Supplementary Material to *A Study of Mexican Free-Tailed Bat Chirp Syllables: Bayesian Functional Mixed Modeling of Nonstationary Acoustic Time Series*

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1 Simulation of Synthetic Chirps

1.1 Illustration of Variable Overlap

We generate synthetic chirps to test our variable window overlap approach to define spectrograms on a relative signal scale. This method, as explained in the main text, varies the size of the overlap when producing spectrograms. This process allows for the comparison of sound signals in the time-frequency domain via spectrograms that are of disproportionate size. To allow for a fair comparison between existing methods and our new procedure, we generate a synthetic chirp with a frequency signal that evolves as a sine wave from beginning to end. The signal is engineered so that its amplitude expands and contracts as that of bat chirp, from beginning to end, its frequency range always from 180 to 237 Hertz, and the signal hits the trough and peak of this range in the same location along its evolution, regardless of its length. The trough is reached 36.5% along the signal evolution and the peak 65.8% along its evolution. Figure 1 shows two simulated chirp signals, one of length 10 seconds and the other of length 30 seconds. Both signals contain the same frequency information along the same portions of the chirps as it evolves from beginning to end. To demonstrate this, we calculate the respective spectrograms of these two chirps and show them as plots (A) and (B) in Figure 2. In this section, all our spectrogram calculations are based on a window size to $N = 256$ and the spacing between windows to $N_m = 46$. One can see that the frequency information remains the same regardless of the length of the chirp; however, the size of the spectrograms is very different. The dimensions of the spectrogram associated with the shorter signal are 128×173 while those of the longer signal are 128×546, where rows represents the frequency resolution and the columns the time resolution. The frequency resolution is determined by the size of the window $N/2$, while the time resolution is dictated by the spacing between successive windows and the number samples in the signal. The substantial difference in the time resolution does not allow for the direct comparison of these two spectrogram matrices. To be able to compare the two spectrograms we must use a procedure to increase the dimensionality of the shorter signal, or decrease the dimensionality of the longer signal. There are several methods available to carry out this type of procedure, including up and down sampling as well as interpolation schemes. We propose the use of our new variable overlap procedure to facilitate this process, and will compare its performance with the mentioned, competing methods; however, first, we illustrate the applicability of our method using the two chirps shown in Figure 1.

To begin, we remind the reader that the shorter signal spans 10 seconds while the longer signal spans 30 seconds of time. Because we keep a constant sampling rate constant, of 819.2

Hz as we generate the chirps, this implies that the number of samples in the second signal is 3 times that of the first. To reconcile this drastic difference, we propose the simple scheme of changing the overlap in the computation of the spectrogram so that the time resolution is changed to a desired value. For this illustration, we will increase the size of the time grid of the spectrogram corresponding to the shorter signal, so that it matches that of the longer signal and vice versa.

For the purposes of this illustration we increase the time resolution of the spectrogram associated with the shorter signal from 173 columns to 546 columns, and decrease the time resolution of the spectrogram associated with the longer signal from 546 columns to 173 columns. The aim is to be able to compare the time-frequency information of the shorter signal at the time resolution of the longer signal and vice versa. Figure 2 shows the resulting spectrograms in plots (C) and (D). Plot (C) of Figure 2 is the lower resolution spectrogram of the longer signal with dimensions 128×173, that spans 30 seconds, while plot (D) is the higher resolution spectrogram of the shorter signal with dimensions 128×546, that spans 10 seconds. Recall that the actual time axis that would be used is a relative time axis on a scale of 0 to 1, intrepretable as relative signal position. We plot on the 10s and 30s time axis here to denote whether the grid size corresponding to the 10s or 30s signal is used.

Because 173 windows are used to generate the spectrogram in plot (C), a time grid with 173 time points is obtained, and similarly 546 windows are used to generate the spectrogram in plot (D), hence it contains 546 time values. Now that we have increased the time resolution in the spectrogram of the shorter signal we can compare it directly to the natural spectrogram of the longer signal. We take that difference, (D)-(B), and show the result in plot (F) of Figure 2. We see good agreement in the first third of the chirp with minor discrepancies on the remaining part of the chirp. We perform the same comparison using the resulting lower resolution spectrogram of the longer chirp with the natural spectrogram of the shorter chirp, taking the difference (A)-(C) and displaying it in plot (E) of Figure 2. This difference shows minor discrepancies between the two spectrograms throughout the evolution of the chirp. This illustration demonstrates the simple yet practical and efficacious nature of our proposed method, allowing for the comparison of signals that differ in length by a large margin. Next, we compare our method to cubic spline interpolation as well as up and down sampling, all of which are used to match time resolution issues when computing spectrograms of signals that differ in length.

1.2 Elongation and Compression of Time Resolution in Spectrogram Generation

As we illustrate in the previous section, reconciliation of time resolution in the generation of spectrograms of signals of different lengths is an important issue and one we address by introducing the variable overlap method. To demonstrate its validity we carry out a simulation analysis where we compare a fixed signal against longer signals. All signals are generated as explained above, introducing the same frequency information along the same part of the chirp as it evolves. The fixed signal generated is 10 seconds long and contains 8192 samples, the longer signals are 1.2, 1.4, 1.6, 1.8, 2.0 and 3.0 times the length of the fixed signal. Because we use the same sampling rate, 819.2 Hz, when generating all signals, the number of samples in the longer signals are 9830, 11468, 13107, 14745, 16384 and 24576, spanning 12, 14, 16, 18, 20 and 30 seconds, respectively.

We will conduct two parallel analyses: 1) where we elongate the fixed, 10 second signal to match the time resolution of the longer signals, and 2) where we compress the time resolution of each of the longer signals to match the time resolution of the fixed, shorter one. Elongation is performed using upsampling, cubic spline interpolation and variable overlap, while compression is performed using downsampling, cubic spline interpolation and variable overlap. It is important to understand that the time resolution adjustment, whether elongating or compressing, is accomplished differently in upsampling, downsampling and cubic spline interpolation than in variable overlap. Variable overlap adjusts the resolution when computing the spectrogram and does not change the original signal. In contrast, competing methods such as upsampling, downsampling and any interpolation method removes/introduces observations from/to the original signal, thereby altering the data and introducing artifacts that can hinder interpretation. Specifically, aliasing, which is the introduction of frequencies not present in the original signal, is a common issue in these competing methods, and one that must be addressed by using filters to remove such artifacts. Moreover, once the elongation or compression of the signal is achieved to match a target time resolution, the resulting spectrogram has a different frequency resolution than the target; therefore, a second interpolation must be carried out to match the frequency resolution to the target signal. Additionally, use of downsampling as well as upsampling requires either a clever method to introduce or remove the samples, or that e, the fractional increase in signal length of the longer signal when compared to the shorter one, equal $f/(f-1)$, where f is a positive integer greater than 2. These issues make the use of competing methods much more cumbersome; in contrast, our method does not require any additional steps or clever schemes in order to compare the resulting spectrograms.

In our implementation of downsampling as well as upsampling we used a scheme to choose the removal/introduction of the candidate samples by the following rule: select a set of integers I_i so that their average equals the fraction $F = LONG/(LONG - SHORT)$, where $LONG$ is the number of samples of the longer sequence and $SHORT$ is the number of samples of the shorter sequence. Selecting the set I_i is a difficult task, in particular because they do not have to be distinct. However, once found they will simplify the issue of deciding which samples to remove/introduce in order to cover the entire signal. Examples:

1. $F = 2.5$ and $I_1 = 2$ and $I_2 = 3$, so that we alternate in spacing between values that will be removed in the downsampling, $2, 3, 2, 3, 2, 3, \ldots$ until we reach the desired number.

2. $F = 3.5$ and $I_1 = 3$ and $I_2 = 4$, so that we alternate in spacing between values that will be removed in the downsampling, $3, 4, 3, 4, 3, 4, \ldots$ until we reach the desired number.

3. $F = 2.66$ and $I_1 = 3$, $I_2 = 2$ and $I_3 = 3$ so that we alternate in spacing between values that will be removed in the downsampling, $3, 2, 3, 3, 2, 3, \ldots$ until we reach the desired number.

4. $F = 2.25$ and $I_1 = 2$, $I_2 = 2$, $I_3 = 2$ and $I_4 = 3$ so that we alternate in spacing between values that will be removed in the downsampling, $2, 2, 2, 3, 2, 2, 2, 3, \ldots$ until we reach the desired number.

It is important to understand that the order of the set of integers in the scheme above does not affect the result; however one should try to keep spacing as uniform as possible to avoid large gaps when removing samples from the original signal. Also, the examples mention downsampling; however, one can also use the scheme to introduce samples when upsampling.

Whether we use downsampling, upsampling or cubic spline interpolation to adjust the time resolution to compare signals of different lengths, we must use a filter in order to remove aliasing in the resulting spectrogram. We choose to use a third order, Chebyshev type I filter with low dB pass band and varying critical frequencies depending on the signal. Moreover, either low-pass, high-pass or both are used depending on the signal.

Lastly, when reconciling differences in frequency resolution between the target and resulting spectrograms after upsampling, downsampling or cubic spline interpolation, we use linear interpolation and interpolate up to match the lower frequency resolution with the higher frequency resolution, as well as interpolate down to do the opposite.

Results of our elongation and compression analyses are presented as images of differences between the resultant spectrogram and the target spectrogram. Figures 3, 4 and 5 show the results of the elongation analysis using upsampling, cubic spline interpolation and variable overlap, respectively. The images presented in these Figures interpolate up the frequencies in the spectrograms resulting from the upsampling and cubic spline interpolation procedures. Results are similar when we interpolate frequencies down and therefore do not display the associated images. Mean square error values of each method are presented in Table 1. The results in Table 1 present both interpolated up as well as interpolated down frequency resolutions for the spectrograms resulting from upsampling and cubic spline interpolation. Similarly, Figures 6, 7 and 8 show the results of the compression analysis using downsampling, cubic spline interpolation and variable overlap, respectively. The images presented in these Figures interpolate up the frequencies in the spectrograms resulting from the upsampling and cubic spline interpolation procedures. Results are similar when we interpolate frequencies down and therefore do not display the associated images. Mean square error values of each method are presented in Table 2. The results in Table 2 present both interpolated up as well as interpolated down frequency resolutions for the spectrograms resulting from downsampling and cubic spline interpolation. Figures as well as mean square error results show that as the elongation and compression increase, the error in competing methods increases when compared to variable overlap. Moreover, although elongation of signals seems to be possible at any magnitude, compression using either of the competing schemes has its limits. Because frequency resolution is dictated by the sampling frequency and the number of samples removed in the compression, it is not possible to capture all the features of the frequency signature, in the signal, once compression is above 44%. This compression limit corresponds to signals that are 1.8, 2 and 3 times larger than the fixed signal, see plots (D)-(F) in Figures 7 and 8. Moreover, when using either of the competing methods, cubic spline interpolation or up or down sampling, we can see the smearing of frequencies along the frequency axis. Although, variable overlap shows minor discrepancies it captures the target frequencies and avoids smearing along the frequency axis. Lastly, an important aspect of an algorithm is its complexity, variable overlap is much simpler to implement than either of the competing methods. To carry out cubic spline interpolation or up or down sampling, not only is it necessary to reconcile discrepancies in the time axis, one must also interpolate in the frequency axis to match different frequency resolutions introduced by the methods. In our data of 788 samples, that would imply reconciling 788 distinct frequency resolutions. In contrast, variable overlap only requires knowledge of the length of the original signal and the target time resolution in the desired spectrogram. Hence, due to its simplicity, accuracy and fidelity in the frequency domain, we employ variable overlap to process our data.

