$R$	$CC$	$(R, Y)$	$R$	$CC$	$(R, Y)$	$CC/R$	$(R, Y)/R$	$(R, Y)/CC$
$\alpha = 6.55, \beta = 15.28^*$										
0.90	0.82	0.82	0.82	0.82	0.01	0.50	0.51	77.19	77.64	1.01
0.80	0.69	0.69	0.69	0.69	0.01	0.61	0.61	67.99	68.55	1.01
0.70	0.58	0.58	0.58	0.58	0.01	0.61	0.62	64.54	65.20	1.01
0.60	0.47	0.47	0.47	0.47	0.01	0.56	0.57	59.14	59.86	1.01
$\alpha = 1, \beta = 2.33^{**}$										
0.90	0.64	0.64	0.64	0.64	0.05	0.78	0.81	14.68	15.22	1.04
0.80	0.50	0.50	0.50	0.50	0.05	0.57	0.60	11.27	12.04	1.07
0.70	0.40	0.40	0.40	0.40	0.04	0.40	0.44	9.94	10.92	1.10
0.60	0.31	0.31	0.31	0.31	0.03	0.28	0.32	9.22	10.37	1.13
$\alpha = 0.3, \beta = 0.7^{***}$										
0.90	0.44	0.44	0.44	0.44	0.11	0.44	0.53	4.19	5.05	1.21
0.80	0.34	0.34	0.34	0.34	0.08	0.22	0.31	2.75	3.89	1.42
0.70	0.27	0.27	0.27	0.27	0.06	0.12	0.20	2.02	3.36	1.66
0.60	0.22	0.22	0.22	0.22	0.04	0.06	0.13	1.57	3.10	1.97

\* AUC=0.63, \*\* AUC=0.79 \*\*\* AUC= 0.92

$\S$ ARE = asymptotic relative efficiency, the ratio of the influence functions based variances

$CC/R = \text{var}(T_{CC})/\text{var}(T_R)$ ,  $(R, Y)/R = \text{var}(T_{(R, Y)})/\text{var}(T_R)$ ,  $(R, Y)/CC = \text{var}(T_{(R, Y)})/\text{var}(T_{CC})$ , where  $T = PCF$  or  $T = PNF$  respectively

Table 2. Mean values of  $iPCF$  and  $iPNF$  estimated using: the population risk distribution  $F(R)$  assuming that the model is well calibrated; risks in a case-control study and known disease prevalence  $\mu$ ; observations of  $(R, Y)$  in the population. Results are based on 500 simulations for each set of parameters  $(\alpha, \beta)$  for the beta distribution and values of  $q$  and  $p$ . Each simulation has  $N = 10000$  samples with  $\mu = 0.3$ .  $ARE$ s are computed as the ratio of the influence function based variances.

$p^*$	$iPCF$	$\widehat{iPCF}$			$var(\widehat{iPCF})$			$ARE$		
	true	$R$	$CC$	$(R, Y)$	$R$	$CC$	$(R, Y)$	$CC/R$	$(R, Y)/R$	$(R, Y)/CC$
$\alpha = 6.55, \beta = 15.28^*$										
0.10	0.58	0.58	0.58	0.58	0	0.15	0.15	44.61	45.51	1.02
0.20	0.56	0.56	0.56	0.56	0	0.14	0.14	49.48	50.09	1.01
0.30	0.52	0.52	0.52	0.52	0	0.11	0.11	49.30	50.51	1.02
0.40	0.48	0.48	0.48	0.48	0	0.08	0.08	58.35	58.22	1.00
$\alpha = 1, \beta = 2.33^{**}$										
0.10	0.69	0.69	0.69	0.69	0.01	0.11	0.12	7.74	9.00	1.16
0.20	0.66	0.66	0.66	0.66	0.01	0.09	0.10	9.05	10.45	1.16
0.30	0.61	0.61	0.61	0.61	0.01	0.06	0.07	9.66	10.53	1.09
0.40	0.54	0.54	0.54	0.54	0	0.04	0.04	10.34	10.79	1.04
$q^*$	$iPNF$	$\widehat{iPNF}$			$var(\widehat{iPNF})$			$ARE$		
	true	$R$	$CC$	$(R, Y)$	$R$	$CC$	$(R, Y)$	$CC/R$	$(R, Y)/R$	$(R, Y)/CC$
$\alpha = 6.55, \beta = 15.28^*$										
0.60	0.88	0.88	0.88	0.88	0	0.06	0.06	51.68	52.65	1.02
0.70	0.93	0.93	0.93	0.93	0	0.03	0.03	53.34	54.00	1.01
0.80	0.97	0.97	0.97	0.97	0	0.01	0.01	60.50	60.89	1.01
0.90	0.99	0.99	0.99	0.99	0	0	0	65.63	65.43	1.00
$\alpha = 1, \beta = 2.33^{**}$										
0.60	0.81	0.81	0.81	0.81	0.01	0.06	0.07	10.31	11.67	1.13
0.70	0.88	0.88	0.88	0.88	0.0006	0.04	0.04	10.02	10.90	1.09
0.80	0.93	0.93	0.93	0.93	0.0002	0.02	0.02	9.47	9.90	1.05
0.90	0.98	0.98	0.98	0.98	0	0.01	0.01	14.67	15.09	1.03
$\alpha = 0.3, \beta = 0.7^{**}$										
0.60	0.15	0.15	0.15	0.15	0.01	0.03	0.04	2.30	3.18	1.38
0.70	0.12	0.12	0.12	0.12	0.01	0.02	0.03	2.68	3.72	1.39
0.80	0.09	0.09	0.09	0.09	0	0.01	0.02	3.62	4.40	1.21
0.90	0.05	0.05	0.05	0.05	0	0.01	0.01	5.77	6.37	1.10

\* AUC=0.63, \*\* AUC=0.79 \*\*\* AUC= 0.92

$\S$ ARE = asymptotic relative efficiency, the ratio of the influence functions based variances

$CC/R = \text{var}(T_{CC})/\text{var}(T_R)$ ,  $(R, Y)/R = \text{var}(T_{(R, Y)})/\text{var}(T_R)$ ,  $(R, Y)/CC = \text{var}(T_{(R, Y)})/\text{var}(T_{CC})$ , where  $T = PCF$  or  $T = PNF$  respectively