Supplementary Information

# Flexible surface acoustic wave resonators built on disposable plastic film for electronics and lab-on-a-chip applications

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1. **The mean crystallite grain size**, D, was estimated using the Debye–Scherrer formula[30] as 72.3 nm

$$D = K\lambda/(\beta cos\theta) \quad \text{(S1)}$$

where K is the shape factor of the average crystallite with a value of 0.94, λ the X-ray wavelength (1.5405 Å for Cu target), β the FWHM in radians, θ the Bragg angle. The mean grain size is about 72.3 nm for a 4 μm thick ZnO film, comparable to or larger than most of the ZnO previously deposited on solid substrates.

2. Tables.

**Table S1**. The effect of wavelength on the characteristics of flexible SAW devices with a 4 μm thick ZnO film.

| sample no. | λ (μm) | $f_0$ (MHz) | $v_{p0}$ ($ms^{-1}$) | $f_1$ (MHz) | $v_{p1}$ ($ms^{-1}$) |
|---|---|---|---|---|---|
| A1 | 10 | 198.1 | 1981 | 447 | 4470 |
| A2 | 12 | 161 | 1932 | 399.2 | 4790.4 |
| A3 | 16 | 101.5 | 1624 | 313.1 | 5009.6 |
| A4 | 20 | 75 | 1500 | 253 | 5060 |
| A5 | 24 | 56.4 | 1353.6 | 211.1 | 5066.4 |
| A6 | 32 | 34.4 | 1100.8 | 158.5 | 5072 |

**Table S2**. The effect of ZnO film thickness on the characteristics of the flexible SAW devices with different wavelengths.

| Sample | λ (μm) | ZnO (μm) | $f_0$ (MHz) | $v_{p0}$ ($ms^{-1}$) | $f_1$ (MHz) | $v_{p1}$ ($ms^{-1}$) |
|---|---|---|---|---|---|---|
| B1 | 16 | 1.7 | 58.6 | 937.6 | 323.9 | 5182.4 |
| B2 | | 2.3 | 68.5 | 1096 | 321.2 | 5139.2 |
| B3 | | 2.9 | 85.5 | 1368 | 319.6 | 5113.6 |
| B4 | | 4 | 101.5 | 1624 | 313.1 | 5009.6 |
| C1 | 12 | 1.7 | 93.5 | 1122 | 427 | 5124 |
| C2 | | 2.3 | 107 | 1284 | 426.8 | 5121.6 |
| C3 | | 2.9 | 127 | 1524 | 423.1 | 5077.2 |
| C4 | | 4 | 161 | 1932 | 399.2 | 4790.4 |

**3. The phase velocity of the polyimide.** It can be estimated by the following equation,

$$v_p = 0.93\sqrt{E/2(1+\sigma)\rho_0} \qquad \text{(S2)}$$

where E is the Young' modulus, σ the Poisson's ratio and $\rho_0$ the density of materials. The polyimide has a Young's modulus E of 2.5 GPa, σ of 0.34, and $\rho_0$ of 1.42×103 kg/$m^3$ (from the datasheet of Dupont Kapton® 100H). The theoretical Rayleigh wave velocity of the polyimide substrate is thus 754 m/s, much smaller than that in ZnO. The thickness, Young's modulus, Poisson's ratio and density of the Al electrode were set to be 100 nm, 70 GPa, 0.35 and 2.7 g/$cm^3$, respectively, while the material constants of the ZnO are taken from Ref.38.

**4. SAW device modeling.** Finite element analysis was used to model the transmission characteristics of the layered structure SAW devices using the COMSOL Multiphysics software. For the modeling, the actual layer thicknesses and material properties of the polyimide from the maker's data sheet and the ZnO layer from Ref.38, and the Al layer (The Young's modulus, Poisson's ratio and density are set to be 70 GPa, 0.35 and 2.7 g/cm$^3$) were used to model the phase velocities and resonant frequencies of the Al/ZnO/Polyimide SAW devices.

**5. Theoretical analysis.** To theoretically analyze the Rayleigh and Lamb waves propagated in a ZnO thin film on a PI substrate, a schematic structure and the coordinate system are presented in figure S1, where the ZnO is a thin layer and the polyimide is taken as an isotropic half space considering its thickness much larger than the ZnO layer. The contact boundary condition between them is assumed to be welded.

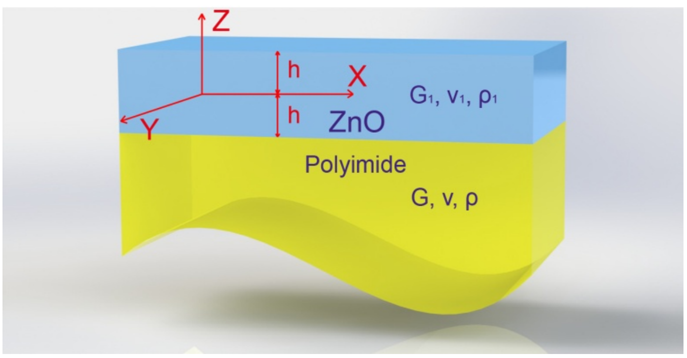


**Figure S1.** Schematic of an ZnO thin film on PI substrate for theoretical model analysis of Rayleigh and Lamb waves.

The displacement components in the $x$ and $z$ directions are denoted as $u$ and $w$, respectively, whereas the displacement in $y$ direction vanishes in this condition. Assuming the thickness of ZnO is $2h$, the general displacements in the ZnO layer can be written as