2 Sensitivity Analysis

2.1 Window Size (N) and Spacing Between Windows (M)

After choice of the window $w(s)$, the key parameters to specify for spectrogram calculation include the window size N and successive spacing between window placement M, in the sequential computation of discrete Fourier transforms to obtain the spectrogram. We explore different window size values, N, as well as spacings between successive windows, M, in order to discern which are best suited for our application. We calculate spectrograms for a simulated signal generated as above using window size values of 128, 256, 512 and 1024 samples, and increasing spacing between windows which are equivalent to 4.5%, 9%, 18%, 36%, and 72% the size of the window used. Figures 9, 10, 11 and 12 show the resulting spectrograms using window size 128, 256, 512 and 1024. These plots show that, as expected, the size of the window corresponds with the frequency resolution, with larger window size providing larger frequency resolution and vice versa. Moreover, the spacing between successive windows determines the resolution in time, with larger spacing providing less resolution and smaller spacing more resolution. Our experimentation shows that frequency resolution is tied to time resolution, with higher frequency resolution requiring higher resolution in time. Frequency resolutions corresponding to 1024, 512 and 256 require time resolutions corresponding to spacing between windows of 4.5%, 4.5%-9% and 4.5%-18%. Any choice in this set will provide adequate resolution in time as well as frequency. We verify these choices by applying the mentioned window size and spacing parameters in a simulation where the size of the chirp is varied, but the same frequency information is kept along the progression of the chirp. We register all 400 simulated chirps using our variable overlap method and calculate the average spectrogram of all these and find that results do not change from what is shown in Figures 9, 10, 11 and 12. Our variable overlap method is applied by calculating an average number of columns, using all 400 simulated chirps, our parameter choices and equation **??**. We then use this common time resolution to register the data in the time-frequency domain via spectrograms.

2.2 Time Resolution in our Application

As mentioned above, the frequency resolution for each spectrogram is dictated by N, the size of the window, and the time resolution is determined by the spacing between the windows, M. The frequency resolution, set by $N = 256$, is sufficient to capture the features in these data, and increasing the resolution from 256 to 512 would only result in slightly less smoothing in the frequency domain. Recall that our strategy for dealing with different chirp size involves varying the window overlap M_i over chirps according to a relative time grid of specified size T. Here, we assess sensitivity of our results to this parameter. The only requirement on the spacing between windows, M_i, is that it be smaller than N, which requires that $T > N/\min(\mathcal{N}_i)$, where $\mathcal{N}_i$ is the length of signal i. To explore how sensitive our results are to a change in time resolution, we consider T values of $\{35, 43, 56, 80, 141\}$, which correspond to average window spacings of roughly $M_i = \{106, 86, 66, 46, 26\}$. We fit our model to each of the five sets of spectrograms resulting from these settings in time resolution, and compare the results. Figures 13 and 14 show the corresponding probability discovery images and their 15% Bayesian FDR counter parts.

We can see that time resolution behaves as above, with larger spacing providing less resolution in time. However, we still observe a frequency shift between the Austin and College Station bats, where Austin bats prefer the lower frequency range of 20-30 kHz, while College Station bats prefer that of 30-50 kHz. The two sets of bats still parallel each other in that their frequency preference decreases from the start to the middle of the chirp. As mentioned above, for a window size of 256, a spacing that corresponds to 4.5% - 18% of that window size, will give an appropriate time resolution. In this illustration, the spacing values, $M_i = \{106, 86, 66, 46, 26\}$, correspond to 41%-10% of the window size. Making a time resolution of 80 columns and 141 columns appropriate for our analysis. Even though an increase in time resolution from 80 columns to 141 columns does enable us to capture subtle changes in frequency preferences between the two groups, the differences are not great enough to raise serious concerns about our procedure. Although care must be exercised in choice of a time resolution to match the frequency resolution given by the choice of N, so that results given the spectrogram are optimal for display, these results verify that our analysis is robust to time resolution and thus validates the registration procedure used to generate the fixed column spectrograms. Moreover, we adjusted the fold change used to determine the difference between the Austin and College Station bats to include 1.75 and 2.00. As before, we find that Austin bats consistently prefer lower frequencies around 20-30 kHz in the middle part of the chirp, and College Station bats prefer higher frequencies around 30-50 kHz at the beginning of the chirp. These preferences are present at every fold change: from 1.5 to 2.0. Figures 13, 15 and 16 show plots of these results which correspond with 1.5, 1.75 and 2.00 fold change.

3 Correlation of Random Effects

In addition to our analysis of variance, we also generated random effects, in the wavelet space, using parameter values estimated via our Bayesian approach. We took the correlation of these random effects and show the resulting matrix in Figure 17. Recall that there are 13 College Station bats and 14 Austin bats, in that order. Our correlation matrix supports results from our spectrogram analysis demonstrating strong association between bats in College Station or between bats in Austin, but very week correlation between the two distinct geographic locations.

4 Length of Chirps Included in WFMM as a Predictor

The model presented in the main text only includes covariates that correspond to geographic location: Austin and College Station.

Since our variable overlap procedure filters out direct signal duration information, it may be important to consider this effect in the modeling.

However, because the length of the chirp may be an important piece of information, which was ignored in the original model, we add it as a covariate and show the resulting contrast plots corresponding to a chirp length of 0, 3440, 3905 and 4336 samples which correspond to 0, 13.7, 15.6 and 17.3 $X10^{-3}$ seconds, see Figure 18. Note that chirp length zero is added for reference only, as this is far outside the range of observed chirp lengths (plus chirp length of zero would not make sense!). We see that incorporation of this effect does not appreciably affect the region results, with Austin bats preferring lower frequencies than College Station bats. Furthermore, as we increase the size of the chirp, results differ from those where the size is zero, showing a shift in frequency preference in Austin bats from the beginning to the middle of the chirp. College Station bats still prefer higher frequencies than Austin bats; however, there is a shift in preference of lower frequencies toward the beginning of the chirp.

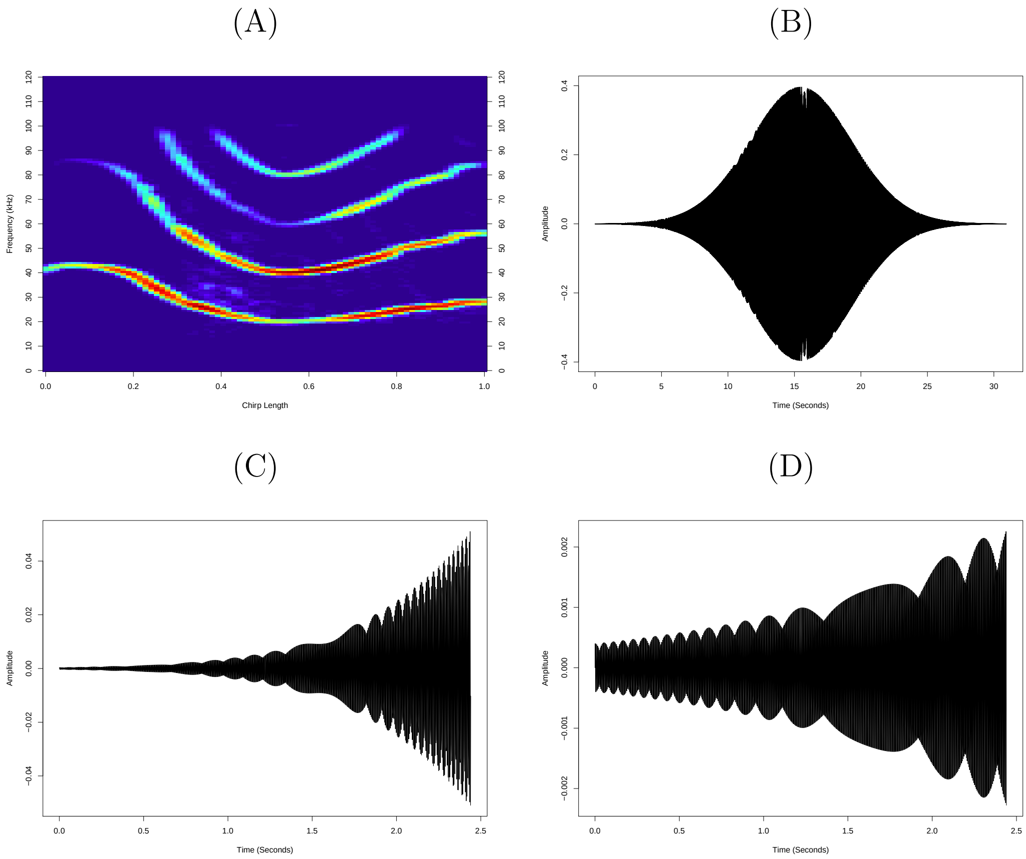

Figure 1: Simulated chirps. Plot (A) shows chirp 1, which lasts 10 seconds, and (B) shows chirp 2, which lasts 30 seconds. Both chirps contain frequencies that range from 180 to 237 Hertz along the same piece of the chirp as it progresses in time. Plots (C) and (D) show the first 2.5 seconds of each chirp, respectively.

