# Supporting Information

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## SI Text

**Specularity Content of Radar Returns.** We use the angular distribution of energy in radar returns to identify and characterize water systems beneath Thwaites Glacier, West Antarctica. We exploit the radar scattering character of the planar interfaces of distributed subglacial water systems to identify them from the narrow angular distribution of their returned radar energy and to distinguish them from concentrated subglacial water systems with cylindrical interfaces that scatter energy over a wide range of angles. The metric we use to describe the angular distribution of echo energy on a continuum between specular reflection and diffuse scattering is the specularity content. In this work, we define the specularity content, $S_C$, in terms of two quantities, the specular energy, $S$, which is the radar energy returned in a narrow angular distribution around the specular direction, and the diffuse energy, $D$, which is the total energy isotropically scattered over the 180° half-space (1). The specularity content is then the fraction of the total energy that is contributed by the specular component:

$$S_C = \frac{S}{S+D}, \tag{S1}$$

so that a purely specular (or mirror-like) surface will have a specularity content of 1 and a purely diffuse surface (i.e., a point-scatter) will have a specularity content of 0.

**Focusing with Different Apertures.** In this work, we calculate the specularity content of radar returns by performing range-migrated synthetic aperture radar focusing on airborne ice-penetrating radar data using two different focusing correlation aperture lengths, $L_1$ and $L_2$ [after Peters et al. (2)]. By focusing with two different apertures, we produce bed echoes that include radar energy that has been focused across two different spans of scattering angles, $\phi_1$ and $\phi_2$. These angles can be determined from Snell's law by solving

$$\frac{\frac{L}{2}-x}{\sqrt{h^2+\left(\frac{L}{2}-x\right)^2}} = \sqrt{\varepsilon_r}\;\frac{x}{\sqrt{d^2+x^2}} \tag{S2}$$

for the refraction point at the surface, $x$, where $h$ is the survey height, $d$ is the ice thickness, and $\varepsilon_r$ is the relative permittivity of ice [after Hélière et al. (3)]. The range of scattering angles spanned by the focusing aperture is then given by

$$\phi = 2\tan(x/d). \tag{S3}$$

The focused radar echo strengths for each aperture will include the energy scattered within the angle $\pm\phi/2$ of nadir. By comparing these amplitudes for different focusing apertures, we constrain the angular distribution of echo energy and calculate the specularity content of the radar return from the bed. The specularity content is only a function of the angular distribution of echo energy and is therefore independent of attenuation ambiguities from uncertain ice temperature and chemistry (4, 5).

**Calculating Specularity Content.** We calculate the specularity content of bed echoes for Thwaites Glacier, West Antarctica by focusing airborne ice-penetrating radar data with two different focusing apertures, $L_1 = 700$ m and $L_2 = 2$ km. In calculating the specularity content, we assume that the entirety of the energy from the specular component is contained within the range of angles spanned by aperture $L_1$, which is a conservative assumption for the bed slopes (~0–6°) and ice thicknesses (~1–4 km) (6) that typify our study area. The echo strength, $E_1$, of radar returns focused using aperture $L_1$will therefore include all of the energy in the specular component and a portion of the energy in the diffuse component (corresponding to the fraction of the half-space spanned by $\phi_1$) and given by

$$E_1 = S + D\frac{\phi_1}{180^\circ}. \tag{S4}$$

Likewise, the echo strength, $E_2$, of radar returns focused using aperture $L_2$will include all of the energy in the specular component and a larger portion of the energy in the diffuse component (corresponding to the fraction of the half-space spanned by $\phi_2$) and given by

$$E_2 = S + D\frac{\phi_2}{180^\circ}. \tag{S5}$$

The difference between these two echo strengths, $\Delta E$, is the fraction of diffuse energy that was scattered at an angle greater than $\pm\phi_1/2$ but smaller than $\pm\phi_2/2$ . The total energy in the diffuse component, $D$, can be calculated from

$$\Delta E = E_2 - E_1 = D\frac{\phi_2-\phi_1}{180^\circ} \tag{S6}$$

as

$$D = \frac{180^\circ}{\phi_2-\phi_1}(E_2 - E_1), \tag{S7}$$

and the specular component, $S$, can be calculated using the echo strengths from either focusing aperture by combining Eq. **S7** with Eq. **S4** or **S5**, giving

$$S = E_2 - D\frac{\phi_2}{180^\circ} = E_1 - D\frac{\phi_1}{180^\circ}. \tag{S8}$$

**Interpreting Specularity from a Gridded Survey.** In this work, we calculate the specularity content for bed echoes along the survey lines for the gridded Airborne Geophysical Survey of the Amundsen Sea Embayment airborne ice-penetrating radar survey (6). We produce 5- × 5-km gridded data products for each of the orthogonal survey directions. By comparing the specularity content for these two orthogonal directions, we are able to infer the anisotropy of the specularly reflecting interfaces at the bed (and infer that the distributed water systems at the bed are anisotropic). We also average gridded specularity values from the two orthogonal directions, which provides a rough constraint on the spatial distribution of specular interfaces across the catchment. This average specularity is a function of both the portion of the grid cell that is covered by specular reflecting interfaces (or distributed water) and the orientation of the survey grid with respect to any anisotropy in these interfaces. In other words, a change in the specularity along either axis will result from a change in the along-track length of a reflecting interface (7) for any change in grid orientation. However, because our survey grid is orthogonal, the along-track reflector length and the specularity along the two axes of the grid will change complimentarily (with one growing as the other shrinks), so the resulting "average" specularity will be much less sensitive to orientation than to the existence and areal extent of distributed water.

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