$$u_1 = [A_1 e^{-kq_1 z} + B_1 e^{-kq_1 z}]e^{ik(x-ct)} - [C_1 e^{-ks_1 z} - D_1 e^{-ks_1 z}]e^{ik(x-ct)} \quad \text{(S3)}$$

$$w_1 = iq[A_1 e^{-kq_1 z} - B_1 e^{-kq_1 z}]e^{ik(x-ct)} - (i/s_1)[C_1 e^{-ks_1 z} + D_1 e^{-ks_1 z}]e^{ik(x-ct)} \quad \text{(S4)}$$

where $k$ is the wave vector and $c$ is the phase velocity, and $A_1$, $B_1$, $C_1$, $D_1$ are the unknowns to be determined. Intermediate variables $q_1$ and $s_1$ are defined as

$$q_1 = (1 - \alpha^2 \frac{\theta}{\gamma} \beta^2)^{1/2},$$

$$s_1 = (1 - \frac{\theta}{\gamma} \beta^2)^{1/2},$$

where $\beta = c/c_T$, $\theta = \rho_1/\rho$, $\gamma = G_1/G$, $\alpha_1^2 = (1 - 2v_1)/2(1 - v_1)$, and $c_T$ is the transverse wave velocity of the material. In the above equations, subscript "1" is used to distinguish the ZnO layer from the polyimide substrate, for which no subscript is used.

The general displacements in the polyimide substrate can be also written as

$$u = [Be^{kq(z+h)} + De^{ks(z+h)}]e^{ik(x-ct)} \quad \text{(S5)}$$

$$w = -i[qBe^{kq(z+h)} + (1 + s)De^{ks(z+h)}]e^{ik(x-ct)} \quad \text{(S6)}$$

wherein $s = (1 - \beta^2)^{1/2}$, $q = (1 - \alpha^2\beta^2)^{1/2}$, $\alpha^2 = (1 - 2v)/2(1 - v)$, $B$ and $D$ are also the unknowns to be determined. By substituting the displacement solution of the top layer (Eqs. S3 and S4) and the bottom half space (Eqs. S5 and S6) to the following stress-strain relations,

$$\sigma_x = \frac{2G(1-v)}{1-2v}\frac{\partial u}{\partial x} + \frac{2Gv}{1-2v}\frac{\partial w}{\partial x}, \sigma_z = \frac{2G(1-v)}{1-2v}\frac{\partial w}{\partial z} + \frac{2Gv}{1-2v}\frac{\partial u}{\partial x}, \sigma_{xz} = G(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x})$$

where $G$ and $\nu$ represent the shear modulus and Poisson's ratio, respectively, we can get the stress components of each part. Because of the welded contact boundary conditions, the stress and the displacement are continuous in the interface, which can be written as follows:

At $z=h$, $[\sigma_z]_1 = 0$ and $[\sigma_{zx}]_1 = 0$.

At $z=-h$, $[\sigma_z]_1 = \sigma_z$ , $[\sigma_{zx}]_1 = \sigma_{zx}$, $u_1 = u$ , and $w_1 = w$.

By substituting the displacement and stress solution to the boundary conditions, we get a set of six equations with six unknowns ($A_1$, $B_1$, $C_1$, $D_1$, $B$ and $D$) for nontrivial solutions. The determinant of the coefficients matrix must be zero, which leads to the characteristic equation shown in Eq.S7, wherein $\xi = kh$ is the dimensionless wave vector. By numerically solving the transcendental equation in Eq.S7 with geometrical and material parameters of our fabricated SAW devices, we can obtain the dispersion relation. The phase velocity of the Rayleigh wave in the ZnO layer decreases and approaches that in the polymer substrate, while that of the Lamb wave approaches that in the ZnO layer as the wavelength increases. The theoretical and experimental results are also shown in figure 4b for comparison, showing a good agreement for both the Rayleigh and Lamb waves. These also agree well with that obtained by finite elemental analysis as shown in figure 4b.

$$\begin{vmatrix} (1+s_1^2)e^{-\xi q_1} & (1+s_1^2)e^{\xi q_1} & -2e^{-\xi s_1} & 2e^{\xi s_1} & 0 & 0 \\ -2q_1 e^{-\xi q_1} & 2q_1 e^{\xi q_1} & (s_1+\frac{1}{s_1})e^{-\xi s_1} & (s_1+\frac{1}{s_1})e^{\xi s_1} & 0 & 0 \\ (1+s_1^2)e^{\xi q_1} & (1+s_1^2)e^{-\xi q_1} & -2e^{\xi s_1} & 2e^{-\xi s_1} & \frac{(1+s_1^2)}{\gamma} & \frac{2}{\gamma} \\ -2q_1 e^{\xi q_1} & 2q_1 e^{-\xi q_1} & (s_1+\frac{1}{s_1})e^{\xi s_1} & (s_1+\frac{1}{s_1})e^{-\xi s_1} & \frac{2q}{\gamma} & \frac{(s+\frac{1}{s})}{\gamma} \\ e^{\xi q_1} & e^{-\xi q_1} & -e^{-\xi s_1} & e^{\xi s_1} & 1 & 1 \\ q_1 e^{\xi q_1} & -q_1 e^{-\xi q_1} & -\frac{1}{s_1}e^{\xi s_1} & -\frac{1}{s_1}e^{-\xi s_1} & -q & -\frac{1}{S} \end{vmatrix} = 0 \qquad \text{(S7)}$$