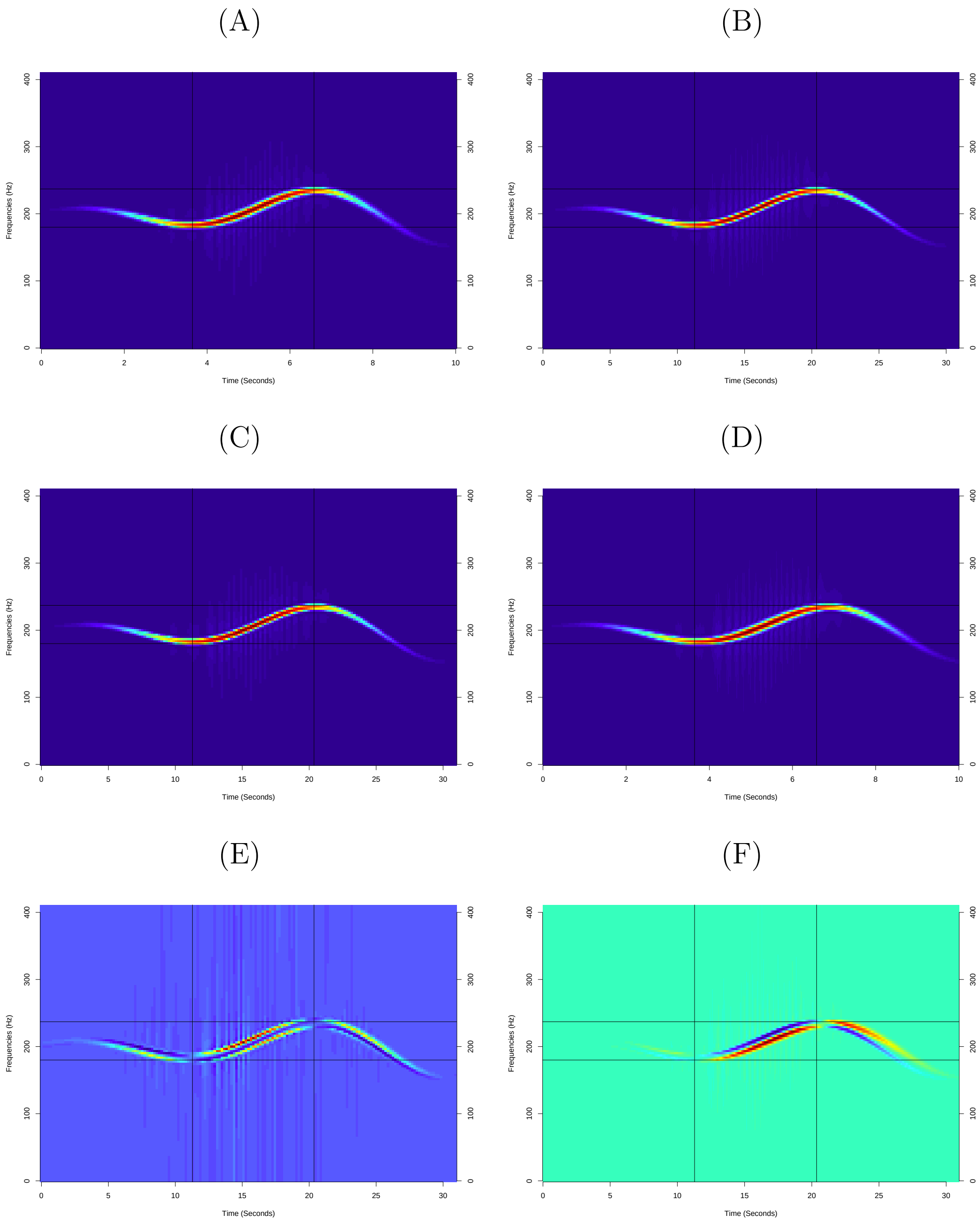

Figure 2: Application of our variable overlap procedure to elongate the spectrogram for chirp 1 and shrink the spectrogram for chirp 2, both shown in Figure 1. Spectrograms of chirp 1 and chirp 2 are shown in plots (A) and (B). The elongated spectrogram for chirp 1 is shown in plot (D) and the shrunken spectrogram for chirp 2 in plot (C). The difference between (A) and (C) is shown in plot (E) and the difference between (B) and (D) is shown in plot (F).

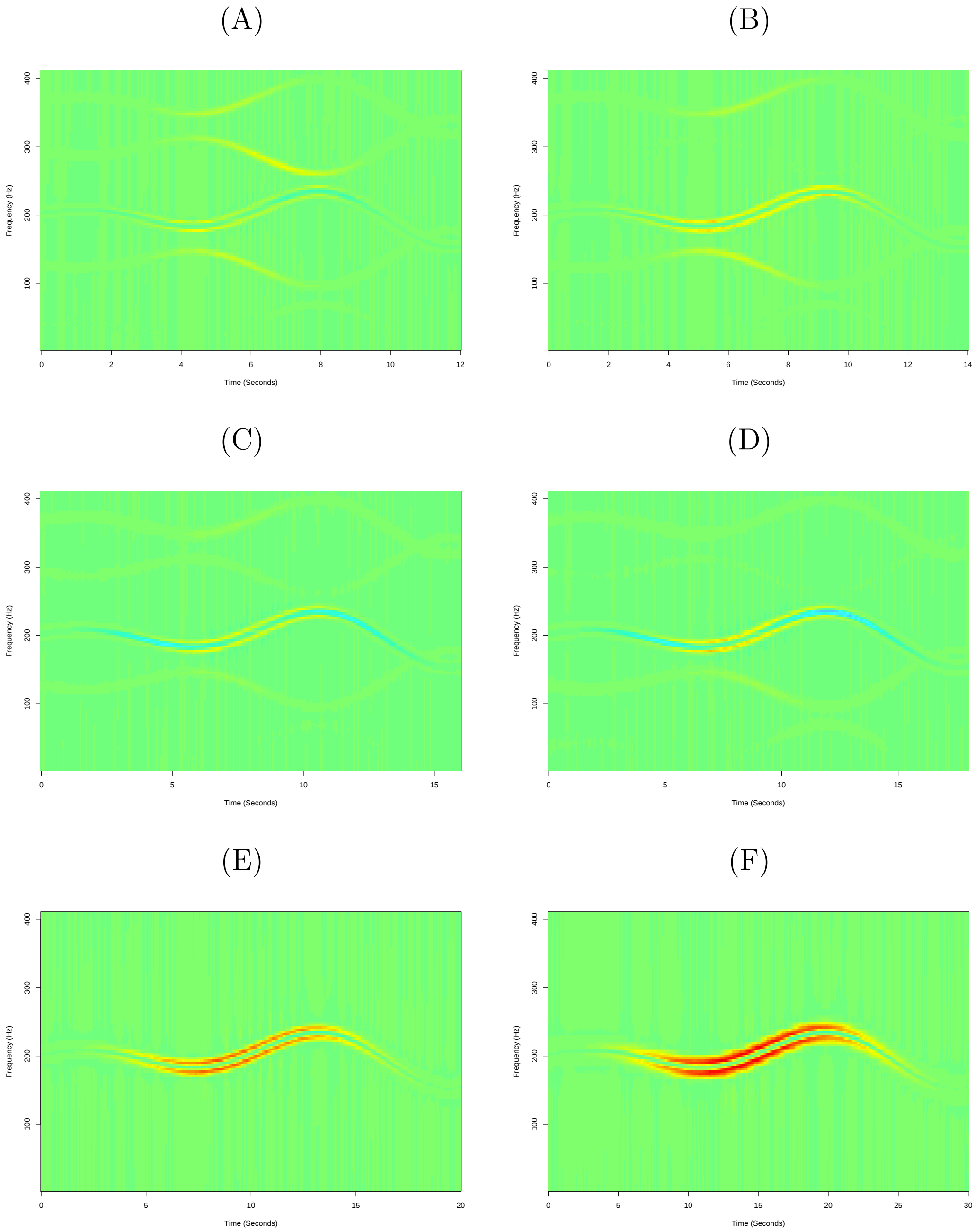

Figure 3: Difference of simulated spectrograms in elongation analysis. We use upsampling to compare a short signal with 6 longer signals. The short signal has a fixed length of 8192 samples and the long signals are: 1) 20%, 2) 40%, 3) 60%, 4) 80%, 5) 100%, 6) 200% larger than that. Plot (A) shows the difference between the resulting spectrograms of the upsampled short signal and the first longer signal, (short-long). Plots (B)-(F) show similar differences for the 5 remaining comparisons.

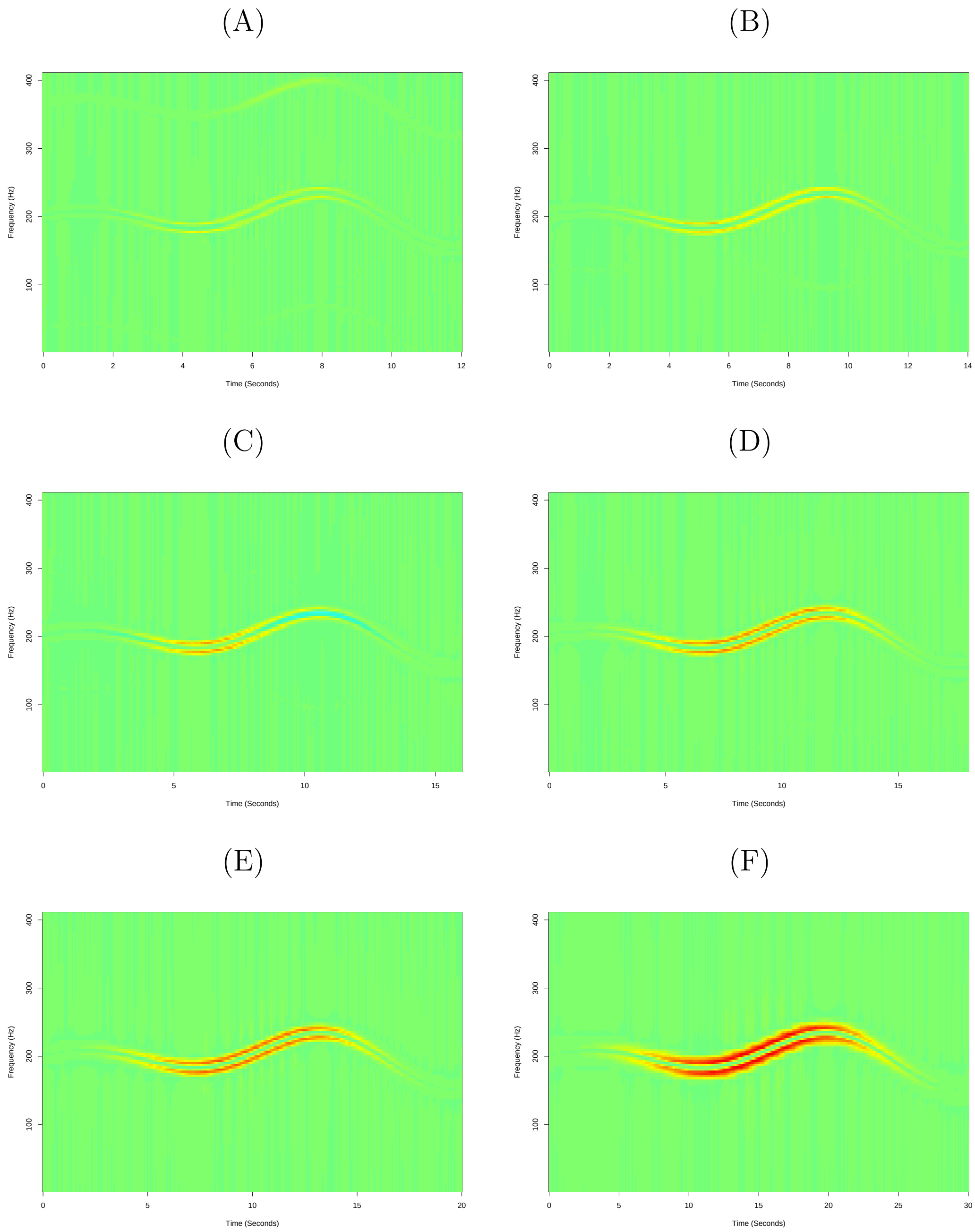

Figure 4: Difference of simulated spectrograms in elongation analysis. We use cubic spline interpolation to compare a short signal with 6 longer signals. The short signal has a fixed length of 8192 samples and the long signals are: 1) 20%, 2) 40%, 3) 60%, 4) 80%, 5) 100%, 6) 200% larger than 8192 samples. Plot (A) shows the difference between the resulting spectrograms of the interpolated short signal and the first longer signal, (short-long). Plots (B)-(F) show similar differences for the 5 remaining comparisons.

