Appendix I: Estimation Error of Sensitivity Indices in Models with Dependent Parameters

For a model $y = g(x_1,..,x_n)$ with dependent parameters, the model output can be decomposed for a subset of independent model parameters (x_{sub}) as follows,

$$y = u(x_{sub}) + \xi_{(-x_{sub})}$$

where

$$u(x_{sub}) = E(g(x_1,...,x_n) \mid x_{sub})$$

and $\xi_{(-x_{sub})}$ is a random error arising due to uncertainties in all parameters except x_{sub} with

$$E(\xi_{(-x_{sub})}) = 0$$
$$V(\xi_{(-x_{sub})}) = \mathrm{V(y)} - \mathrm{V}(E(g(x_1,...,x_n) \mid x_{sub})).$$

$V(\xi_{(-x_{sub})})$ is equal to the partial variance contributed by the subset of dependent parameters. In this way, we are able to quantify partial variances in the framework for stochastic models in eq. (H1) assuming $g(x_1,..,x_n) = u(x_{sub})$ and $\xi = \xi_{(-x_{sub})}$. The additional estimation bias introduced by not including the dependent parameters is $\frac{V(\xi_{(-x_{sub})})}{N}$ (see eq.(H3) in Appendix H in supplementary materials).

One key point in the above analysis framework is to draw samples for correlated parameters. There were many sampling approaches available (e.g., the Markov Chain Monte Carlo and rejection sampling) to draw random samples if we know the joint probability density function for all parameters (Liu 2001). However, it could be difficult and infeasible to obtain joint distribution estimates for models with many parameters (e.g., 100 parameters). Xu and Gertner (2007, 2008a) used a simple

random reordering approach to generate samples for parameters with a specific rank correlation structure. The main idea is that, after generating grid samples for $(\theta_1,..,\theta_n)$ and the corresponding samples for parameters $(x_1,..,x_n)$, sampled parameter values are reordered to honor a specified correlation structure using the Iman and Conover's method (Iman and Conover 1982). Samples of $(\theta_1,..,\theta_n)$ are reordered correspondingly based on the order of parameter samples. Based on the reordered samples of $(\theta_1,..,\theta_n)$, partial variances contributed by main effects as well as interaction effects within the subset of independent model parameters (x_{sub}) can be estimated with eq. (30). If all parameters are dependent, then x_{sub} contains only one parameter and the multiple Fourier transformation for $u(x_{sub})$ is reduced to a one-dimensional Fourier transformation with eq. (12).

The current framework for models with correlated parameters does not allow for the calculation of partial variances contributed by interaction effects among correlated parameters. Since correlations are common in different kinds of models, it would be desirable to explore interaction effects for correlated parameters using the FAST framework. Future development to incorporate interaction effects for correlated parameters will be of interest.

There are also other approaches available for conducting sensitivity analysis of models with dependent parameters (Iman et al. 1985, Bedford 1998, Iman et al. 2002, Fang et al. 2004, Sulieman et al. 2004, Xu and Gertner 2008b). For a specific comparison of different approaches for a test model, please refer to Sulieman et al. (2009) .

References

Bedford, T. 1998. Sensitivity indices for (Tree) - dependent variables. Pages 17-20 *in* Proceedings of the Second International Symposium on Sensitivity Analysis of Model Output, Venice, Italy.

Fang, S., G. Z. Gertner, and A. Anderson. 2004. Estimation of sensitivity coefficients of nonlinear model input parameters which have a multinormal distribution. Computer Physics Communications 157:9-16.

Iman, R. L., and W. J. Conover. 1982. A distribution-free approach to inducing rank correlation among input variables. Communications in Statistics-Simulation and Computation 11:311-334.

Iman, R. L., M. E. Johnson, and T. A. Schroeder. 2002. Assessing hurricane effects. Part 1. Sensitivity analysis. Reliab. Eng. Syst. Safe. 78:131-145.

Iman, R. L., M. J. Shortencarier, and J. D. Jonson. 1985. A FORTRAN 77 and user's guide for calculation of partial correlation function and standardized coefficient. Sandia National Laboratories, Albuquerque, NM.

Liu, J. S. 2001. Monte Carlo strategies in scientific computing. Springer, New York.

Sulieman, H., I. Kucuk, and P. J. McLellan. 2009. Parametric sensitivity: A case study comparison. Comput. Stat. Data. An. 53:2640-2652.

Sulieman, H., P. J. McLellan, and D. W. Bacon. 2004. A profile-based approach to parametric sensitivity in multiresponse regression models. Comput. Stat. Data. An. 45:721-740.

Xu, C., and G. Z. Gertner. 2007. Extending a global sensitivity analysis technique to models with correlated parameters. Comput. Stat. Data An. 51:5579-5590.

Xu, C., and G. Z. Gertner. 2008a. A general first-order global sensitivity analysis method. Reliab. Eng. Syst. Safe. 93:1060-1071.

Xu, C., and G. Z. Gertner. 2008b. Uncertainty and sensitivity analysis for models with correlated parameters. Reliab. Eng. Syst. Safe. 93:1563-1573.