Appendix E1

Scaling Power Laws

We defined a proximal vessel segment as a stem and the tree distal to the stem as a crown, as shown in Figure E1. A tree structure (eg, the epicardial coronary artery tree) has many stem-crown units. In a stem-crown unit, the crown volume (V_c, in milliliters) is defined as the sum of the intravascular volumes of each vessel segment and the crown length (L_c, in centimeters) as the sum of the lengths of each vessel segment in the crown from the stem to the most distal vessels. Here, the smallest stem-crown unit corresponds to a terminal bifurcation (ie, $Stem_n$ and $Crown_n$ in Fig E1) of the epicardial coronary arterial tree obtained from CT angiography with the diameter of terminal vessels in the range of 0.6–1 mm.

To derive the volume-length scaling power law, a cost function for an integrated system of stem-crown units was proposed, which consists of two terms: viscous and metabolic power dissipation. The cost function, F_c (in ergs), is written as follows (22):

$$F_c = Q_s \cdot \Delta P_c + K_m V_c \text{, [A1]}$$

where Q_s and $\Delta P_c = Q_s \cdot R_c$ are the flow rate through the stem (in milliliters per second) and the pressure drop in the distal crown (in dynes per square centimeter), respectively. K_m is a metabolic constant of blood in a crown (in dynes/cm^2 · sec). Two important structure-structure scaling power laws as well as a flow-structure scaling power law are needed to perform the minimum energy analysis in the cost function. First, we have shown that the resistance of a crown (in dynes · sec/cm^5) has the following form (22):

$$R_c = K_R \frac{L_c}{{D_s}^4} \text{, [A2]}$$

where D_s is the stem diameter (in centimeters) and K_R a flow resistance constant in a crown (in dynes · sec/cm^2), which depends on the branching ratio and total number of tree generations in a crown. Second, the crown volume is found to scale with the stem diameter, as follows (21):

$$V_c = K_{\text{VD}} D_s^3 \text{, [A3]}$$

where K_{VD} is a morphometric constant in a crown. Finally, the flow-length scaling power law is given as follows (21):

$$Q_s = K_{\text{QL}} L_c \text{, [A4]}$$

where K_{QL} is a functional constant in a crown (in square centimeters per second).

When resistance (Eq [A2]), volume-diameter (Eq [A3]), and flow-length (Eq [A4]) scaling power laws are substituted into the energy cost function, Equation [A1] can be written as follows:

$$F_c = Q_s^2 \cdot R_c + K_m V_c = \left(K_R \cdot K_{\text{QL}}^2 \cdot K_{\text{VD}}^{4/3}\right) \frac{L_c^3}{V_c^{4/3}} + K_m V_c \text{. [A5]}$$

Similar to the approach used by Murray (31), we minimized the cost function with respect to crown volume at a fixed crown length to obtain the following equation (21):

$$\frac{\partial F_c}{\partial [V_c]} = 0 \Rightarrow -\frac{4}{3}\left(K_R \cdot K_{\mathrm{QL}}^2 \cdot K_{\mathrm{VD}}^{4/3}\right)\frac{L_c^3}{V_c^{7/3}} = -K_m \text{. [A6]}$$

Equation [A6] can be written as follows:

$$L_c = \sqrt[3]{\frac{3K_m}{4K_R \cdot K_{\mathrm{QL}}^2 \cdot K_{\mathrm{VD}}^{4/3}}} V_c^{7/9} = K_{\mathrm{LV}} V_c^{7/9} \text{, [A7]}$$

where

$$K_{\mathrm{LV}} = \sqrt[3]{\frac{3K_m}{4K_R \cdot K_{\mathrm{QL}}^2 \cdot K_{\mathrm{VD}}^{4/3}}} \text{ [A8]}$$

is a constant (in $\mathrm{cm}^{-4/3}$). Equation [A7] provides the length-volume scaling power law, which forms the theoretic basis for the diagnosis of diffuse CAD. Moreover, a combination of Equations [A3] and [A7] results in

$$L_c = K_{\mathrm{LA}} A_s^{7/6} \text{, [A9]}$$

where A_s is the cross-sectional area of the stem (in square centimeters) and K_{LA} is a constant (in $\mathrm{cm}^{-4/3}$). Equation [A9] refers to the length-area scaling power law.

Reference

31. Murray CD. The physiological principle of minimum work. I. The vascular system and the cost of blood volume. Proc Natl Acad Sci U S A 1926;12(3):207–214.