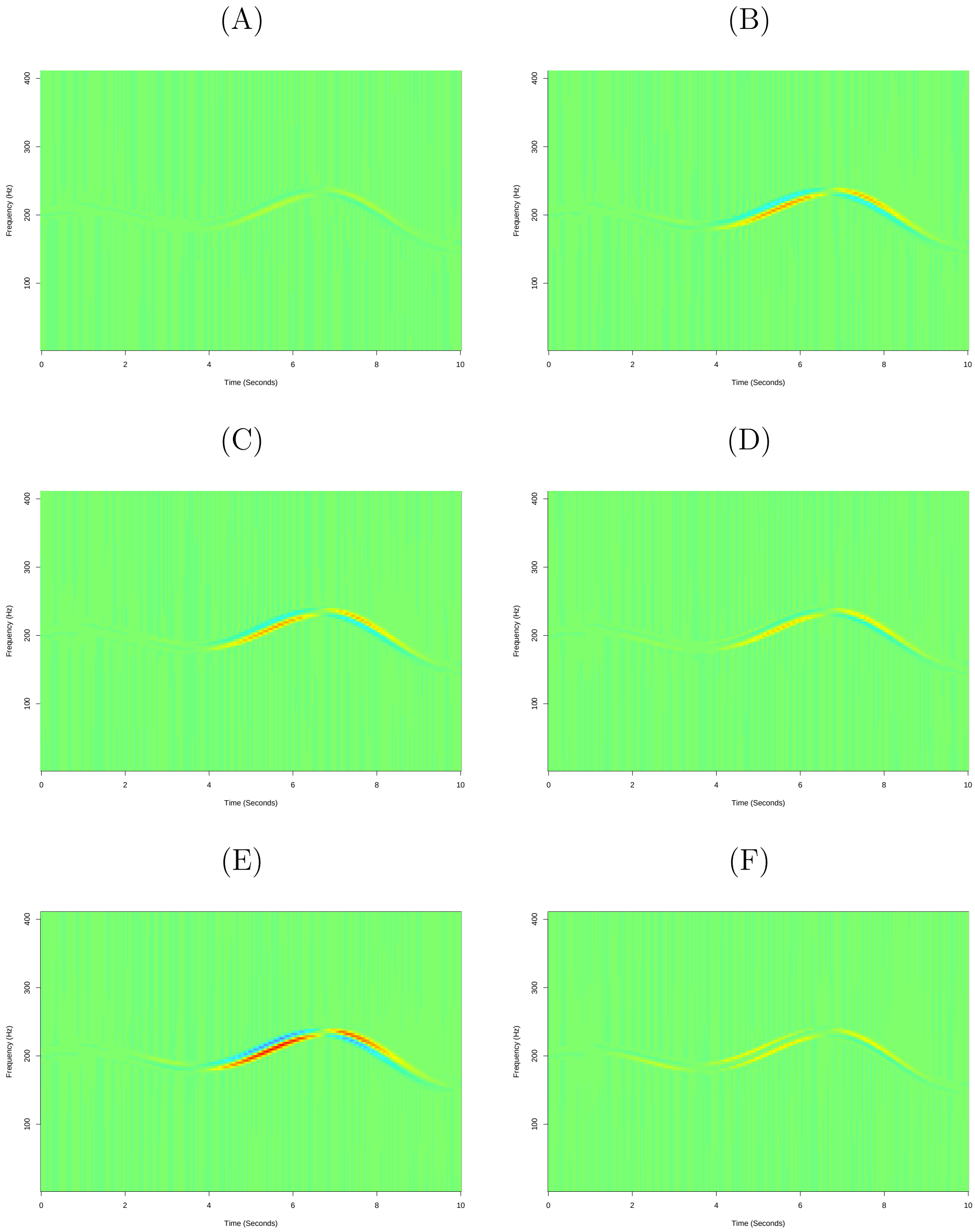

Figure 5: Difference of simulated spectrograms in elongation analysis. We use our variable overlap method to compare a short signal with 6 longer signals. The short signal has a fixed length of 8192 samples and the long signals are: 1) 20%, 2) 40%, 3) 60%, 4) 80%, 5) 100%, 6) 200% larger than 8192 samples. Plot (A) shows the difference between the resulting spectrograms of the short signal and the first longer signal, (short-long). Plots (B)-(F) show similar differences for the 5 remaining comparisons.

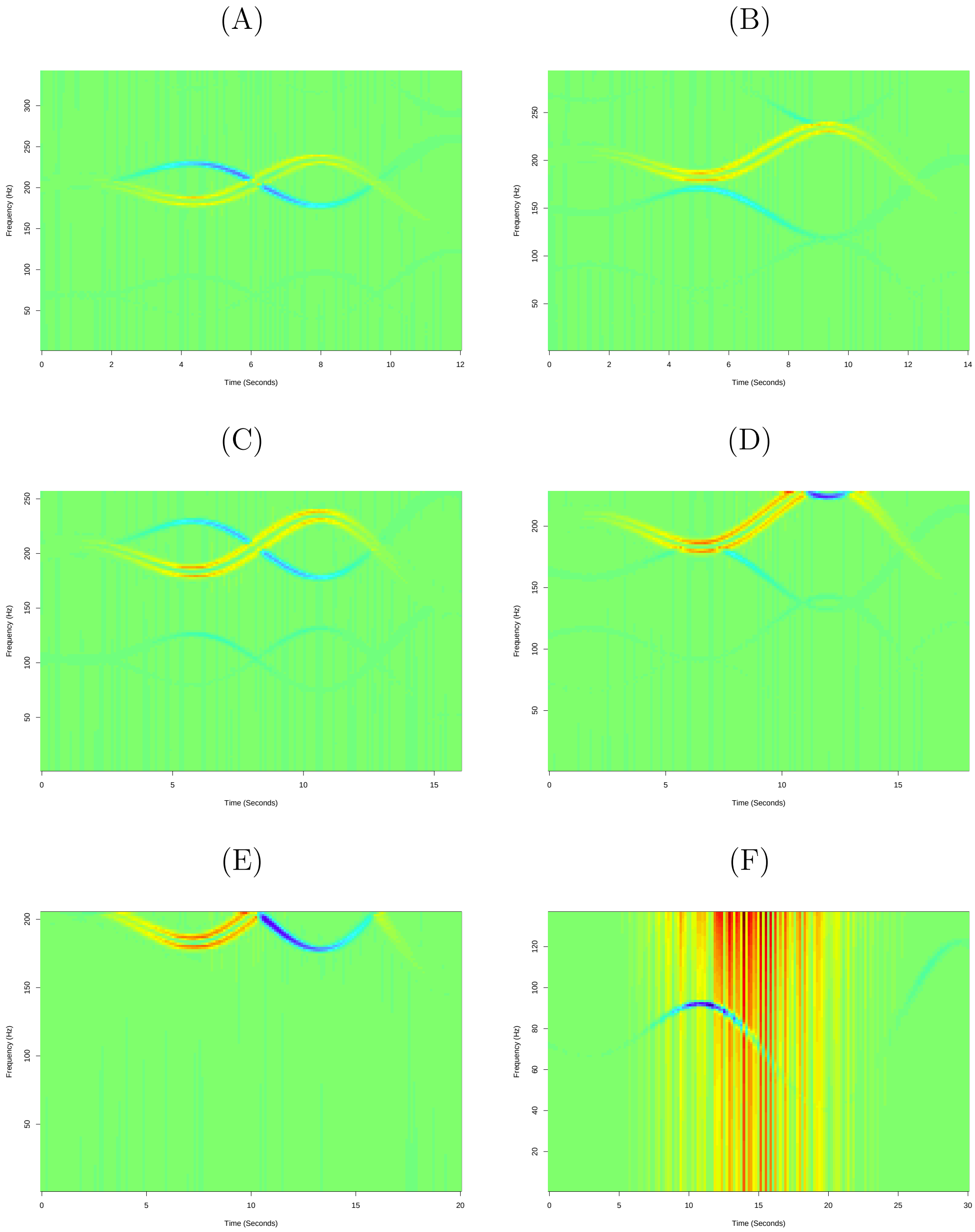

Figure 6: Difference of simulated spectrograms in compression analysis. We use downsampling to compare 6 long signals with a fixed short signal. The short signal has a fixed length of 8192 samples and the long signals are: 1) 20%, 2) 40%, 3) 60%, 4) 80%, 5) 100%, 6) 200% larger than that. Plot (A) shows the difference between resulting spectrograms of the fixed, short signal and the downsampled first long signal, (short-long). Plots (B)-(F) show similar differences for the 5 remaining comparisons.

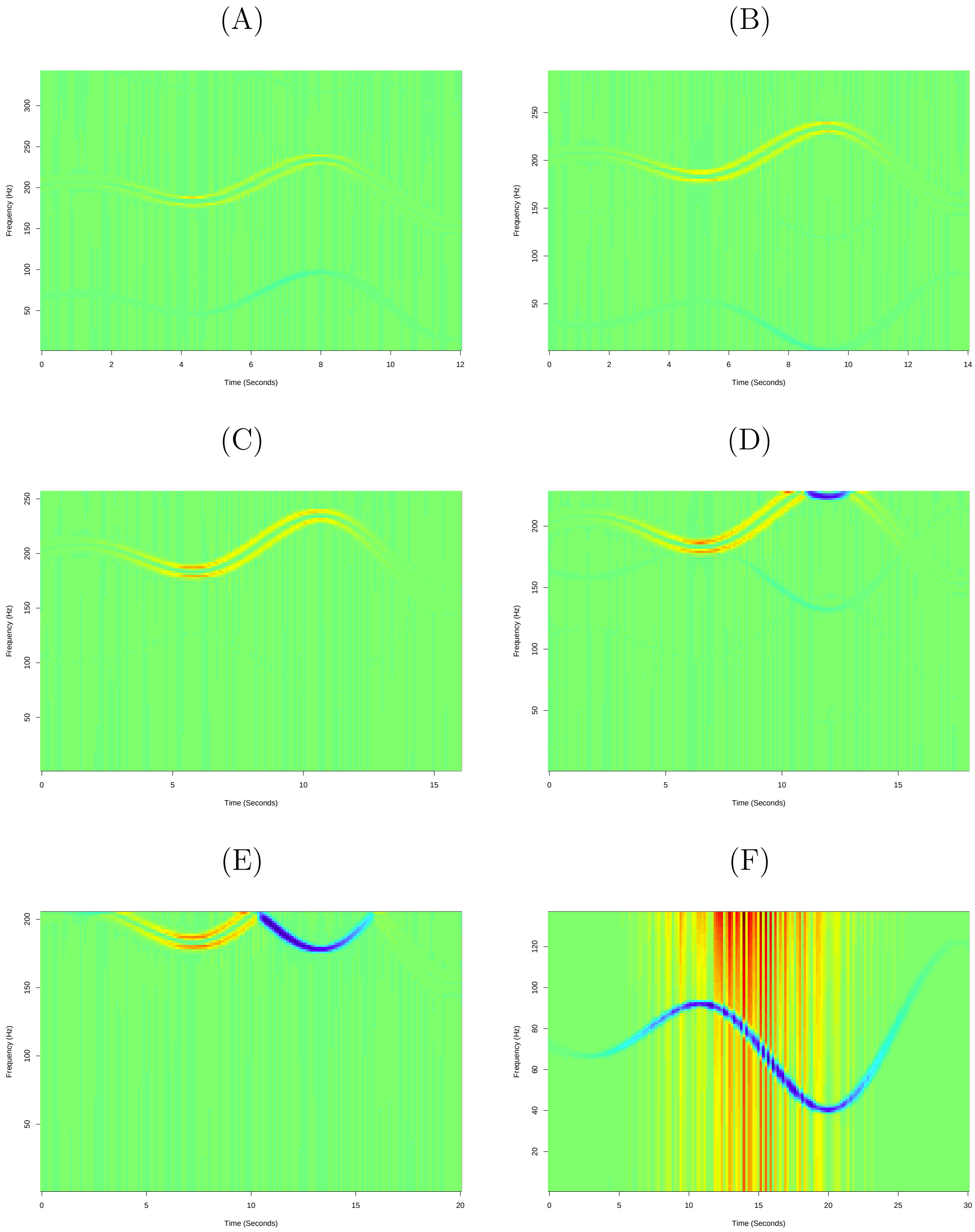

Figure 7: Difference of simulated spectrograms in compression analysis. We use cubic spline interpolation to compare 6 long signals with a fixed short signal. The short signal has a fixed length of 8192 samples and the long signals are: 1) 20%, 2) 40%, 3) 60%, 4) 80%, 5) 100%, 6) 200% larger than that. Plot (A) shows the difference between resulting spectrograms of the fixed, short signal and the interpolated first long signal, (short-long). Plots (B)-(F) show similar differences for the 5 remaining comparisons.

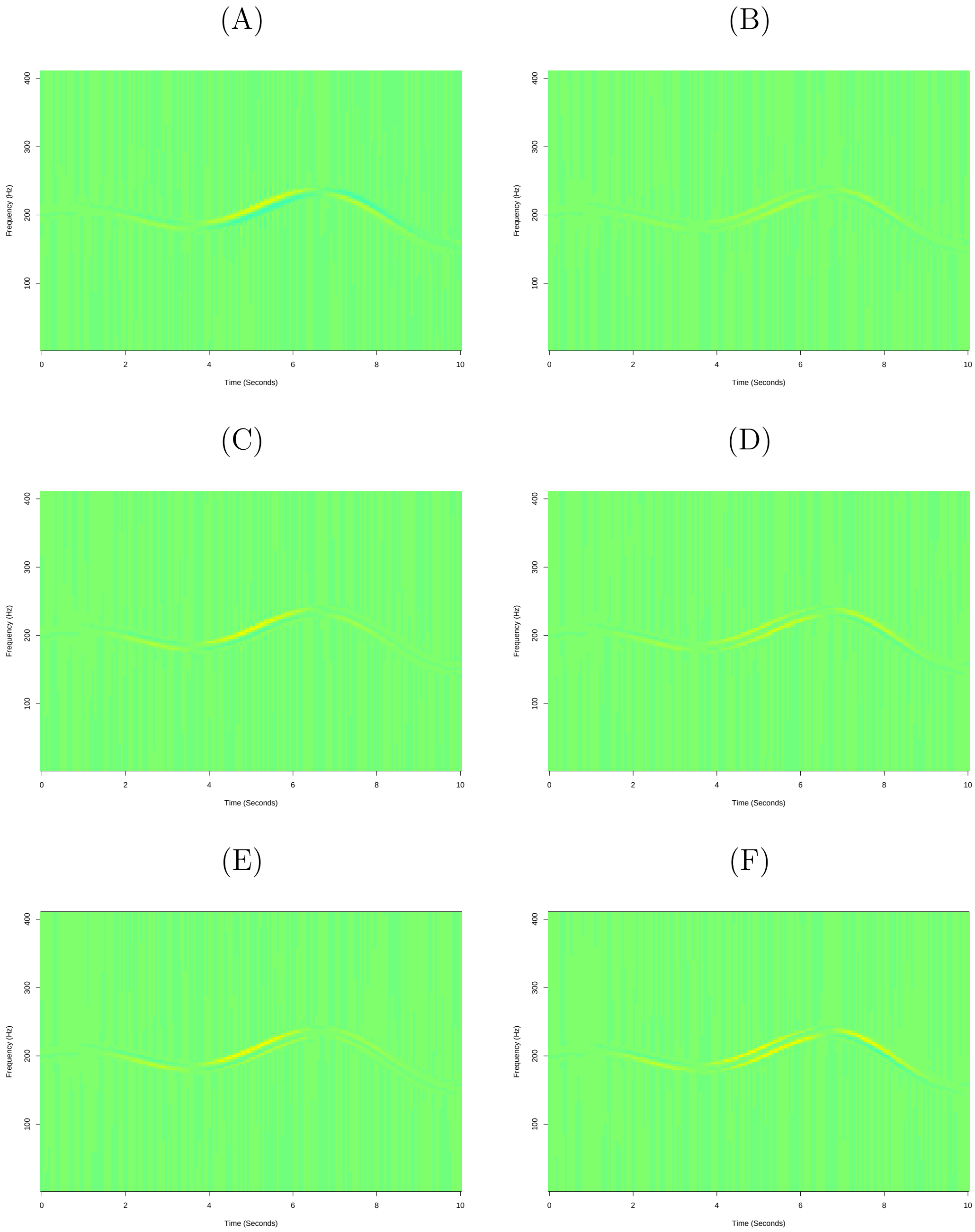

Figure 8: Difference of simulated spectrograms in compression analysis. We use our variable overlap method to compare 6 long signals with a fixed short signal. The short signal has a fixed length of 8192 samples and the long signals are: 1) 20%, 2) 40%, 3) 60%, 4) 80%, 5) 100%, 6) 200% larger than that. Plot (A) shows the difference between resulting spectrograms of the fixed, short signal and the first long signal, (short-long). Plots (B)-(F) show similar differences for the 5 remaining comparisons.

Percent Increase	Up Sampling		Cubic Spline Interpolation		Variable Overlap
	Down(10^{-4})	Up(10^{-4})	Down(10^{-4})	Up(10^{-4})	(10^{-4})
1.2	3.87	4.37	1.73	2.59	0.659
1.4	5.83	6.82	5.39	6.39	8.28
1.6	3.00	5.61	6.59	9.24	6.87
1.8	5.29	8.73	18.23	19.53	3.59
2.0	24.31	24.73	28.39	28.41	19.63
3.0	74.75	68.16	83.31	75.90	2.82
1.2	5.87	6.63	2.63	3.93	
1.4	0.70	0.82	0.65	0.77	
1.6	0.44	0.82	0.96	1.34	
1.8	1.47	2.43	5.08	5.44	
2.0	1.24	1.26	1.45	1.45	
3.0	26.51	24.17	29.54	26.91	

Table 1: **Elongation analysis**: A short signal is processed using upsampling, cubic spline interpolation and variable overlap to compare to a set of signals that are 1.2, 1.4, 1.6, 1.8, 2 and 3 times larger than the short signal. Interpolated up and interpolated down frequency discrepancies are presented under the appropriate heading. **Upper table**: Mean Square Error of each method. **Lower**: Error fold change (Competing Method/Variable Overlap).

Percent Increase	Down Sampling		Cubic Spline Interpolation		Variable Overlap
	Down(10^{-4})	Up(10^{-4})	Down(10^{-4})	Up(10^{-4})	(10^{-4})
1.2	13.70	14.10	1.73	2.28	1.93
1.4	10.41	11.08	4.15	5.44	0.557
1.6	17.55	18.96	8.28	9.61	1.48
1.8	20.90	22.42	17.87	19.90	1.28
2.0	32.19	31.69	47.19	46.58	1.67
3.0	327.0	329.0	353.0	355.0	2.73
1.2	7.10	7.31	0.90	1.18	
1.4	18.69	19.89	7.45	9.77	
1.6	11.86	12.81	5.59	6.49	
1.8	16.33	17.52	13.96	15.55	
2.0	19.28	18.98	28.26	27.89	
3.0	119.78	120.51	129.30	130.04	

Table 2: **Compression analysis**: A set of long signals are processed using downsampling, cubic spline interpolation and variable overlap to compare to a fixed short signal. Comparison is carried out using the resulting spectrograms of the processed signals. The longer signals are 1.2, 1.4, 1.6, 1.8, 2 and 3 times larger than the fixed short signal. Interpolated up and interpolated down frequency discrepancies are presented under the appropriate heading. **Upper table**: Mean Square Error of each method. **Lower**: Error fold change (Competing Method/Variable Overlap).

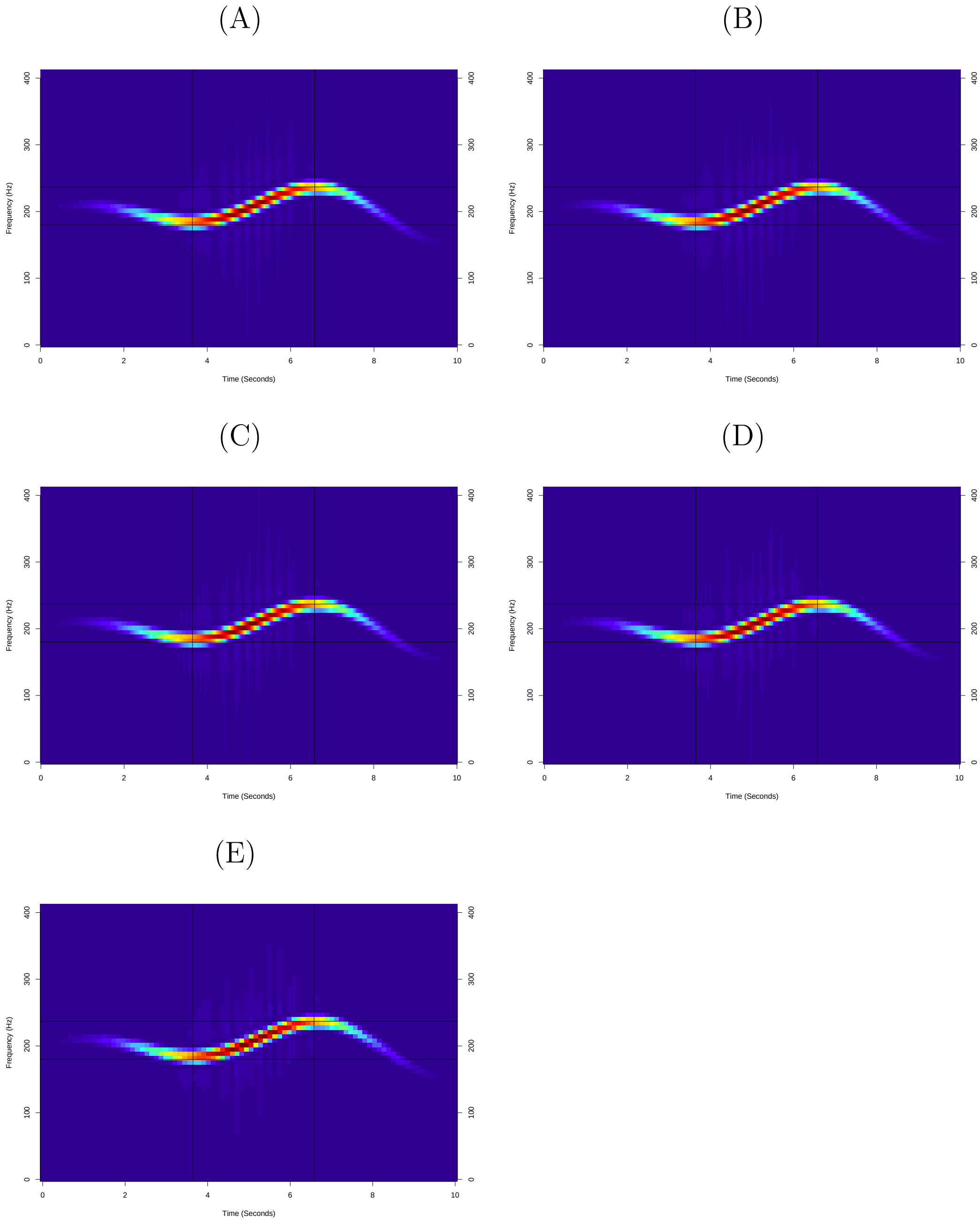

Figure 9: Each plot corresponds to a different size in spacing between successive windows in the generation of spectrograms, M. Window size used in this illustration is 128, with spacing between successive windows, M, equalling: A) 6, B) 12, C) 23, D) 46, and E) 92. The spacing size corresponds to 4.5%, 9%, 18%, 36%, and 72% the size of the window.

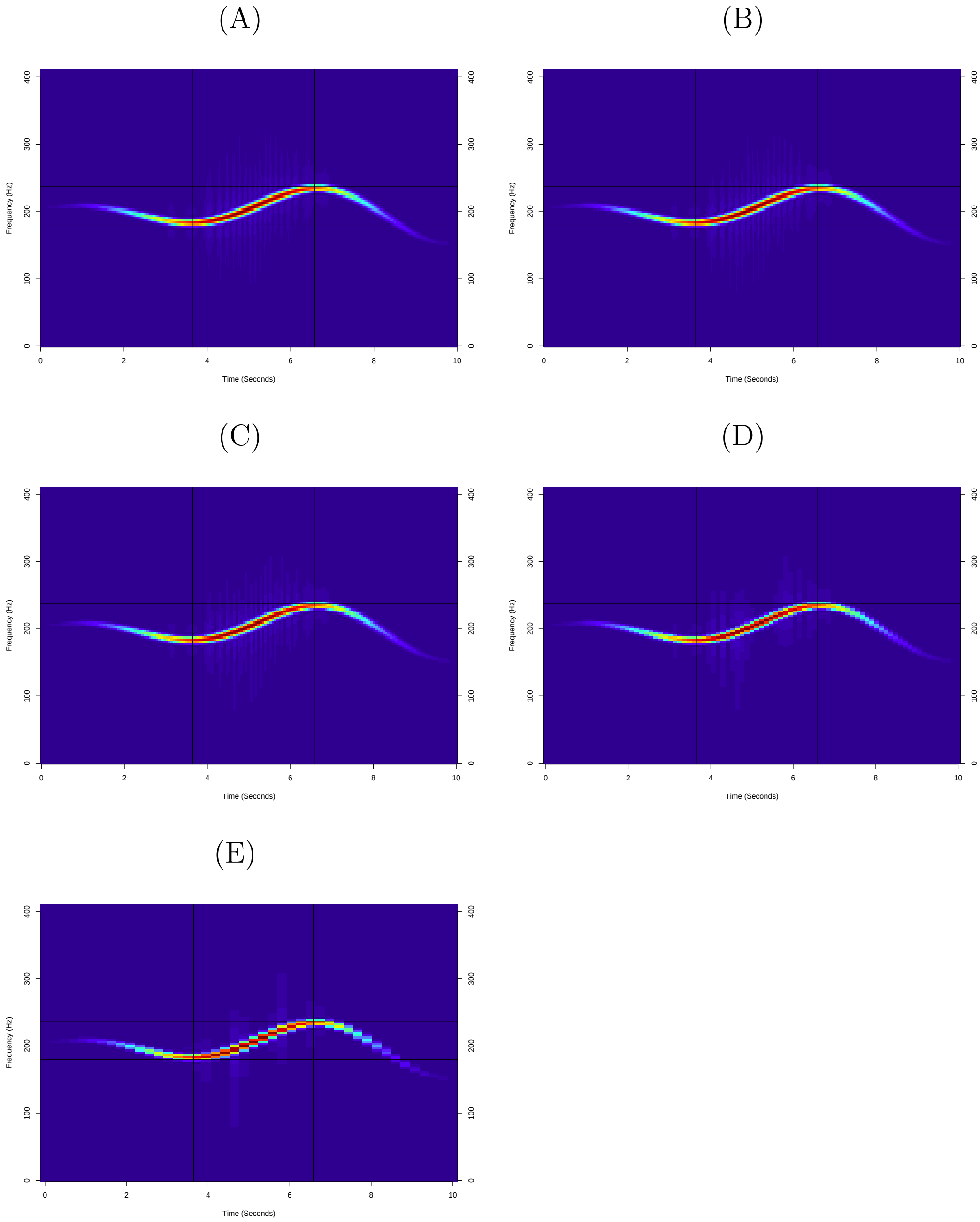

Figure 10: Each plot corresponds to a different size in spacing between successive windows in the generation of spectrograms, M. Window size used in this illustration is 256, with spacing between successive windows, M, equalling: A) 12, B) 23, C) 46, D) 92, and E) 184. The spacing size corresponds to 4.5%, 9%, 18%, 36%, and 72% the size of the window.

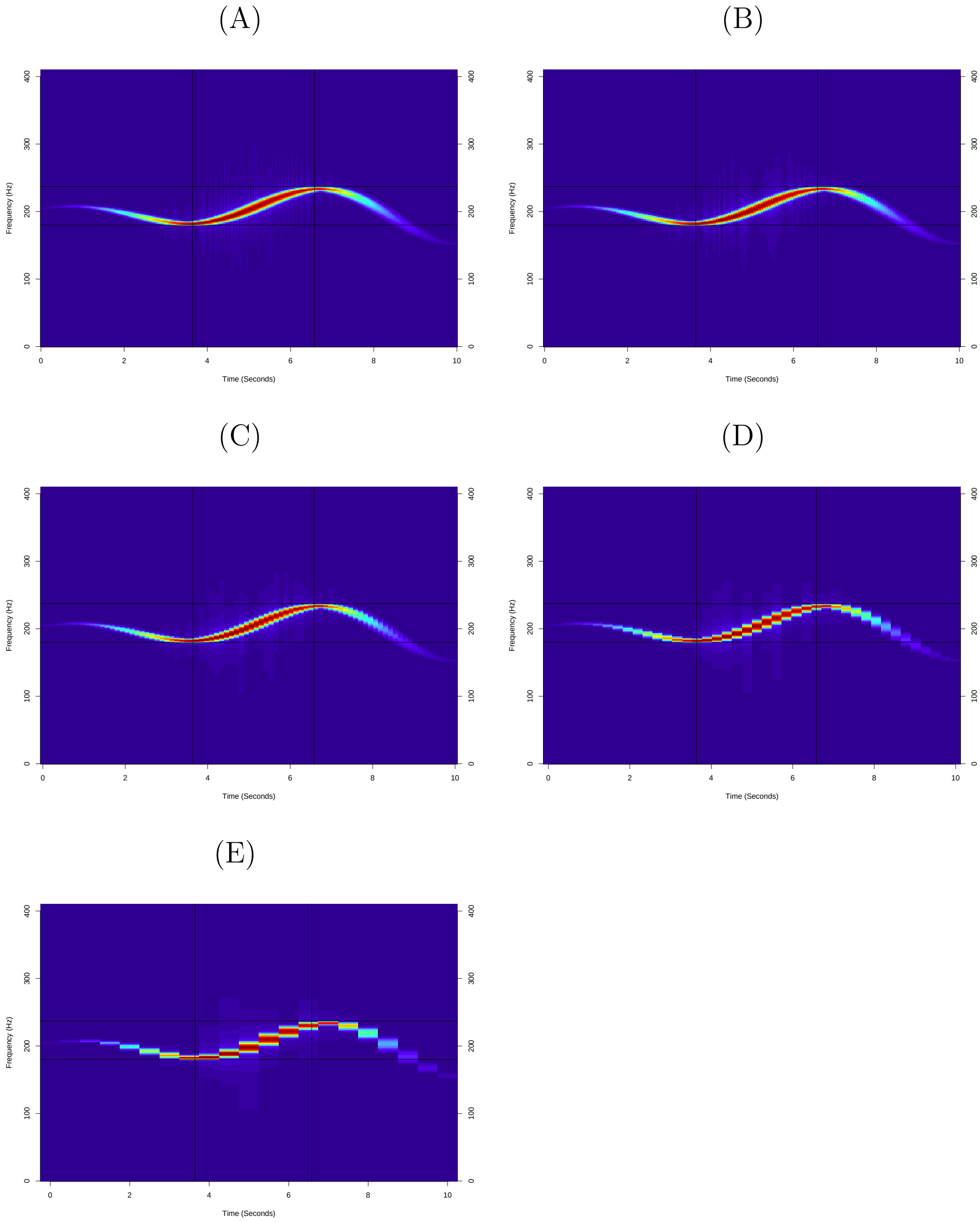

Figure 11: Each plot corresponds to a different size in spacing between successive windows in the generation of spectrograms, M. Window size used in this illustration is 512, with spacing between successive windows, M, equalling: A) 23, B) 46, C) 92, D) 184, and E) 368. The spacing size corresponds to 4.5%, 9%, 18%, 36%, and 72% the size of the window.

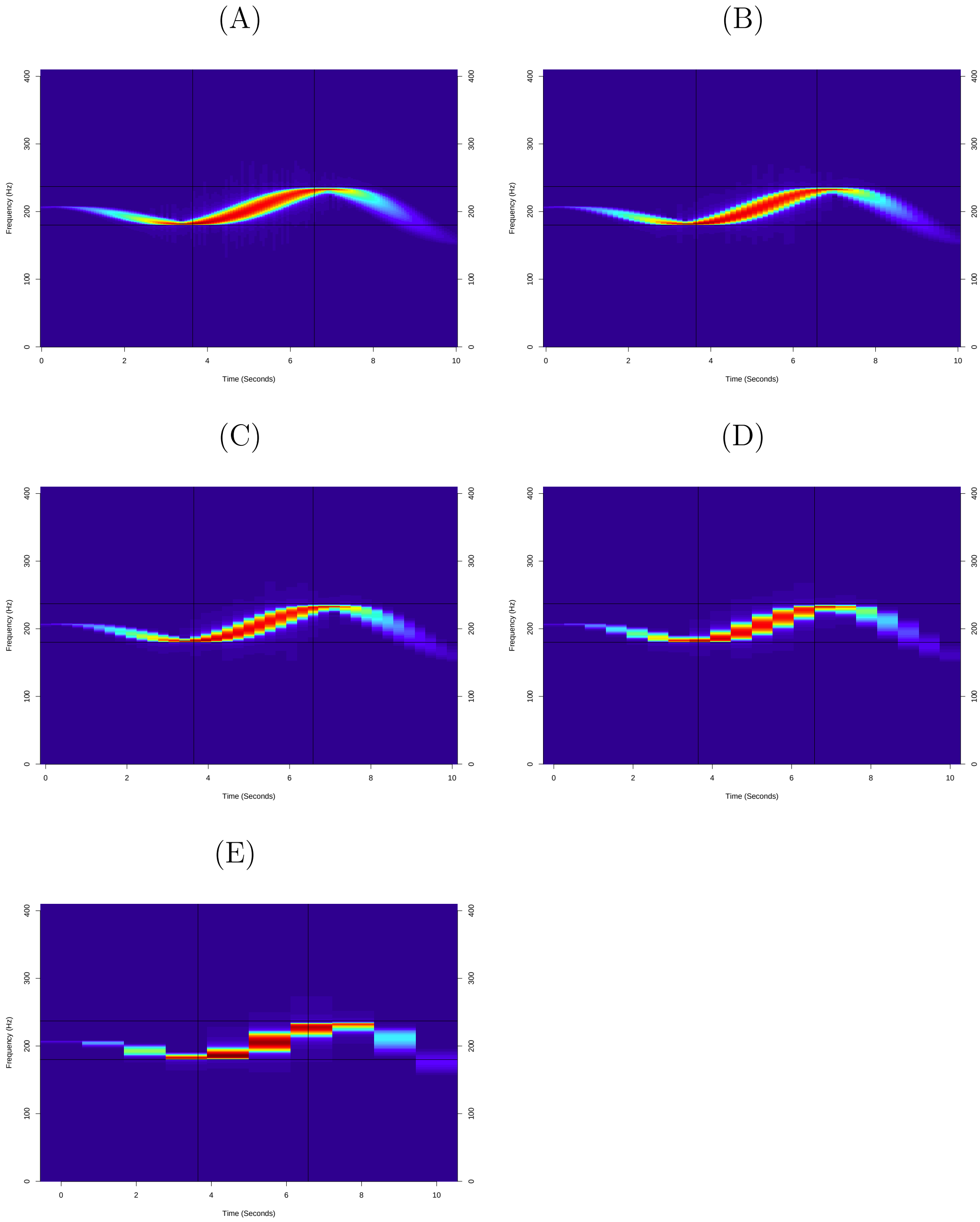

Figure 12: Each plot corresponds to a different size in spacing between successive windows in the generation of spectrograms, M. Window size used in this illustration is 1024, with spacing between successive windows, M, equalling: A) 46, B) 92, C) 184, D) 368, and E) 736. The spacing size corresponds to 4.5%, 9%, 18%, 36%, and 72% the size of the window.

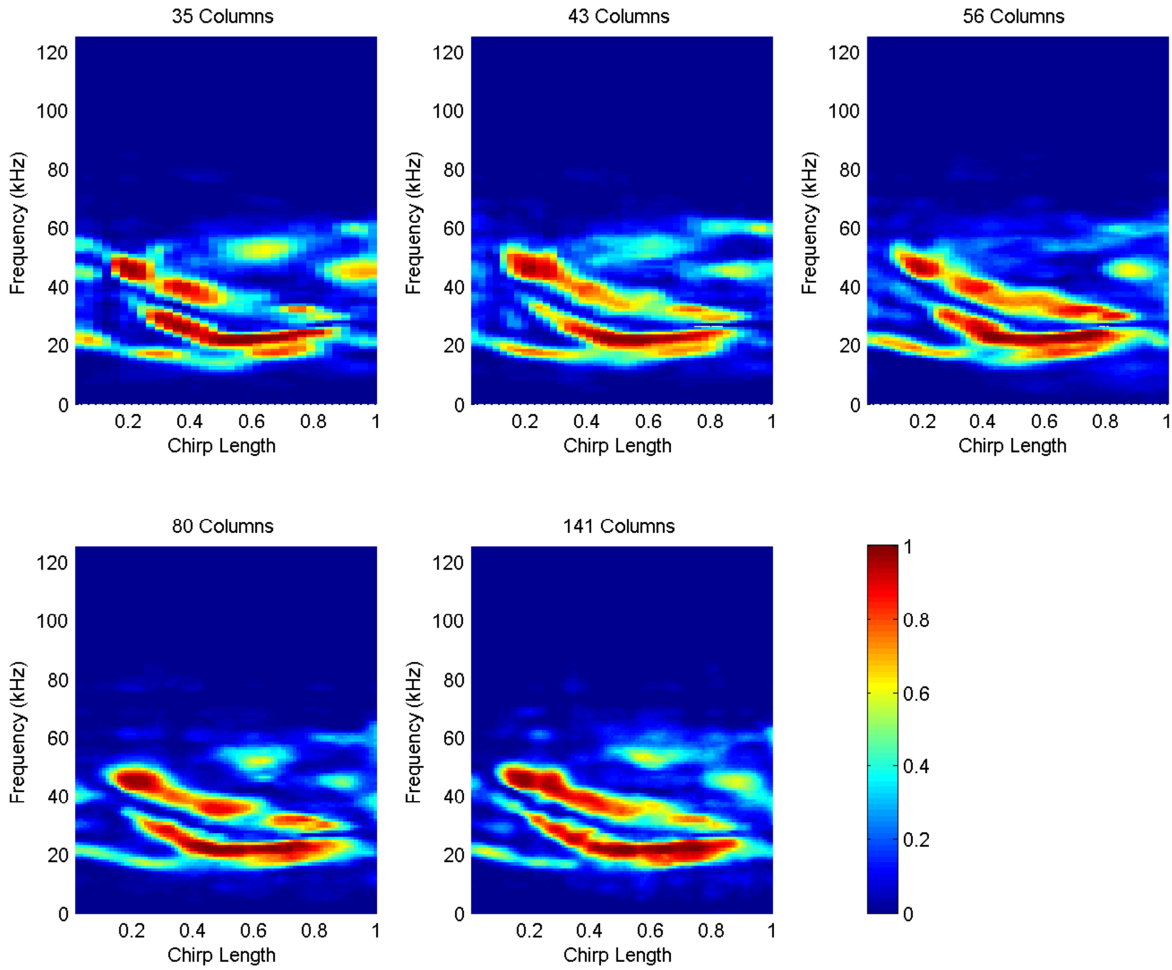

Figure 13: Posterior probability plot indicating the regions of most pronounced differences between Austin and College Station bats. The probability that there is a 1.5 fold difference between Austin and College Station bats is shown. High probability indicates a very likely chance that the is a difference observed between Austin and College Station bats, and where that difference is located in terms of frequency (kHz) and location, along the chirp. We fit our model using 5 different time resolution bands of 35, 43, 56, 80 and 141 values along the length of the chirp.

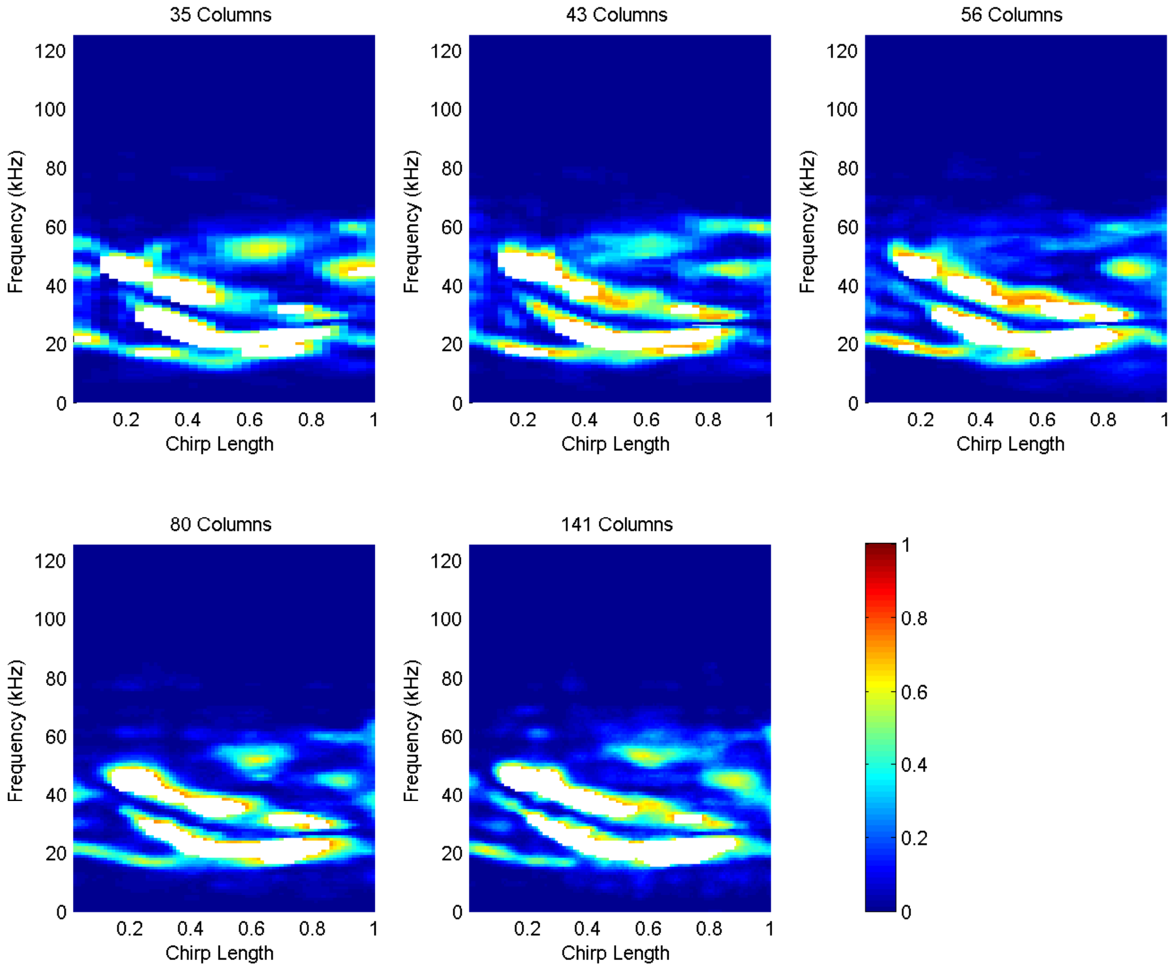

Figure 14: Spectrogram regions where highly significant differences between Austin and College Station bats exist. The cut-outs shown above are the regions with highest probability of a 1.5 fold difference, between Austin and College Station bats, that also have a controlled Bayesian false discovery rate. We control the false discovery rate to 15%, which means that, on average, the ratio of the size of falsely discovered regions to the size of the discovered regions will be no more than 0.15. In other words, we will control the rate at which we will falsely determine weather a region in the spectrogram is important. We fit our model using 5 different time resolution bands of 35, 43, 56, 80 and 141 values along the length of the chirp.

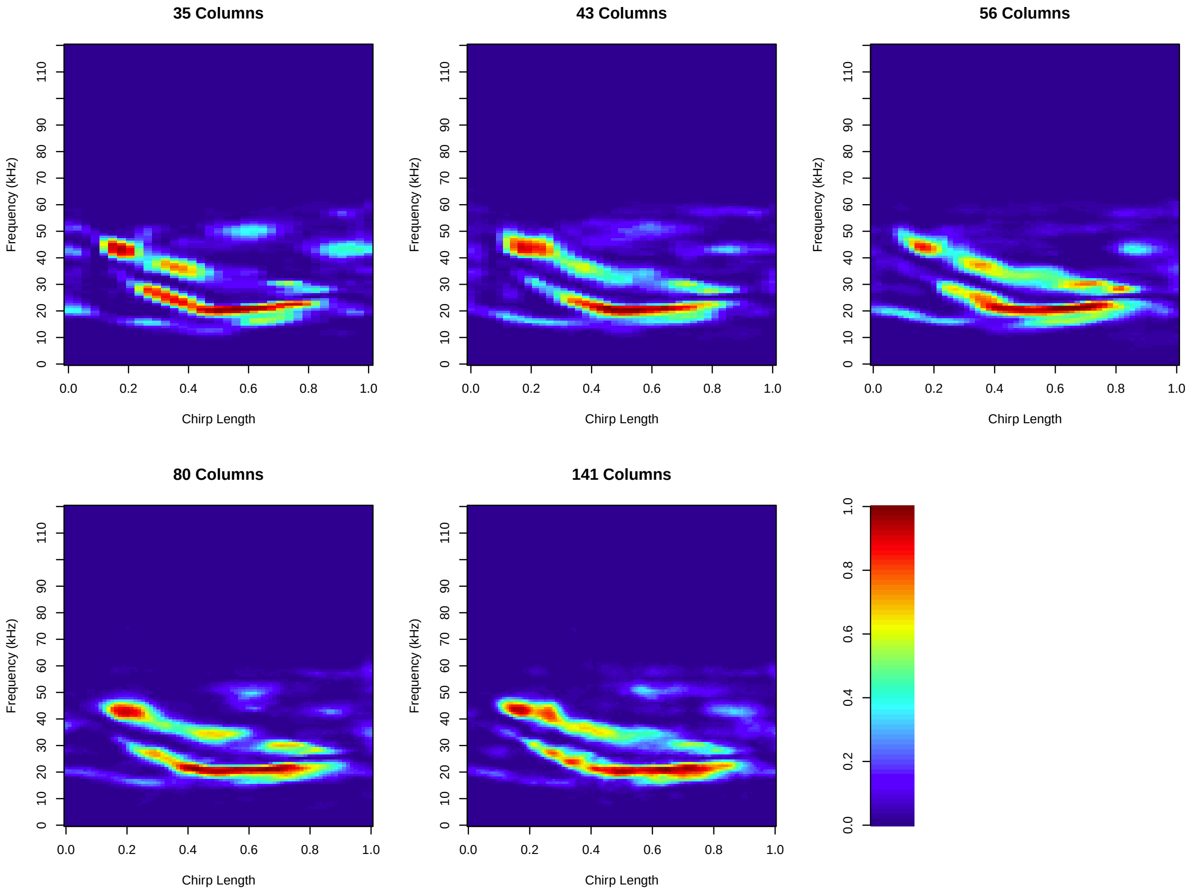

Figure 15: Posterior probability plot indicating the regions of most pronounced differences between Austin and College Station bats. The probability that there is a 1.75 fold difference between Austin and College Station bats is shown. High probability indicates a very likely chance that there is a difference observed between Austin and College Station bats, and where that difference is located in terms of frequency (kHz) and location, along the chirp. We fit our model using 5 different time resolution bands of 35, 43, 56, 80 and 141 values along the length of the chirp.

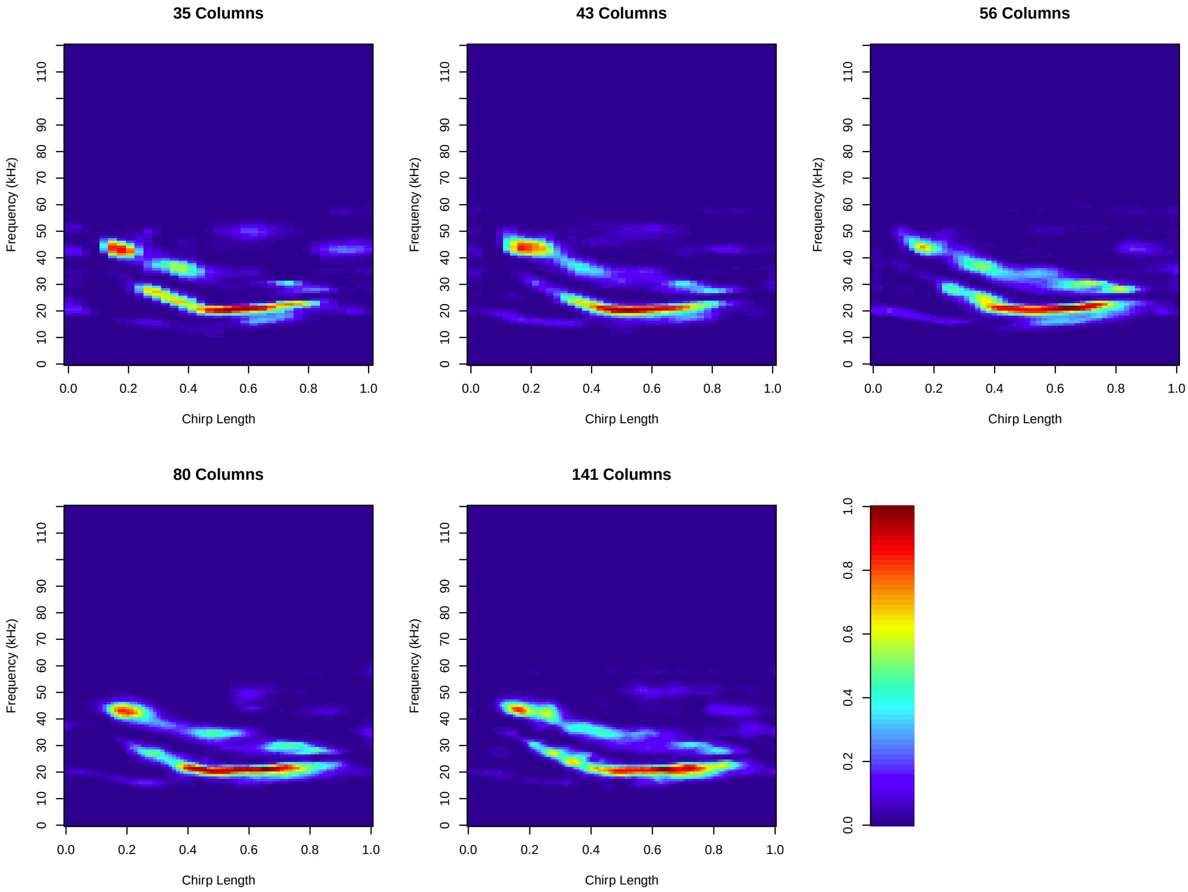

Figure 16: Posterior probability plot indicating the regions of most pronounced differences between Austin and College Station bats. The probability that there is a 2 fold difference between Austin and College Station bats is shown. High probability indicates a very likely chance that there is a difference observed between Austin and College Station bats, and where that difference is located in terms of frequency (kHz) and location, along the chirp. We fit our model using 5 different time resolution bands of 35, 43, 56, 80 and 141 values along the length of the chirp.

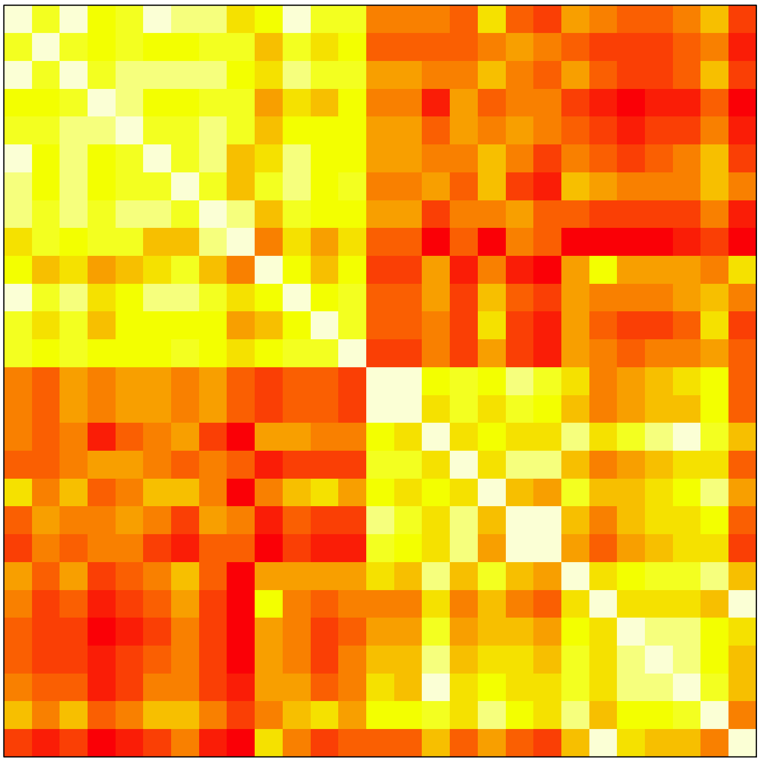

Figure 17: Correlation among random effects in wavelet space. First 13 rows are occupied by College Station bats and last 14 rows are occupied by Austin bats.

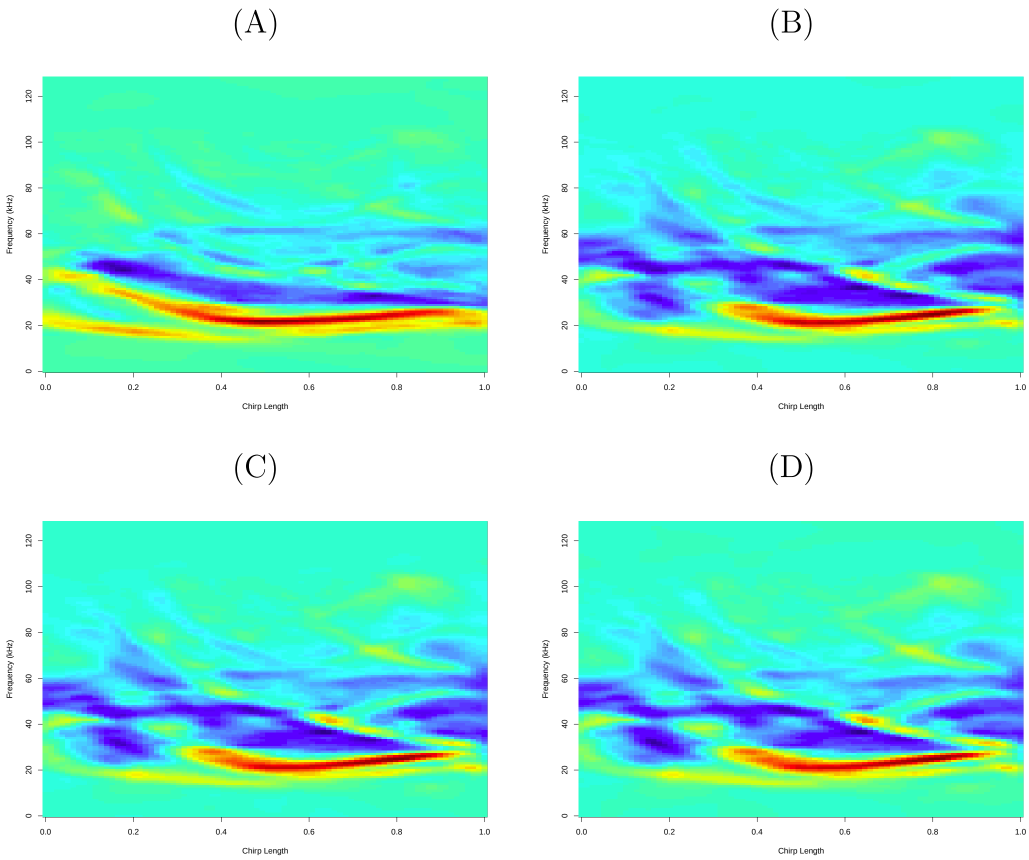

Figure 18: The length of the chirp is added to the WFMM as a predictor and we present results as contrast plots, taking the difference between mean Austin spectrogram minus the mean College Station spectrogram. Plot (A) shows the results at chirp length of 0, plot (B) shows the results at chirp length that corresponds to the first quartile of the data (3440), plot (C) shows the results at chirp length that corresponds to the median of the data (3905), and plot (D) shows the results at chirp length that corresponds to the third quartile of the data (4336).

References

Ziemer, R. E., Tranter, W. H. and Fannin, D. R. (1983). *Signals and Systems*, New York: Macmillan.